Estimating the Output Gap for the Indian Economy

Comparing Results from Unobserved-Components Models and the Hodrick-Prescott Filter

Vineet Virmani

vineet@iimahd.ernet.in

Address for Correspondence

Dorm 13, Room 2

Indian Institute of Management (IIM)

Vastrapur

Ahmedabad – 380015

Ph: 079-2632-7302

Indian Institute of Management, Ahmedabad

Title

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Abstract

Output gap estimates are constructed for India using unobserved components model (UCM) approach on the lines of Watson (1986) and Kuttner (1994). Results from UCMs are not found to be any less sensitive to data revisions when compared to those from the Hodrick-Prescott filter. This, however, could be because of lack of sufficient 'revised-data' on which the sensitivity of the results can be tested. Based on standard deviation of change in potential output to data revisions and its 'economic' content, the UCM using trimmed mean as the numeraire for inflation comes forth as the best choice. Alternative estimates of "core" inflation, included as a state variable in one of the UCMs, are also provided

I. Introduction

Importance of a potential output series for analysing macroeconomic phenomena cannot be overemphasized. It not only enables policy evaluation studies (e.g. analysis of Taylor-type rules for monetary policy), it also helps in 'what if' analysis in both structural and reduced form models (e.g. in VARs for monetary policy analysis, modeling inflation using structural models). Also many phenomena are much better understood with output taken as deviation from a long-run trend (e.g. Phillips curve trade off studies).

Unlike in developed countries like Canada, England, and the US, as of now no official output gap series exist for India. Rao, Fernandes and Deshpande (1990) earlier estimated potential output for India, but no attempts were made to extend the series beyond '90s. In this study, taking output series constructed by Virmani and Kapoor (2003), an unobserved component model *(UCM)* approach is used to create a potential output series for India for the period 1983Q1 – 2001Q4.

The plan of the paper is as follows. After a brief literature review in **section II**, unobserved components models *(UCMs)* as used in the study are specified in **section III**. Results are presented in **section IV.** Sensitivity analysis of the estimated trend using the three *UCMs* to data revisions and comparisons with the Hodrick and Prescott (1980, hereafter *HP*) and the modified *HPA* filter is done in **section V. Section VI** concludes.

II. Estimating Potential Output

Ideally one would like to have a series for potential output which truly captures the steady state level of the economy corresponding to the long run aggregate supply curve. It is not surprising, however, that this approach is not in vogue. Not only are the data requirements stupendous, the size of a structural econometric model required for such a study, lags associated with the measurement of the variables (not to mention the noise and the data revisions problems) makes it both unwieldy and impracticable.

For reasons of speed and ease of estimation time-series based methods have gained popularity, most popular being the *HP* filter and the approximate band pass filter of

Baxter and King (1995, hereafter *BP*). Taken together, Kuttner (1994), Amant and van Norden (1997), Kichian (1999), Domenech and Gomez (2003) and Rennison (2003) provide a comprehensive survey of techniques used in literature for estimating potential output.

The problem in using filters of the likes of *HP* and *BP* is that they are purely empirical in nature and are essentially *ad hoc* solution to the problem of trend estimation. If only estimation of a long-run trend was the concern, the time series based techniques of *HP*-based filters provide quite quick and reliable estimates[1]. However, as Kuttner (1994) argues, *"main drawback to all these is the lack of substantive economic content.[2]"* He uses a latent variables approach to model the unobserved potential output.

Watson (1986) and Clark (1987) were amongst the first to use *UCM* approach to estimate potential output. Kuttner (1994) extended the idea and specified *"potential as the level of output at which inflation is constant.[3]"* Thus, by exploiting a backward looking Phillips curve, Kuttner (1994) explicitly modeled inflation as a function of the output gap, thereby giving an economic interpretation to the measure thus constructed.

Furthering the idea of Kuttner (1994), Domenech and Gomez (2003) include "core" inflation, the *NAIRU*, and the structural investment rate also in their state space formulation (hereafter SSF). Thus, using an extended *UCM*, they are able to extract information about cyclical output from unemployment and investment series also, thereby adding to the economic content of the model.

