Real Assets, Financial Assets, Liquidity, and *Lemon* Problem

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**Abstract**

Return obtained by diversification is based on average quality. Similarly, under asymmetric information, the price at which an asset can be sold reflects the average quality of assets. Therefore, under some conditions, sale of an asset under asymmetric information is a useful alternative to diversification. This idea is developed with a model that incorporates a liquidity shock. One key result is that investment in real assets is higher under asymmetric information than under symmetric information. The model can explain why the ratio of real assets to financial assets is higher in emerging economies than in developed countries.

Key words: Diversification, asymmetric information, real asset, financial asset, liquidity, and cost of delegation.

JEL Classifications: D82, G00, O16.

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1 Introduction

It is well known that the level of financial development in emerging economies is low compared to that in developed countries. Typically, the ratio of real assets (RAs) to financial assets (FAs) is higher in emerging economies than in developed countries. One way to view RAs and FAs is to distinguish between owner-managed projects and delegated projects\(^2\). Delegation may be through financial intermediaries or through financial markets. In this paper, we will view an owner-managed project as an RA\(^3\), and a delegated project as an FA. One way in which financial development manifests itself is that there is a shift from RAs to FAs. In this context, it is important to understand the nature of RAs. Are RAs inferior to FAs? If RAs have their drawbacks, then why are they very important in emerging economies? These are the issues that have motivated this paper.

If technology is more advanced in the case of delegated projects than in the case of owner-managed projects, then the technology gap between the developed countries and emerging economies can explain the greater role of owner-managed projects in emerging economies than in developed countries (Iyigun and Owen, 1999). In an alternative explanation, Locay (1990) assumes that altruistic households have less need to monitor their members than firms, giving households a comparative advantage over professionally managed firms (at low levels of output).

The focus of this paper is diversification. The role of diversification is well known (Samuelson, 1967). In our model, an agent can manage only one project. But all projects are risky, and agents are risk-averse. On the other hand, it is possible for an agent to diversify by investing in several projects through a

\(^2\)One measure of financial development is domestic assets of deposit money banks plus stock market capitalization together as a proportion of gross domestic product. This measure is 0.62 for an emerging economy like India and, 1.53 for a developed country like USA. Data for other emerging economies and developed countries tells a similar story (Demirguc-Kunt and Levine, 2001). In India, ‘... the contribution of the unorganized sector to the total net value added (NVA) stood at about 64% in the recent years.’ (Jacob, 1997). Much of the investment in the unorganized sector is in owner-managed firms.

\(^3\)An alternative definition of RA can be one which includes not only owner-managed projects but also real estate, gold, etc. These are absent in our model.
financial intermediary or through a financial market. Thus, an agent can get the benefit of diversification by investing in FAs. But there is a cost of investing in FAs. This is the cost of delegation, or cost of separation between ownership and management. This cost is zero in the case of RAs. This cost of delegation can explain the investment in RAs.

This paper will go beyond the cost of delegation. One way to get the benefit of diversification is to actually diversify across several projects. When an agent invests in several projects, she receives a price (per unit of investment) that reflects the average quality of projects. This paper shows that, under some conditions, there is an alternative to actual diversification. If RAs are sold under asymmetric information (AI), then the alternative to actual diversification is to invest in RA and sell the same under AI. The price of RA reflects the average quality of projects put up for sale. Observe that in both methods of getting the benefit of diversification, an investor receives a return that reflects average quality. Hence, under AI, it is sometimes possible to get the benefit of diversification without investing in several projects.

A brief of outline of the model in this paper is as follows. To begin with, agents are identical. Thereafter, there are two types of agents - type 1 agents who face a liquidity shock, and type 2 agents who do not face such a shock. There are two assets - RA and FA. When some agents face a liquidity shock, they can sell their assets in the secondary market. After an agent invests and before the projects mature, there is a possibility of a liquidity shock (in which case there are forced sales), or a trading opportunity under AI (in which case there are strategic sales).

Key results of this paper are as follows. Investment in RAs is higher under AI than under symmetric information (SI). Further, investment in RAs increases with the cost of delegation, and with the probability of a liquidity shock, given that RAs are traded under AI. The model can be useful in explaining why the ratio of RAs to FAs is higher in emerging economies than in developed countries.

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4. We will show that this result holds even if strategic trades occur.

5. This follows Diamond and Dybvig (1983) but the focus of this paper is entirely different.

6. This analysis is somewhat similar to that in Merton (1971), which has similarly shown that portfolio behavior for inter-temporal maximization will be significantly different when an agent faces a changing investment opportunity set instead of a constant one.
The plan of this paper is as follows. In section 2, we set out the model. In section 3, we study the following three cases of portfolio choice:

Case (1) - SI on quality of RA,
Case (2) - AI on quality of RA, and SI on type of agent, and
Case (3) - AI on quality of RA, and AI on type of agent.

In section 4, we compare the three cases. Section 5 has some concluding remarks. The formal proofs for all the results are shown in the appendix.

2 The Model

Investment takes place at date 0, and projects mature at date 1. We will describe the interim period later. There is a continuum of agents in $[0, 1]$. All these agents are identical at date 0. Henceforth, we consider a representative agent. Each agent has an endowment at date 0 only. All agents need to consume in future only. The endowment is one unit of funds, which can be invested. There are two assets - RA and FA. Both RA and FA give returns at date 1 only. If any project is physically liquidated before date 1, it gives a return of zero. An RA is an owner-managed project. It can be of good quality or of bad quality. An RA yields a risky return, $R$, at date 1, where

$$R = \begin{cases} R, & \text{if the RA is good,} \\ R', & \text{if the RA is bad,} \end{cases}$$

where $0 < R' < R < \infty$. Assume that RA is good with probability $\beta$, and that the RA is bad with probability $(1 - \beta)$, where $0 < \beta < 1$. Since each project has the same probabilities of success and failure, and project returns are independent, we may interpret as follows. $\beta$ proportion of the projects will give a return $R$, and $(1 - \beta)$ proportion will give a return $R'$. Let

$$\triangle R \equiv R - R'.$$

Let $R^c$ and $R^v$ denote the mean and variance of $R$. Clearly,

$$R^c = \beta R + (1 - \beta)R' = R + \beta(\triangle R),$$

We will later clarify why there is only one case of SI on quality of RA.
where the second equality follows after using (2). Similarly,
\[
R'' = \beta(R - R^c)^2 + (1 - \beta)(R - R^c)^2 = \beta(1 - \beta)(\Delta R)^2, \tag{4}
\]
where the second equality follows after using (2) and (3).

Each project needs to be managed. An agent can manage only one project. Therefore, an agent can not invest in more than one RA. In addition, an agent can invest in an FA, which is issued by a representative competitive financial intermediary. The latter can invest in several projects. So an agent can delegate management of several projects, but there is a cost of delegation. Let \( m \) denote the cost of delegation per unit of investment in delegated projects. The intermediary invests in (independent) risky projects, some of which may succeed and others may fail. However, given its ability to diversify risk, the financial intermediary is able to guarantee a return to its clients equal to the mean return of all the projects that it funds, minus the cost of delegation. We assume that the technology used in the projects is the same, whether these are owner-managed projects or delegated projects. Hence, the net return on FA is \((R^e - m)\). We assume that \((R^e - m) > 0\).

Projects require active management in early stages. Over time, the need for active management becomes less. For simplicity, assume that projects need to be looked after from date 0 to some date \( X \), where \( 0 < X < 1 \). Thereafter, the project does not need to be looked after at all, though it will mature at date 1 only. It is assumed that the cost of effort involved in management is zero.

To begin with, agents are identical. Thereafter, there are two types of agents - type 1 agents who face a liquidity shock and need to consume before the projects mature, and type 2 agents who need to consume at date 1. The proportion of type 1 agents is \( t \), where \( 0 < t < 1 \). The proportion of type 2 agents is \((1 - t)\). Type 1 agents face a liquidity shock at date \( Z \). Formally,
\[
U(c_Z, c_1) = \begin{cases} 
  u(c_Z), & \text{if agent is type 1}, \\
  u(c_1), & \text{if agent is type 2},
\end{cases} \tag{5}
\]
where \( c_Z \) and \( c_1 \) are consumption at date \( Z \) and at date 1 respectively. Liquidity shock occurs just before projects mature i.e. date \( Z \) is very close to date 1. This enables us to assume that the discount rate for buyers is zero. This abstraction helps us to focus on the effect of AI on the portfolio choice.
Assume that $X < Z$. This assumption implies that when an agent is hit by a liquidity shock and sells her RA, then the buyer does not have to manage the project. She just needs to wait till the project matures at date 1.

Projects mature at date 1. There is no trade at date 1. Type 2 agents consume what their assets yield at date 1. Type 1 agents need to consume at date $Z$. They can sell their assets in the secondary market, where some agents buy at date $Z$. These buyers have ‘deep pockets’ at date $Z$ and are risk neutral agents. These agents are different from the agents who invest at date 0. We will refer to these risk neutral traders as agents $N$. The latter are all identical at date $Z$, unlike the risk-averse investors, who are type 1 agents or type 2 agents at date $Z$. The only role of agents $N$ in our model is that they buy assets in the secondary market at date $Z$. Henceforth, our focus will be on the risk-averse agents. In what follows, unless otherwise specified, an agent will mean a risk-averse agent.

All agents have access to storage technology at zero cost. It is not necessary to assume that storage is costless from date 0 to date 1. We need only assume that storage is costless from date $Z$ to date 1.

At date 0, agents know that $t$ proportion of agents will be type 1 agents. Each agent can face a liquidity shock with probability $t$ but there is no aggregate uncertainty i.e. $t$ is known at date 0. This paper will study the case of SI as well as the case of AI. At date $Z$, under SI on type of agent, each agent knows the type of all agents. Under AI on the type of agent, each agent knows her own type but not the type of others at date $Z$. Next, recall that some agents will have good RAs and others will have bad RAs. It follows that at date $Z$, there can be four types of agents viz. $1G$, $1B$, $2G$ and $2B$. Agent $ij$ is a type $i$ agent ($i = 1, 2$) who has an RA which has quality $j$, where $j = G, B$. $G$ denotes an agent with a good project, and $B$ denotes an agent with a bad project. At date $Z$, under SI on the quality of RA, both the buyers and sellers know the quality of RA. Under AI on the quality of RA, the seller knows the quality of RA but the buyer does not know the same. Given the nature of FAs in our model, these are traded under SI in all cases.

Four cases are possible. These are: SI on quality of RA and SI on type of

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*These agents $N$ are similar to the ‘speculators’ in Allen and Gale (1998).*
agent, SI on quality of RA but AI on type of agent, AI on quality of RA and SI on type of agent, and finally, AI on quality of RA and AI on type of agent.

Observe that in the first two cases, we have SI on quality of RA. Then it does not matter whether or not we have SI on type of agent. The reason is very simple. The price of RA depends on the quality of RA. Since this is known to the buyer, the type of agent is irrelevant. It follows that we can treat the first two cases as one case. Therefore, effectively we have the three cases that we had listed (see the end of section 1).

Since all agents are identical at date 0, they take the same decision on the amount to be invested in RA. The problem is analyzed in two stages. In stage 1, portfolio choice is made at date 0. In stage 2, trades and prices at date $Z$ are determined, for a given portfolio choice at date 0. We will first analyze stage 2, given the portfolio choice. Thereafter, we examine portfolio choice in stage 1.

At date $Z$, as mentioned already, agents $N$ are potential buyers of assets. Another group of buyers are the risk-averse agents themselves, who do not have any endowment of liquidity at date $Z$ but can obtain it by selling some of their assets. We will later see why they may do so under AI. We will assume that agents submit their sale and purchase orders and the net position is calculated, and settlements made. In other words, there is no margin requirement in trade. Notice that the net demand for liquidity at date $Z$ comes from type 1 agents. Any type 2 agent who sells her asset(s) is not a net demander of liquidity as she sells some assets and then uses the proceeds to buy other assets. It follows that there are two kinds of sales at date $Z$ - forced sales by type 1 agents, and possible strategic sales by type 2 agents.

Throughout we assume that markets are competitive and that there are zero transactions costs. Thus all buyers and sellers take prices as given and, the buying price of an asset is the same as its selling price. We have two markets - one for RA and the other for FA. An agent can sell or buy in any market.

Let $Y^r_k$ denote the return on RA that an investor receives in case $k$, where $k = 1, 2, 3$. Table 1 shows the probability density function (pdf) of $Y^r_k$. In Table 1, column 1 lists the four states of the world viz. $1G$, $1B$, $2G$ and $2B$. In our model, the type of an agent is independent of the quality of RA that is held. It

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9This does not contradict the assumption that $m > 0$. See section 5.
follows that the probabilities of the four states of the world viz. 1$_G$, 1$_B$, 2$_G$ and 2$_B$ are $t\beta$, $t(1-\beta)$, $(1-t)\beta$ and $(1-t)(1-\beta)$ respectively. Column 2 shows the probabilities. Columns 3, 4 and 5 show the values that $Y^r_1$, $Y^r_2$ and $Y^r_3$ can take respectively. We will explain the entries in these three columns later. Since, in our model, the probability of liquidity shock is the same as the fraction of type 1 agents, we will use the two expressions interchangeably.

Observe that $R$ and $Y^r_k$ are, what we may call, the technological return on RA, and the realized return on RA respectively. The former is the same across the three cases, whereas the latter can vary. $R$ is relevant at date 1 only, whereas $Y^r_k$ is relevant at date $Z$ as well. We will later see that, in general, $Y^r_k \neq R$, even when discount rate for buyers is zero. For comparison with $Y^r_k$, we show the values of $R$ in column 6 in Table 1.

Since trades take place at date $Z$ only, prices are applicable to date $Z$ only. Let $P^r_k$ denote the price of an RA in case $k$. Given (5), it follows that type 1 agents sell their assets at date $Z$. Hence, the return on unit investment in RA for type 1 agents is the same as the price of RA. Formally,

$$Y^r_k = P^r_k, \text{ if agent is type } 1,$$

where $k = 1, 2, 3$. See Table 1. We will later see that $P^r_k$ differs from one case to another, since the expected return on RAs put up for sale in the market at date $Z$ will depend on the information in each case. But the price of FA ($P^f_k$) is the same in each case. It is equal to the return on FA at date 1 i.e.

$$P^f_k = R^e - m, \quad k = 1, 2, 3.$$

Table 1: Probability Density Function (pdf) of $Y^r_1$, $Y^r_2$, $Y^r_3$ and $R$

<table>
<thead>
<tr>
<th>State of the world</th>
<th>Probability</th>
<th>$Y^r_1$</th>
<th>$Y^r_2$</th>
<th>$Y^r_3$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$_G$</td>
<td>$t\beta$</td>
<td>$P^r_1$ = $R$</td>
<td>$P^r_2$ = $R^e$</td>
<td>$P^r_3$ = $R'$</td>
<td>$R$</td>
</tr>
<tr>
<td>1$_B$</td>
<td>$t(1-\beta)$</td>
<td>$P^r_1$ = $R$</td>
<td>$P^r_2$ = $R^e$</td>
<td>$P^r_3$ = $R'$</td>
<td>$R$</td>
</tr>
<tr>
<td>2$_G$</td>
<td>$(1-t)\beta$</td>
<td>$R$</td>
<td>$R$</td>
<td>$R$</td>
<td>$R$</td>
</tr>
<tr>
<td>2$_B$</td>
<td>$(1-t)(1-\beta)$</td>
<td>$R$</td>
<td>$R$</td>
<td>$P^r_3 = R'$</td>
<td>$R$</td>
</tr>
</tbody>
</table>

This is because in each case, at date 1, there is a certain return of $R^e - m$ on
FA, and at date $Z$, FA is traded under SI, the buyers of assets have ‘adequate endowment’, and have zero discount rate.

Let $a_k$ denote the investment in RA in case $k$. It follows that $(1 - a_k)$ is invested in FA in case $k$. Let $Y_k$ denote the return on portfolio in case $k$. It follows from the above discussion that

$$Y_k = a_k Y_{k}^{r} + (1 - a_k)(R^e - m).$$

(8)

Let $\rho$ denote the absolute risk aversion. Let $W_k$ denote expected utility in case $k$. Further, let $E[Y_k]$ and $V[Y_k]$ denote the mean of $Y_k$, and the variance of $Y_k$ respectively. For explicit solutions and comparisons, we will use the following assumption:

A.1 $W_k = E[Y_k] - \frac{1}{2} \rho V[Y_k]$.