In this study two different *UCMs* are used on the lines of Watson (1986) and Clark (1987) and Kuttner (1994) and Domenech and Gomez (2003). Details follow in the next section.

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[1] The estimates are still sensitive to the end-of-the-sample problems

[2] K. Kuttner (1994), "Estimating Potential Output as a Latent Variable," *Journal of Business and Economic Statistics,* 12, 3, p. 362

[3] *ibid,* p. 364

III. Model Specification

- *Output:* Following Watson (1986), output is separated into a trend and a cycle. The trend component is assumed to follow a random walk with drift and the cyclical component is assumed to follow an *AR (2)* process (much popular with the real business cyclical theorists; see Romer, 1996, Ch. 4). Thus, (natural logarithm of) output is specified as:

$$
\begin{aligned}
& y_t = y_t^* + z_t \\
& y_t^* = a + y_{t-1}^* + \varepsilon_t \\
& z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \eta_t \\
& \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \\
& \eta_t \sim N(0, \sigma_\eta^2)
\end{aligned}
\qquad [1]
$$

- *Inflation:* As found by Kuttner (1994) for the U.S., a parsimonious backward looking Phillips curve specification with *MA(2)* errors fits well for inflation in India too[4]:

$$
\begin{aligned}
& \pi_t = \pi_t^* + \beta z_{t-1} + \nu_t + \delta_1 \nu_{t-1} + \delta_2 \nu_{t-2} \\
& \pi_t^* = \pi_{t-1}^* + \varsigma_t \\
& \nu_t \sim N(0, \sigma_\nu^2) \\
& \varsigma_t \sim N(0, \sigma_\varsigma^2)
\end{aligned}
\qquad [2]
$$

where, following Domenech and Gomez (2003), core inflation (π_t^*) is modeled as a random walk without drift.

Note how 'restriction' on the coefficient of core inflation as above allows for its interpretation as that level of inflation when the output gap, z_{t-1} is zero. If in first equation in *[2]*, z_{t-1} is 0, with $E(\nu_t) = 0$, it follows that $E(\pi_t) = \pi_t^*$.

4 Other specification for inflation were also checked; *MA(2)* was selected using the *general to specific* criterion

Equations *[1]* and *[2]* can be conveniently cast as a State Space Model (SSM), facilitating estimation of the latent variables by Maximum Likelihood (ML) using Kalman Filter. Details can be found in Harvey (1993). For above specification, the SSM is:

$$\alpha_t = c + A\alpha_{t\text{-}1} + \xi_t$$

$$y_t = H\alpha_t \qquad [3]$$

with the state vectors and system matrices in the two models given as below:

- *UCM-1:* On the lines of Watson (1986) and Clark (1987)

$$\alpha_t = \begin{bmatrix} y_t^* \\ z_t \\ z_{t-1} \\ z_{t-2} \end{bmatrix} \quad c = \begin{bmatrix} a \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \xi_t = \begin{bmatrix} \varepsilon_\tau \\ \eta_\tau \\ 0 \\ 0 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$$

- *UCM-2* and *UCM-3:* On the lines of Kuttner (1994) and Domenech and Gomez (2003), with inflation alternatively based on WPI-All Commodities Index and a Trimmed Mean

$$\alpha_t = \begin{bmatrix} y_t^* \\ z_t \\ z_{t-1} \\ z_{t-2} \\ \pi_t^* \\ v_t \\ v_{t-1} \\ v_{t-2} \end{bmatrix} \quad c = \begin{bmatrix} a \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \xi_t = \begin{bmatrix} \varepsilon_t \\ \eta_t \\ 0 \\ 0 \\ \varsigma_t \\ v_t \\ 0 \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 1 & 1 & \delta_1 & \delta_2 \end{bmatrix}$$

IV. Estimation and Results

After running the Kalman Filter recursions as given in Harvey (1993), the state vector along with their associated Mean Squared Errors *(MSEs)* and the hyperparameters can be estimated using ML. The likelihood function is proportional to:

$$L(\theta) = -ln\,|\,F\,| - \sum_{t=1}^{n} e_t^{'} F_t^{-1} e_t \qquad [4]$$

where θ is the vector of the hyperparameters, and *F* is the *MSE* associated with error, *e*.