This utility function has some limitations but it can serve as a rough approximation to a general utility function in a broad variety of cases (see Samuelson (1970)). From (8) and A.1, we have

$$W_k = a_k E[Y_k^r] + (1 - a_k)(R^e - m) - \frac{1}{2} \rho a_k^2 V[Y_k^r].$$

(9)

after observing that the mean and variance of return on FA is $(R^e - m)$ and 0 respectively, and that the covariance between the return on RA and that on FA is 0. In what follows, we will use the superscript $*$ to indicate the equilibrium value. Short-selling is absent in our model i.e. $0 \leq a_k \leq 1$. Optimization with respect to $a_k$ yields

$$a_k^* = \max \left\{ 0, \min \left\{ \frac{E[Y_k^r] - R^e + m}{\rho V[Y_k^r]}, 1 \right\} \right\}.$$

(10)

To ensure interior solution, we will need the following parametric restriction.

A.2 $0 < m < \rho R^v$.

We have so far described the model. Next, we will study the three cases. In each case, we will study comparative statics, where our focus will be on the effect of a change in the cost of delegation, and that in the probability of liquidity shock.
3 Portfolio Choice - Real Assets and Financial Assets

In this section, we will study each case more or less in isolation. In each case, we will first study the outcomes at date $Z$ (see Lemma 1, Lemma 3 and Lemma 5 below), and thereafter, the outcomes at date 1 (see Lemma 2, Lemma 4 and Lemma 6 below). In the next section, we will compare the three cases.

3.1 Symmetric Information on Quality of Real Asset

In this case, both the owners and the buyers know the quality of RAs at date $Z$. Recall that the type of an agent at date $Z$ is irrelevant in this case. The next two lemmas are straightforward but they will be useful as a benchmark for a comparison with the cases of AI.

**Lemma 1** Assume that $0 < a_1 < 1$. In equilibrium, there are no strategic trades, $P_r^1 = R$, and $\frac{\partial P_r^1}{\partial t} = 0$.

Lemma 1 states that in case (1), strategic trades are absent. This is obvious, given SI. Further, the price of each asset at date $Z$ is equal to its return at date 1. This is also straightforward, given zero discount rate of buyers. Finally, the price of an RA is invariant to the probability of liquidity shock. In other words, the price of an RA is not affected by the number of RAs for sale. This is due to ‘adequate endowment’ with the buyers, and competition between them.

It follows from Lemma 1, (1) and (6) that $Y_r^1 = \overline{R}$ for type 1G agents, and $Y_r^1 = \overline{R}$ for type 1B agents. Given the technology, and the absence of strategic trades, it follows that $Y_r^1 = \overline{R}$ for type 2G agents, and $Y_r^1 = \overline{R}$ for type 2B agents. For the pdf of $Y_r^1$, see columns 1, 2 and 3 in Table 1. It follows from Table 1 that

$$Y_r^1 = \begin{cases} \overline{R}, & \text{with probability } t\beta + (1 - t)\beta = \beta, \\ \overline{R}, & \text{with probability } t(1 - \beta) + (1 - t)(1 - \beta) = 1 - \beta. \end{cases} \quad (11)$$

Observe that $t$ is finally irrelevant in the pdf of $Y_r^1$. We can now examine the optimal portfolio and the expected utility at date 0.
Lemma 2 Let A.1 and A.2 hold. Then, $0 < a_1^* < 1$, $\frac{\partial a_1^*}{\partial m} > 0$, $\frac{\partial a_1^*}{\partial t} = 0$, $\frac{\partial W_1^*}{\partial m} < 0$, and $\frac{\partial W_1^*}{\partial t} = 0$.

Lemma 2 states that, under SI, we have an interior solution i.e. some funds are invested in RA and others are invested in FA. This follows from the assumption that the cost of delegation is positive but not ‘large’ (see A.2). Further, under SI, investment in RA increases with the cost of delegation i.e. investment in FA (delegated projects) decreases with the cost of delegation, and expected utility decreases with the cost of delegation. This is intuitively straightforward but has important policy implications. In LDCs, the cost of delegation tends to be substantially more than that in developed countries, and this can be attributed to the economic policies of the governments (see section 5). This explains why the ratio of RAs to FAs is higher in emerging economies than in developed countries.

Lemma 2 states that, under SI, a change in the probability of liquidity shock has no effect on the portfolio choice, or on the expected utility. This is because returns on assets are invariant to the probability of liquidity shock (see (7) and Lemma 1). We will later see that under AI, the probability of liquidity shock also plays a role in determining the portfolio choice and the expected utility.

3.2 Asymmetric Information on Quality of Real Asset and Symmetric Information on Type of Agent

In the previous subsection, we studied case (1). In this subsection, we study case (2) - AI on quality of RA and SI on the type of agent. We will begin by analysis of trades at date $Z$.

Lemma 3 Assume that $0 < a_2 < 1$. In equilibrium, there are no strategic trades, $P^*_2 = R^*$, and $\frac{\partial P^*_2}{\partial t} = 0$.

Lemma 3 states that under AI on quality of RA only, there are no strategic trades in equilibrium. This is because the quality of RA can be inferred, given SI on type of agent. We will later see that when there is AI on both quality of RA and on type of agent, then strategic trades occur.
Lemma 3 brings out the role of, what we may call, the facility of diversification under asymmetric information (FDAI). To see this, observe that there is one price for RA for the simple reason that there is AI on quality of RA and that all (good or bad) RAs sell for one and the same price. The type 1 agents get a certain return on both RA and on FA. There is a certain return on FA because the financial intermediary invests in several projects and hence is able to diversify. The return on RA is certain at date Z because an RA is sold under AI at a price that reflects the average quality of RAs put up for sale. This has an interesting implication.

It is possible to get the benefit of diversification in two ways. One is by actually diversifying across projects (investing in FA). The other is by investing in RA and selling the same under AI. This is, what we have called, FDAI. Observe that $P_{f1} = R_e - m < P_{r2} = R_e$ (see (7) and Lemma 3) i.e. the return on FA is less than that on RA at date Z. In other words, the return on actual diversification across several projects is less than the return, which an agent can get by utilizing FDAI. This implies that FDAI is better than actual diversification. Why then do agents invest in FA at all?

Observe that AI is necessary for FDAI. Since AI is relevant at date Z only, and only type 1 agents sell their assets at date Z in case (2), it follows that FDAI is for type 1 agents only. Ex-ante, an agent does not know her type. She only knows the probability of her being type 1. This explains why an agent will not always fully invest in RA (see Lemma 4 below).

Lemma 3 states that the price of an RA is invariant to the fraction of type 1 agents. As in the previous case, this is due to ‘adequate endowment’ with the buyers, and competition between them.

It follows from (6) and Lemma 3 that $Y_{r2}^r = R_e$ for type 1 agents. Given the technology, and the absence of strategic trades, it follows that $Y_{f2}^f = \overline{R}$ for type 2G agents, and $Y_{f2}^g = \underline{R}$ for type 2B agents. For the pdf of $Y_{r2}^r$, see columns 1, 2 and 4 in Table 1.

Before we analyze choice at date 0, let us define $\overline{t}_2$ as follows:

$$\overline{t}_2 = 1 - \frac{m}{\rho R^v}, \quad (12)$$
Lemma 4 Let A.1 and A.2 hold. Then, \(0 < a^*_2 \leq 1, 0 < \bar{t}_2 < 1, \frac{\partial \bar{t}_2}{\partial m} < 0,\) and \(\frac{\partial W^*_2}{\partial t} > 0.\) Further, given that \(0 < a^*_2 < 1,\) we have \(\frac{\partial a^*_2}{\partial m} > 0, \frac{\partial a^*_2}{\partial t} > 0,\) and \(\frac{\partial W^*_2}{\partial m} < 0.\) \(^{10}\)

Lemma 4 is comparable to Lemma 2. In both cases, investment in RA is positive under the assumption that the cost of delegation is positive (see A.2). Lemma 4, however, states that, under AI, an agent may invest all her funds in one RA only. This is in contrast to Lemma 2, which stated that, under SI, investment in RA is less than 1. Observe that in both cases, it is assumed that the cost of delegation is not ‘large’ i.e. \(m < \rho R^*\) (see A.2). It is interesting that a risk averse agent may invest all her funds in one RA. The intuition lies in FDAI. Next, observe that \(\bar{t}_2\) is a cut-off level, which lies between 0 and 1. If the fraction of type 1 agents is greater than this cut-off, then all funds are invested in RA. This cut-off level is decreasing in \(m.\) The intuition is that as the cost of delegation increases, investment in RA becomes more attractive. One way in which this is manifested is that \(\bar{t}_2\) decreases as the cost of delegation increases. Finally, the expected utility of an agent increases with the fraction of type 1 agents. The intuition is that in this case, FDAI is available for type 1 agents only, and that the benefit from FDAI increases with the fraction of type 1 agents.

Let us now discuss the second part of Lemma 4, which focuses an interior solution. Investment in RA increases with the cost of delegation, and expected utility decreases with the cost of delegation. The intuition is the same as in case (1). Next, investment in RA increases with the fraction of type 1 agents. The intuition again lies in FDAI.

We have seen some interesting results in this subsection. It may seem that these results are due to the absence of strategic trades. In the next case, we will see that strategic trades are possible and yet the results seen in this subsection hold. In addition, there are other interesting results.

\(^{10}\) Obviously \(\frac{\partial a^*_2}{\partial m} = \frac{\partial a^*_2}{\partial t} = 0,\) if \(a^*_2 = 1.\) Further, \(\frac{\partial W^*_2}{\partial m} = 0,\) if \(a^*_2 = 1,\) or \(\bar{t}_2 \leq t < 1\) (see (24)) in the appendix.


3.3 Asymmetric Information on both the Quality of Real Asset and on the Type of Agent

We will now consider the last case i.e. case (3). In this case, as we will see, there is a possibility of strategic trades at date $Z$. If $a_3 > 0$ and $P_r^3 > R$, then an owner of a bad RA will try to sell her asset to an uninformed buyer. This can lead to $\beta' < \beta$, where $\beta'$ denotes the probability of a good RA amongst the RAs put up for sale at date $Z$. Recall that $\beta$ denotes the probability of a good RA amongst the RAs with the investors at date 0.

It follows from (5) that a type 1 agent would sell her FA at date $Z$, since $P_f^3 > 0$ (see (7)). If $P_r^3 > 0$, she would sell her RA as well. If $R < P_r^3 < R$, type 2G agents will retain their RAs, and type 2B agents will sell their RAs. Suppose, for the moment, that these conditions on price hold. Then, the supply of bad RAs comes from both type 1 and type 2 agents. Therefore, given that an agent has invested $a_3$ in RA at date 0, the total supply of bad RAs at date $Z$ is $a_3(1 - \beta)t + a_3(1 - \beta)(1 - t) = a_3(1 - \beta)$. The supply of good RAs comes from type 1 agents only. Therefore, total supply of good RAs is $a_3\beta t$. It follows that the total supply of (bad or good) RAs is $a_3[1 - \beta + \beta t]$. Thus,

$$\beta' = \frac{\beta t}{1 - \beta + \beta t} \quad (13)$$

Let $R'$ denote the expected return on an RA put up for sale at date $Z$. Clearly,

$$R' = \beta' R + (1 - \beta') R = R + \beta'(\Delta R) \quad (14)$$

where the second equality follows after using (2). Given $0 < t < 1$, $0 < \beta < 1$, $0 < R < R$, it is easy to check from (13) and (14) that the following properties hold:

$$0 < \beta' < \beta, \quad \frac{\partial \beta'}{\partial t} > 0, \quad R < R' < R^c, \quad \text{and} \quad \frac{\partial R'}{\partial t} > 0. \quad (15)$$

We can now analyze date $Z$ trades.

**Lemma 5** Let A.1 hold, and assume that $0 < a_3 < 1$. In equilibrium, there are strategic trades, $P_r^3 = R'$, and $\frac{\partial P_r^3}{\partial t} > 0$.

As compared to the results in Lemma 1 and those in Lemma 3, the novel part in Lemma 5 is that now strategic trades occur, and interestingly, the price
of RA at date $Z$ increases with the fraction of agents who are hit by liquidity shock. We will comment on these results in the next section.

It follows from (6) and Lemma 5 that $Y^*_Z = R'$ for all type 1 and type 2B agents. Finally, given that a type 2G agent retains her RA, it follows that $Y^*_Z = R$ for a type 2G agent. This completes the description of pdf of $Y^*_Z$. See columns 1, 2 and 5 in Table 1.

We can now analyze choice at date 0. Define $\bar{t}_3$ as follows:

$$\bar{t}_3 = \frac{\rho R^v (1 - \beta) - m (1 - \beta)}{\beta m + (1 - \beta) \rho R^v}. \quad (16)$$

**Lemma 6** Let A.1 and A.2 hold. Then, $0 < a^*_3 < 1$, $0 < t_3 < 1$, $\frac{\partial a^*_3}{\partial m} < 0$, and $\frac{\partial W^*_3}{\partial m} > 0$. Further, given that $0 < a^*_3 < 1$, we have $\frac{\partial a^*_3}{\partial t} > 0$, $\frac{\partial a^*_3}{\partial t} > 0$, and $\frac{\partial W^*_3}{\partial m} < 0$.\footnote{Obviously, $\frac{\partial a^*_3}{\partial m} = \frac{\partial a^*_3}{\partial m} = 0$, if $a^*_3 = 1$. Further, $\frac{\partial W^*_3}{\partial m} = 0$, if $a^*_3 = 1$, or $\bar{t}_3 \leq t < 1$ (see (28)) in the appendix.}

Observe that the results in Lemma 6 are similar to those in Lemma 4, and we have already discussed the latter case.

An important difference between the results in this subsection and those in the previous subsection is that now strategic trades are possible. Yet the thrust of the results of the previous subsection holds even in the present case. We will elaborate in the next section.

We have so far studied three cases in this section. In the next section, we will compare these cases.

4 A Comparison of the Three Cases

We begin by comparing date $Z$ trades and prices in the three cases. Thereafter, we will consider outcomes at date 1.

**Proposition 1** Let A.1 hold and assume that $0 < a_k < 1$, where $k = 1, 2, 3$.

In equilibrium, strategic trades are possible in case (3) only. Further, $P^*_1 = R$, $P^*_2 = R^c > R' = P^*_3$, and $0 = \frac{\partial P^*_1}{\partial m} = \frac{\partial P^*_2}{\partial m} < \frac{\partial P^*_3}{\partial m}$. \footnote{Obviously, $\frac{\partial a^*_3}{\partial m} = \frac{\partial a^*_3}{\partial m} = 0$, if $a^*_3 = 1$. Further, $\frac{\partial W^*_3}{\partial m} = 0$, if $a^*_3 = 1$, or $\bar{t}_3 \leq t < 1$ (see (28)) in the appendix.}
Strategic trades are impossible not only in case (1) i.e. under SI, but also in case (2) i.e. AI on quality of RA only. Strategic trades are possible in case (3) only, when we have AI on both the quality of RA and on the type of agent.

In case (1), we have two prices for RA (see (1)), whereas in the other two cases, we have only one price for RA. This is because we have SI in case (1) and AI in the other two cases. Further, price in case (3) is less than that in case (2). This is due to strategic trades in case (3).

There are two effects of a change in the fraction of type 1 agents ($t$) on the price of RA ($P_r$). First, for a given quality of assets, if endowment with buyers is inadequate, then $P_r$ will decrease with $t$. This effect is, however, irrelevant in our model, given ‘adequate endowment’ with the buyers. Second, there are forced sales as well as strategic sales in case (3). Only forced sales include sales of good RAs. Furthermore, forced sales are by type 1 agents. So forced sales increase with $t$. It follows that the probability of good quality increases with $t$. Therefore, the expected return on RAs put up for sale (and hence, $P_r$) increases with $t$ in case (3). This effect is, however, nil in the other two cases since strategic trades are absent in those two cases.

We have so far in this section compared date $Z$ trades and prices across the three cases. Hereafter, we will consider outcomes at date 0.

**Remark** $Y_r^1 = R$, whereas, in general, $Y_r^k \neq R$, where $k = 2, 3$.

We have compared the technological return on RA with the realized return on RA. Observe that, under SI, the technological return on RA is equal to the realized return on RA, whereas under AI, in general, the two are not equal (see Table 1) even though the buyers have ‘adequate endowment’ and the discount rate for the buyers is 0. This is due to AI in case (2) and in case (3).

We will now present a comparison of the mean and the variance of return on RA in the three cases.