To estimate the vector of hyperparameters, we minimize the negative of the likelihood function $L(\theta)$ using the Nelder-Mead simplex search method available in MATLAB[5]. Although Nelder-Mead is one of the slower search routines, it is more reliable provided the initial values are not too off-mark, which is not a concern for the problem at hand.

Data

For output, quarterly estimates of GDP at factor cost (1993-94 = 100) constructed by Virmani and Kapoor (2003) have been used after adjusting seasonally by the TRAMO/SEATS[6] method[7]. Inflation is alternatively taken to be based on seasonally adjusted WPI-All Commodities (1993-94 = 100) and 49/50 Trimmed Mean[8].

Initialization of the Hyperparameters

Running the *HP* filter on the output and the inflation series, and estimating *OLS* for models in *[1]* and *[2]* gives initial estimates of the hyperparameters. Results are reported in *Table 1* below.

Table 1
Initialization of the Hyperparameters

Hyperparameter	φ_1	φ_2	a	β	δ_1	δ_2	σ_ε^2	σ_η^2	σ_ς^2	σ_ν^2
Initial Value	0.54	0.18	-0.0094	-0.2	-0.7	-0.2	0.0000012	0.00011	0.0000018	0.002

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5 Using the function *fminsearch* available in the Optimization Toolbox of MATLAB 6.5
6 Time Series Regressions with *ARIMA* Noise/Signal Extraction in *ARIMA* Time Series
7 Using the software DEMETRA made available by the European Statistical Institute (EUROSTAT)
8 See Virmani (2003) for selection of the optimal trimming pattern

Initialization of the State Vector

Since both potential output and core inflation have been modeled as nonstationary, unlike for a stationary state space model, initial conditions for the Kalman Filter are not well defined. However, since we have first estimates for potential output from running the HP filter, and that of output gap from the *OLS* estimates, we can treat the initial condition as 'known' for our purpose. Taking first three values from the *HP* filtered output series, cyclical output is initialized as the residual, $y_t - y_t^*$. For the *MA* terms corresponding to inflation their expectation (zero) is used to for initialization. *MSE* of the initial state vector (taken to be diagonal) are taken from OLS estimates from *[1]* and *[2]*. Since inflation and cyclical output have been modeled as *MA(2)* and *AR(2)* process respectively, essentially filtering starts from the fourth observation. Initial values are reported in *Table 2* below.

Table 2
Initialization of the State Vector

State Variable	y_t^*	z_t	z_{t-1}	z_{t-2}	π_t^*	v_t	v_{t-1}	v_{t-2}
Initial State Value (α_0)	11.67	0.0056	0.0032	0.0093	0.061	0	0	0
Initial State MSE (P_0)	0.096	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0002

Results and Discussion

Results from the three *UCMs* are reported in *Table 3* below[9]. Filtered and Smoothed series from the three models are plotted in *Figure 1,* along with comparisons with results from the HP filter and the modified *HPA* filter. *HPA* is *HP* filter on extended series using a suitably selected *ARIMA* model. Kaiser and Maravall (2000) show using Monte Carlo experiments that *HPA* is less sensitive to end of sample observations. For output data used in the study, an *IMA(1)* was found suitable. "Core" inflation from *UCM-2* and *UCM-3* are compared against estimates from the *HP* filter and the 49/50 Trimmed Mean in the last quadrant of *Figure 1.*

Results from all the models are broadly in agreement, especially at the turning points. Though, there is significant divergence at the end of the sample when results are compared with *HPA*. As would be expected, when trimmed mean is used as the numeraire for inflation, estimates of output gap are quantitatively smaller. From the

[9] For smoothing Fixed-interval algorithm was used

last quadrant in *Figure 1*, a striking feature is 'over-estimation' of "core" inflation when WPI-All Commodities is used as the measure for inflation (in *UCM-2*), suggesting that high noise in the inflation series (as shown by Virmani (2003) in a detailed statistical analysis of the components underlying the WPI-All Commodities index) could have possibly distorted estimates of long-run trend.