Lemma 7 states that the expected return on RA is the same in all cases, and
that the variance of return on RA decreases from case (1) to case (2), and from case (2) to case (3). Let us discuss these two results one by one.

It is interesting that the expected return on investment in RA at date 0 is the same in all cases i.e. it is independent of the information at date $Z$. It is quite obvious that $E[Y_1^r] = E[Y_2^r] = R_e$ (see Table 1, (1) and (3)). In case (3), we have two effects of AI. First, there is a ‘loss’ for type 1 agents. This is the standard lemon effect ($P_1^* = R' < R_e$, see (15)). Second, due to strategic trades, there is ‘gain’ for type 2 agents. The return on RA for a type 2 agent is more than $R_e$ $(\beta R + (1 - \beta)R' > \beta R + (1 - \beta)R_e = R_e$, see (15)). Hence, there is a ‘loss’ for type 1 agents and a ‘gain’ for type 2 agents. Overall, we have $E[Y_3^r] = R_e$. Observe that this is unlike the case of the standard lemon effect (Akerlof, 1970).

The intuition for the result in the second part of Lemma 7 is as follows. Recall that in addition to actual diversification across several projects, there is also the possibility of getting the benefit of diversification in our model by using FDAI. Observe that FDAI is relevant under AI only. Hence, FDAI is irrelevant in case (1) - the case of SI. Since FDAI is not applicable in case (1), the variance of return on RA is ‘high’. On the other hand, in case (2), FDAI is applicable. Hence, the variance of return on RA in case (2) is less than that in case (1).

Observe that the variance of return on RA is zero in both case (2) and in case (3), given that an agent is type 1 (see Table 1). Type 2 agents face a risk on their investment in RA in case (2) - type 2G and type 2B agents get a return of $\bar{R}$ and $\bar{R}$ respectively. In case (3), type 2G and type 2B agents get a return of $\bar{R}$ and $\bar{R}'$ respectively (see Table 1). It follows that there is less variance for an agent in case (3) than in case (2), given that she is a type 2 agent, and given that $\bar{R} < \bar{R}' < \bar{R}$ (see (15) and recall that $R_e < \bar{R}$. All this suggests that the variance of return on RA is less in case (3) than in case (2).

Formally, the second part of Lemma 7 states that the variance of return on RA decreases with a shift from case (1) to case (2), and from case (2) to case (3). We have already seen that FDAI is available in case (2) and not in case (1). We may now say that FDAI is more effective in case (3) than in case (2).

Let us next compare the conditions under which an interior solution to the problem of optimal portfolio choice exists.
Proposition 2 Let $A.1$ hold. Then, (a) $a_k^* > 0$ if and only if $m > 0$, $k = 1, 2, 3$, (b) $a_1^* < 1$ if and only if $m < \rho R^v$, (c) $a_k^* < 1$ if and only if $m < \rho R^v$, and $t < t_k$, where $k = 2, 3$, (d) $\frac{\partial m}{\partial t_k} < 0$, where $k = 2, 3$, and (e) $0 < t_3 < t_2 < 1$.

First, in all three cases, investment in RA is positive if and only if the cost of delegation is positive\(^\text{12}\). This is a reflection of the important role that is played by the cost of delegation in determining whether or not there are RAs in the model economy under consideration. Second, in case (1) i.e. under SI, investment in RA is less than 1 if and only if the cost of delegation is below a certain upper bound ($\rho R^v$). In other words, there will be positive demand for FAs (delegated projects) if and only if the cost of delegation is not ‘large’.

Third, in case (2) and in case (3), i.e. in cases of AI on quality of RA, investment in RA is less than 1 if and only if the cost of delegation is below an upper bound ($\rho R^v$), and the probability of liquidity shock is below a certain cut-off level ($0 < t < t_k$, where $k = 2, 3$). Observe that these conditions are stronger than in case (1) - see Proposition 2(b). Investment of less than 1 in RA is equivalent to investment of more than 0 in FA. So the investment in FA is positive in case (2) and in case (3) under conditions that are stronger than in case (1). We require not only a low cost of delegation but also a low fraction of type 1 agents for seeing positive investment in FA. The intuition for the requirement of a low cost of delegation is straightforward. The intuition for the requirement of a low fraction of type 1 agents lies in the fact that FDAI on investment in RA is available in case (2) and in case (3) but not in case (1).

Finally, let us comment on the cut-off level - $t_k$, where $k = 2, 3$. If the fraction of type 1 agents is above this cut-off level, then investment in FA is zero. This cut-off level decreases with the cost of delegation, and it is lower in case (3) than in case (2). We will return to this aspect.

The next proposition compares the optimal portfolio in the three cases, and looks at some comparative statics results, after assuming that $t < t_3$ - the condition under which an interior solution exists in all three cases.

Proposition 3 Let $A.1$ and $A.2$ hold, and assume that $0 < t < t_3$. Then

\(^{12}\)This is consistent with the standard result that ‘... if a risk is actuarially favorable, then a risk aveter will always accept at least a small amount of it.’ (Mas-Colell, et. al., 1995)
Given an interior solution, Proposition 3(a) states that as we move from symmetry of information (case (1)) to asymmetry of information (case (2)), and from asymmetry of information (case (2)) to what we may call, greater asymmetry of information (case (3)), the investment in RA increases. This is an important result in this paper. This brings out the role of AI in explaining the demand for RAs. The intuition for the result is immediate from Lemma 7. Since the expected return on RA is the same in all cases, the variance of return on RA is decreasing from case (1) to case (2), and from case (2) to case (3), and agents are risk averse, it follows that \( a_1^* < a_2^* < a_3^* \), provided, of course, we have an interior solution in all three cases.

Proposition 3(b) considers the effect of an increase in the cost of delegation on the investment in RA. This effect is positive in all cases. This has an important implication. If the cost of delegation is higher in emerging economies than in developed countries, then investment in RAs is higher in emerging economies than in developed countries. Furthermore, this effect increases as we move from case (1) to case (2), and from case (2) to case (3). The intuition is as follows. In case (1), there is only one way to get the benefit of diversification i.e. by investing in FA. In case (2), FDAI is also available. In case (3), FDAI is even better than in case (2). Recall that investment in FA involves a cost of delegation. The effect of a rise in this cost is as follows. As the alternative to FA for getting the benefit of diversification improves from case (1) to case (2), and from case (2) to case (3), investment in RA becomes more sensitive to a rise in the cost of delegation.

Proposition 3(c) considers the effect of an increase in the fraction of type 1 agents on the investment in RA. This effect is nil in case (1). The intuition is straightforward from (11). The fraction of type 1 agents is effectively irrelevant as far as the probability distribution of return on RA in case (1) is concerned. The effect is positive in case (2) and case (3). Furthermore, this effect is stronger in case (3) than in case (2). The intuition is as follows. In case (2), FDAI is available for type 1 agents only. Furthermore, FDAI is applicable to RAs only.
Hence, in case (2), the investment in RAs increases with the fraction of type 1 agents. In case (3), type 2B agents can undertake strategic trades at date $Z$. The price at which a type 2B agent can sell her RA at date $Z$ is $R'$, and this price increases with the fraction of type 1 agents. On the other hand, in case (2), the price of RA is equal to $R_{e}$, which is invariant to the fraction of type 1 agents (Proposition 1). All this explains why $0 = \frac{\partial a_{1}^{*}}{\partial t} < \frac{\partial a_{2}^{*}}{\partial t} < \frac{\partial a_{3}^{*}}{\partial t}$. The latter result also clarifies why $t_{3} < t_{2}$ (Proposition 2(e)). Recall that $t_{k}$ is the cut-off level in case $k$. If $t \geq t_{k}$, then investment in RA is 1 (see Proposition 2(c)).

We have throughout assumed that $0 < t < 1$. Proposition 3(d) states that as the probability of liquidity shock tends to zero$^{13}$, the portfolio choice under (each case of) AI converges to that under SI. The intuition lies in the following two observations. First, in the limit, all agents are of one type only i.e. type 2. Therefore, in the limit, the problem of portfolio choice in our model resembles the standard textbook problem$^{14}$. Second, the portfolio choice problem under SI in our model is effectively the same as the standard textbook problem. This is because the portfolio choice problem under SI is effectively a case of two states of the world. RA gives a return of $\overline{R}$ with probability $\beta$, and a return of $\underline{R}$ with probability $(1 - \beta)$ (see (11)). FA is safe asset in our model.

The last two propositions compared the optimal portfolio in the three cases, and studied comparative statics with regard to the optimal portfolio. Let us now consider the expected utility of an agent across the three cases.

**Proposition 4** Let A.1 and A.2 hold. Then, (a) $W_{1}^{*} < W_{2}^{*} < W_{3}^{*}$, (b) $W_{2}^{*} \bigg|_{t = t_{2}} = W_{3}^{*} \bigg|_{t = t_{3}}$, (c) $\frac{\partial W_{1}^{*}}{\partial m} < \frac{\partial W_{2}^{*}}{\partial m} < \frac{\partial W_{3}^{*}}{\partial m} < 0$, if $0 < t < t_{a}$, (d) $\frac{\partial W_{1}^{*}}{\partial t} = 0$, $\frac{\partial W_{2}^{*}}{\partial t} > 0$, (e) $W_{2}^{*}, W_{3}^{*} \rightarrow W_{1}^{*}$ as $t \rightarrow 0$, and (f) $W_{2}^{*}, W_{3}^{*} \rightarrow R_{e}$ as $t \rightarrow 1$.

Proposition 4(a) states that as we move from case (1) to case (2), and from

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$^{13}$For completeness, consider the case where $t$ tends to 1. But we have already seen that $a_{1}^{*}$ is constant $\forall$ $t$ (Proposition 3(c)) and that $a_{k}^{*} = 1$ if $t \geq t_{k}$, where $k = 2, 3$ (see Proposition 2(c)).

$^{14}$This problem is as follows. There are two assets - one safe and the other risky. An agent invests a fixed amount at date 0 and obtains returns at date 1. Nothing happens in the interim period (all agents are type 2). What is the optimal portfolio? (Varian, 1992). Observe that the problem in our model reduces to this textbook problem if $t = 0$. 19
case (2) to case (3), the expected utility of an agent increases. This is an
interesting result. The expected utility of an agent increases as we move from
SI to AI, and from AI to, what we may call, greater AI. This is because FDAI
is not available in case (1), and FDAI is better in case (3) than in case (2).

Proposition 4(b) states that the expected utility of an agent in case (2) is
the same as that in case (3), if $t = t_2$ in case (2), and $t = t_3$ in case (3). Recall
that $a^*_2|_{t=t_2} = a^*_3|_{t=t_3} = 1$ (see Proposition 2(c)).

Proposition 4(c) holds if $0 < t < t_3$. Under the latter condition, we have
an interior solution for investment in RA in all three cases (Proposition 3(a)).
In all three cases, expected utility of an agent decreases with the cost of dele-
gation. This is intuitively straightforward. Next, given an interior solution for
investment in RA, there is a decrease in absolute value of the rate of change of
expected utility with respect to the cost of delegation, as we move from case (1)
to case (2), and from case (2) to case (3).

Proposition 4(d) states that expected utility is invariant to the probability
of liquidity shock in case (1), and that the expected utility increases with the
probability of liquidity shock in case (2) and in case (3). The intuition lies in
two observations. First, FDAI is available in case (2) and in case (3) only, and
that FDAI is better in case (3) than in case (2). Second, the usefulness of FDAI
increases with the fraction of type 1 agents in both case (2) and case (3).

Proposition 4(e) is the counterpart of Proposition 3(d).

**FDAI in the extreme case**

Proposition 4(f) states that, under AI$^{15}$, as the fraction of type 1 agents
tends to 1, the expected utility of an agent tends to $R^e$. Recall that this is also
the return per unit of investment in a bundle of independent projects, and that
an agent invests her entire endowment of 1 unit in a single RA, if the fraction of
type 1 agents is large i.e. $t \geq t_k$, where $k = 2, 3$. It is interesting that the agent
under consideration is risk-averse, that she has not actually diversified across
projects, and yet, she gets the benefit of diversification at zero cost$^{16}$. The role
of FDAI is brought out sharply here in this extreme case.

$^{15}$Under SI, the expected utility is invariant to $t$. See Proposition 4(d).

$^{16}$With actual diversification i.e. by investing in FA, an agent gets a return $R^e - m < R^e$. 

20
5 Conclusion

In our model, real assets represent owner-managed projects, and financial assets represent delegated projects. This paper shows that investment in real assets is positive if and only if the cost of delegation is positive. Further, investment in real assets increases with the cost of delegation. Finally, if the cost of delegation is large enough, then there is demand for real assets only.

An important rationale for investment in financial assets is that agents are risk-averse and wish to diversify, and financial assets represent a bundle of projects, whereas a real asset is not diversified. This is, however, not the only way to get the benefit of diversification. Under asymmetric information, there is sometimes an additional way to get the benefit of diversification i.e. by investing in real asset and selling the same under asymmetric information. In both methods of getting the benefit of diversification, the return on an asset reflects the average quality of project(s) underlying the asset, which is put up for sale. This is the reason why the investment in real assets is higher under asymmetric information than under symmetric information.

We have studied two cases of asymmetric information. In one case, we have asymmetric information on the quality of real asset. In the other case, we have asymmetric information on both the quality of the real asset, and on the type of agent i.e. whether or not she faces a liquidity shock. Strategic trades are possible in the latter case only. Despite this, most results in the two cases are qualitatively similar.

We have shown that, under symmetric information, the portfolio choice is invariant to the probability of liquidity shock, whereas under asymmetric information, the portfolio choice varies with the probability of liquidity shock. Investment in real assets increases with the probability of liquidity shock. If probability of liquidity shock is very large, there is demand for real assets only.

It seems obvious that price of a real asset in the secondary market is invariant to the probability of liquidity shock, given that the buyers have deep pockets. This is, however, not always true. An interesting result in the case of information asymmetry on both quality of real asset and the type of agent is that if an agent is hit by a liquidity shock and she sells her real asset, then the price of a real
asset increases with the probability of a liquidity shock.

This paper has focused on two factors to explain why the ratio of real assets to financial assets is higher in emerging economies than in developed countries. One is the cost of delegation, and the other is the liquidity shock. Consider the cost of delegation. Apart from the transactions costs, we have an agency cost in the context of the separation between ownership and management (Jensen and Meckling, 1976). This arises because the managers can get private benefits due to some non-verifiable action. Observe that there is an implicit assumption in the analysis that there exists an impartial court of law, which is accessible at a reasonable cost, and is accessible without much delay. It is this assumption that makes the distinction between verifiable and non-verifiable action meaningful in a developed country. But many emerging economies are different.

It is not always the case in an emerging economy that there effectively exists a court of law\textsuperscript{17}, which is accessible. In such a scenario, it is possible for a manager in an emerging economy to get away not just with some non-verifiable action but also with some verifiable economic offence! So there can be a cost to an investor who has invested in financial assets, even if there is a verifiable offence by a manager\textsuperscript{18}. On the other hand, in the case of a real asset, there is no such separation between ownership and management\textsuperscript{19}. Ex-ante, a rational investor invests less in delegated projects (financial assets) in an emerging economy than in a developed country. Indeed, there are parts of Africa and Asia where there are hardly any financial assets.

It is true that with financial intermediation, the cost of monitoring can be reduced (Diamond, 1984). In any case, financial assets include bank deposits, which are reasonably safe \textit{for depositors} in many emerging economies due to deposit insurance (explicit or implicit)\textsuperscript{20}. But it is only the nominal value that is owed to the depositor, which is insured. If the problem of inflation and its

\textsuperscript{17}There are about 30 million cases pending in various Indian courts. There is only one judge for 90,324 people in South Asia. In the United States, as far back as 1982-3, there were about ten times that number (p. 66, The Mehbub-ul-Haq Human Development Centre, 1999).

\textsuperscript{18}For more on weak investor protection, see, for example, Shleifer and Wolfenzon (2002).

\textsuperscript{19}We assume that protection of physical property is perfect.

\textsuperscript{20}State banks are more common in low income countries. Furthermore, the ratio of deposit insurance coverage to per capita GDP is more than 6 in India as compared to a figure of less than 4 for USA (World Bank, 2001).
variability is more serious in emerging economies than in developed countries\textsuperscript{21},
then this can lead to higher fraction of investment in real assets in emerging
economies than in developed countries.

Let us now consider liquidity shock. There are good reasons to believe that
the probability of liquidity shock is effectively higher in emerging economies
than in developed countries. There can be several reasons. For example, there
is considerable dependence on nature in emerging economies. A harvest failure
can cause a big liquidity shock in an emerging economy. A common person
will typically have less effective access to insurance and banking facilities in an
emerging economy than in a developed country\textsuperscript{22}. In the context of our model,
we may say that the probability of an unguarded liquidity shock is higher in an
emerging economy than in a developed country. Then our model would suggest
that the fraction of investment that takes the form of real assets is higher in
emerging economies than that in developed countries.

Appendix

**Proof of Lemma 1:** Given the technology, an RA yields $R$ at date 1 (see (1)).
Given competition, SI on quality of assets at date $Z$, zero discount rate and
adequate endowment of agents $N$, it follows that $P_{r1} = R$. Since $P_{r1} > 0$, it
follows from (5) that a type 1 agent would sell her RA at date $Z$. Given that
$P_{r1} = R$ at date $Z$, and that an RA yields $R$ at date 1, it follows that there
are no strategic trades\textsuperscript{23}. Finally, it follows from $P_{r1} = R$ that
$\partial P_{r1} / \partial t = 0$ (see (1)).

**Proof of Lemma 2:** It follows from (11) that

$$E[Y_{r1}] = R^e,$$

\textsuperscript{21}Administered nominal rates may adjust slowly and inadequately in emerging economies.
\textsuperscript{22}There is some informal insurance due to social ties in emerging economies. But this
tends to be localized, and most agents within a region or a community face an adverse shock
simultaneously. For a more detailed description of conditions in less developed countries, see
Ray (2002).
\textsuperscript{23}It is easy to check that there is no gain for type 2 agents from selling RA and buying
another RA, or FA, or storing goods from date $Z$ till date 1. Similarly, there is no gain from
selling FA.
after using (3). Next, it follows from (11) and (17) that

$$V[Y_1] = R^e,$$  \hspace{1cm} (18)

after using (4). Using (17) and (18) in (10), we get

$$a_1^* = \frac{m}{\rho R^v}$$  \hspace{1cm} (19)

after using A.2. It follows immediately that $\frac{\partial a_1^*}{\partial m} = 0$, and that $\frac{\partial a_1^*}{\partial m} > 0$, given A.2. From (9), (17), (18) and (19), we get

$$W_1^* = R^e - m + \frac{m^2}{2\rho R^v}$$  \hspace{1cm} (20)

It follows that $\frac{\partial W_1^*}{\partial m} = 0$, and that $\frac{\partial W_1^*}{\partial m} < 0$, given A.2. ||

Proof of Lemma 3: Strategic trades are ruled out by SI on type of agent\(^{24}\). Given (5), it follows that type 1 agents sell their RAs at date Z. Since quality is independent of type of agent, it follows that the probability of good quality in RAs put up for sale is $\frac{\beta t}{\beta t + (1 - \beta)t} = \beta$. Given the technology, an RA yields $R$ at date 1. It follows that the expected return on RA put up for sale at date Z is $R^e$ (see (1) and (3)). Given competition, zero discount rate and adequate endowment of agents $N$, it follows that $P_1^e = R^e$, which implies that $\frac{\partial P_1^e}{\partial t} = 0$ (see (3)). ||

Proof of Lemma 4: It follows from A.2 that $0 < \bar{t}_2 < 1$. It follows from A.2 and (12) that $\frac{\partial \bar{t}_2}{\partial m} < 0$. Given the pdf of $Y_2^r$ in Table 1, it follows that

$$E[Y_2^r] = t\beta R^e + t(1 - \beta)R^e + (1 - t)\beta R + (1 - t)(1 - \beta)R = R^e$$  \hspace{1cm} (21)

where the last equality follows from (3). Similarly, given the pdf of $Y_2^r$ and (21), it follows that

$$V[Y_2^r] = t(R^e - R^e)^2 + (1 - t)\beta [R - R^e]^2 + (1 - t)(1 - \beta)R^e [R - R^e]^2 = (1 - t)R^e$$  \hspace{1cm} (22)

where the last equality follows from (4). Using A.2, (21) and (22) in (10), we get

$$a_2^* = \min \left[ \frac{m}{\rho V[Y_2^r]}, 1 \right] = \min \left[ \frac{m}{\rho R^e (1 - t)}, 1 \right].$$  \hspace{1cm} (23)

\(^{24}\)The footnote in case of proof of Lemma 1 is relevant here as well.
It follows that $\frac{\partial a_2^*}{\partial m} > 0$, if $0 < a_2^* < 1$, after using A.2.

Next, from (9), (21), (22) and (23), we get

$$W_2^* = \begin{cases} 
R^e - m + \frac{m^2 + m_e}{2R^e(1-t)}, & \text{if } 0 < t < \tilde{t}_2, \\
R^e - \frac{\rho R^e(1-t)}{2}, & \text{if } \tilde{t}_2 \leq t < 1,
\end{cases}$$

(24)

where $\tilde{t}_2$ is defined in (12). Observe that $\frac{\partial W_2^*}{\partial m} > 0 \forall t$. It is easy to check that $0 < a_2^* < 1$ if and only if $0 < t < \tilde{t}_2$ (see (12) and (23)). It follows then that $\frac{\partial W_2^*}{\partial t} < 0$, if $0 < a_2^* < 1$.

Proof of Lemma 5: It follows from (5) that a type 1 agent would sell her FA at date $Z$, since $P^f_3 > 0$ (see (7)). If $P^r_3 > 0$, she would sell her RA as well. If $R < P^r_3 < R$, type 2G agents will retain their RAs, and type 2B agents will sell their RAs. Given competition, zero discount rate and ‘adequate endowment’ with agents $N$ at date $Z$, it follows that $P^r_3 = R^e$ (see (14)). Since $P^r_3 = R^e$ and $R < R^e < R$ (see (3) and (15)), it is indeed true that $P^r_3 > 0$, and $R < P^r_3 < R$.

A type 2B agent can choose one of the following: (a) sell the bad RA for another RA which is good with probability $\beta'$, (b) sell the bad RA ($P^r_3 = R^e$), and buy FA, and (c) sell the bad RA for goods, and store the latter. Observe that

$$\beta' u(a_3R + (1 - a_3)(R^e - m)) + (1 - \beta')u(a_3R + (1 - a_3)(R^e - m))$$

$$< u\left(\frac{a_3R^e}{P^r_3}(R^e - m) + (1 - a_3)(R^e - m)\right)$$

$$= u(a_3R^e + (1 - a_3)(R^e - m))$$

where the inequality follows from concavity (A.1) and (14), and the equality follows from (7). Note that we have shown that the expected utility of an agent who chooses (c) is no less than that of an agent who does not choose (c). We have shown that type 2B agents will undertake strategic trade. This is the only strategic trade which pays\textsuperscript{25}.

\textsuperscript{25}The expected return on RA purchased at date $Z$ is $R^e$. Therefore, if a type 2 agent sells her FA and buys RA, then her expected return is $\frac{(1-a_3)R^e}{P^r_3} R^e = (1-a_3)(R^e - m)$, after substituting for $P^r_3 = R^e$, and $P^f_3 = R^e - m$. Thus, the expected return from this strategy is equal to the certain return on retaining FA. Since the agent is risk-averse, she strictly prefers to retain her FAs to exchanging them for RAs. Next, it is obvious that there is no gain from
Since $P_3' = R'$, it follows that $\frac{\partial P_3'}{\partial t} > 0$ after using (15).

Proof of Lemma 6: It follows from A.2 that $0 < t_3 < 1$. It is easy to check from (16) that $\frac{\partial Y_3'}{\partial m} < 0$. Given the pdf of $Y_3'$ in Table 1, and observing that $t\beta + t(1 - \beta) + (1 - t)(1 - \beta) = 1 - \beta + \beta t$, it follows that

$$E[Y_3'] = (1 - \beta + \beta t)R' + (1 - t)\beta R$$

$$= \{(1 - \beta + \beta t)\beta' + (1 - t)\beta\}R + (1 - \beta + \beta t)(1 - \beta')R$$

$$= R', \quad (25)$$

where the second equality follows from (14), and the last equality follows from (3) and (13). Next, given the pdf of $Y_3'$, (25), and observing that $t\beta + t(1 - \beta) + (1 - t)(1 - \beta) = 1 - \beta + \beta t$, it follows that

$$V[Y_3'] = [1 - \beta + \beta t](R' - R')^2 + (1 - t)\beta(R - R')^2.$$

Substituting for $R'$ and $R$ using (3) and (14) respectively, and using (2), we get

$$V[Y_3'] = [1 - \beta + \beta t](\beta' - \beta)^2(\triangle R)^2 + (1 - t)\beta(\beta - \beta')^2(\triangle R)^2,$$

where the last equality follows from (13). After simple manipulation, and using (4) and (13), we get

$$V[Y_3'] = (1 - t)(1 - \beta')(\triangle R)^2. \quad (26)$$

Using A.2, (25) and (26) in (10), we get

$$a_3^* = \min\left[\frac{m}{\rho V[Y_3']}, 1\right] = \min\left[\frac{m}{\rho R^v(1 - t)(1 - \beta')}, 1\right]. \quad (27)$$

It is easy to check that $\frac{\partial a_3^*}{\partial t} > 0$, if $0 < a_3^* < 1$.

Next, from (9), (25), (26) and (27), we get

$$W_3^* = \begin{cases} R^v - m + \frac{m^2}{2\rho R^v(1 - t)(1 - \beta')}, & \text{if } 0 < t < \tilde{t}_3, \\ R - \frac{\rho R^v(1 - t)(1 - \beta')}{2}, & \text{if } \tilde{t}_3 \leq t < 1. \end{cases} \quad (28)$$

selling an FA and buying another FA. Finally, since $P_f = R - m$, and the latter is the same as the return on FA at date 1, a type 2 agent will retain her FA instead of selling the same, and storing the goods from date $Z$ to date 1.
It is easy to check that $\frac{\partial W^3}{\partial t} > 0 \forall t$. It is easy to check from (16) and (27) that $0 < a_3^* < 1$ if and only if $0 < t < \tilde{t}_3$. Given this, it follows from (28) that $\frac{\partial W^3}{\partial m} < 0$, given that $0 < a_3^* < 1$. \[\|\]

**Proof of Proposition 1:** From (15), we have $R' < R^c$. The rest of the proposition follows immediately from Lemma 1, Lemma 3 and Lemma 5. \[\|\]

**Proof of Lemma 7:** From (17), (21) and (25), it follows that

$$E[Y_1'] = E[Y_2'] = E[Y_3'] = R^c.$$ \[(29)\]

Next, it follows from (18), (22) and (26) that

$$V[Y_1'] > V[Y_2'] > V[Y_3'],$$ \[(30)\]

where the inequalities follow from $0 < \beta' < \beta$ (see (15)), and $\beta < 1$ and $0 < t < 1$ (see section 2). \[\|\]

**Proof of Proposition 2:** After substituting for $E[Y_k']$ from (29) in (9), it is easy to check that $\frac{\partial W_k}{\partial a_k} \bigg|_{a_k=0} > 0$ if and only if $m > 0$. This proves Proposition 2(a). After substituting for $E[Y_1']$ and $V[Y_1']$ from (17) and (18) respectively in (9), it is easy to check that $\frac{\partial W_1}{\partial a_1} \bigg|_{a_1=1} < 0$ if and only if $m < \rho R^v$. This proves Proposition 2(b).

After substituting for $E[Y_2']$ and $V[Y_2']$ from (21) and (22) in (9), it is easy to check that $\frac{\partial W_2}{\partial a_2} \bigg|_{a_2=1} < 0$ if and only if $m < \rho R^v$, and $t < \tilde{t}_2$. This proves Proposition 2(c) for $k = 2$. Similarly, using (26) and (29) in (9), it is easy to check that Proposition 2(c) holds for $k = 3$.

From Lemma 4 and Lemma 6, we have $0 < \tilde{t}_k < 1$, and $\frac{\partial \tilde{t}_k}{\partial m} < 0$, where $k = 2, 3$. Finally, it is easy to check from (16) that

$$\tilde{t}_3 < \left[ \frac{\rho R^v - m}{\rho R^v} \right] = \tilde{t}_2$$ \[(31)\]

where the inequality follows from A.2 and the equality follows from (12). \[\|\]

**Proof of Proposition 3:** It follows from (19), (23) and (27) that

$$a_1^* = \frac{m}{\rho R^v} < a_2^* = \frac{m}{\rho R^v(1 - t)} < a_3^* = \frac{m}{\rho R^v(1 - t)(1 - \beta')} < a_3^*, \quad 0 < t < \tilde{t}_3, \quad (32)$$

27
after using (16) and (31). The first inequality follows from \(A.2\) and \(0 < t < 1\), and the second inequality follows from \(0 < \beta' < \beta\) (see (15)) and \(\beta < 1\) (see section 2). Further, it follows from \(A.2\) that \(a_1^* > 0\). Finally, it follows from \(0 < t < \bar{t}_3\) that \(a_3^* < 1\). This proves Proposition 3(a).

Given that \(0 < t < \bar{t}_3\), it follows from (32) that

\[
0 < \frac{\partial a_1^*}{\partial m} = \frac{1}{\rho R^v} < \frac{1}{\rho R^v(1-t)} < \frac{\partial a_2^*}{\partial m} < \frac{1}{\rho R^v(1-t)(1-\beta)} = \frac{\partial a_3^*}{\partial m},
\]

where the second inequality follows from \(0 < t < 1\), and the last inequality follows from (15) and \(0 < \beta < 1\). This proves Proposition 3(b). Next, given that \(0 < t < \bar{t}_3\), it follows from (32) that

\[
\frac{\partial a_1^*}{\partial t} = 0 < \frac{m}{\rho R^v(1-t)^2} = \frac{\partial a_2^*}{\partial t} < \frac{m}{\rho R^v(1-t)(1-\beta)} = \frac{\partial a_3^*}{\partial t},
\]

where the first inequality follows from \(A.2\), and the second inequality follows from \(A.2\) and \(0 < \beta < 1\). This proves Proposition 3(c). Finally, Proposition 3(d) follows from (32) after using (13).

\[
\textbf{Proof of Proposition 4:} \text{ We will prove Proposition 4(a) in three parts i.e. (1) } 0 < t < \bar{t}_3, \\
(2) \bar{t}_3 \leq t < \bar{t}_2, \text{ and (3) } \bar{t}_2 \leq t < 1.
\]

(1) \(0 < t < \bar{t}_3\). From (20), (24) and (28), it follows that \(W_3^* < W_2^* < W_1^*\), after using (31), \(0 < t < 1\) (section 2), and \(0 < \beta' < \beta < 1\) (see (15) and section 2).

(2) \(\bar{t}_3 \leq t < \bar{t}_2\). Substituting for \(\beta'\) from (13) in (28), and thereafter using (16) and \(\bar{t}_3 \leq t\), we get \(W_3^* \geq R^e - \frac{m}{2}\). Next, observe that

\[
R^e - \frac{m}{2} = R^e - m + \frac{m^2}{2\rho R^v(1-\bar{t}_2)} > R^e - m + \frac{m^2}{2\rho R^v(1-t)} = W_2^* > W_1^*
\]

where the first equality follows from (12), the first inequality follows since \(t < \bar{t}_2\), the last equality follows from (24), and the last inequality follows from (20) and \(0 < \bar{t}_3 \leq t < \bar{t}_2 < 1\) (see Proposition 2(e)). The result follows.

(3) \(\bar{t}_2 \leq t < 1\). It follows from (24) and (28) that \(W_3^* > W_2^*\) since \(\bar{t}_3 < \bar{t}_2\) and \(0 < \beta' < 1\) since (15) holds and \(\beta < 1\) by assumption. Next, we have

\[
W_2^* \geq R^e - \frac{\rho R^v(1-\bar{t}_2)}{2} = R^e - \frac{m}{2} > R^e - m + \frac{m^2}{2\rho R^v} = W_1^*
\]
where the weak inequality follows from (24) and \( t \geq t_2 \), the first equality follows from (12), the last inequality follows from A.2, and the last equality follows from (20). The result follows. This completes the proof for Proposition 4(a).

Next consider part (b). After substituting \( t = t_2 \) and \( t = t_3 \) in (24) and (28) respectively, we get \( W^*_2 |_{t=t_2} = W^*_3 |_{t=t_3} \). It is easy to check from (20), (24) and (28) that Proposition 4(c), 4(e) and 4(f) hold after using A.2, (13), (15) and \( 0 < t_3 < t_2 < 1 \) (Proposition 2(e)). Proposition 4(d) follows from Lemma 2, Lemma 4 and Lemma 6.

References