Table 3
Maximum Likelihood Estimates of the Hyperparameters

Hyperparameter	φ_1	φ_2	a	β	δ_1	δ_2	σ_ε^2	σ_η^2	σ_ς^2	σ_ν^2
UCM – 1	0.66	0.21	0.014	-	-	-	0.0000009	0.00013	-	-
UCM – 2	0.57	0.34	0.014	-0.39	0.19	-0.24	0.0000022	0.00013	0.000002	0.0022
UCM – 3	0.63	0.27	0.014	-0.11	0.19	0.22	0.000014	0.00011	0.000006	0.00011

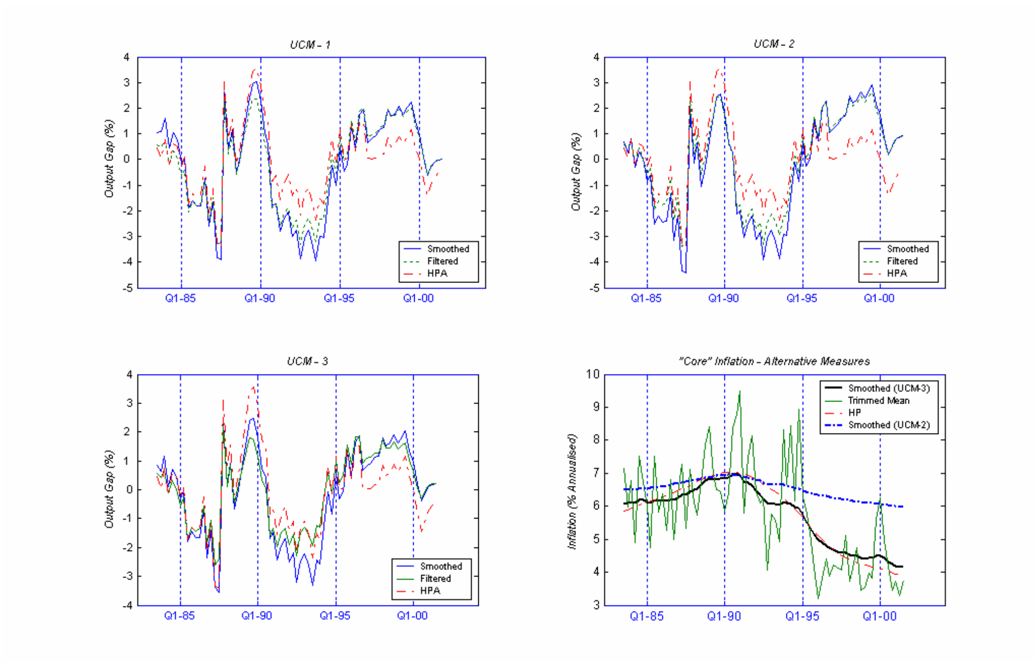

Figure 1

V. Sensitivity Analysis and Out-of-Sample Performance of the *UCMs*

Sensitivity Analysis to Data Revisions

It is well known that contemporaneous national income data undergoes various revisions before finalization. Orphanides and van-Norden (1999) among others have shown that policy suggestions using Taylor-type rules are highly sensitive to vintage of the data used. Thus, it is important to have a measure for potential output that is not very sensitive to data revisions. In this section robustness of different measures of potential output is tested to end-of-sample data revisions. In particular, standard deviation of change in potential output series is calculated for the three *UCMs*, the *HP* filter and the modified *HPA* filter. However, this test on Indian data can at best be illustrative, because it has only been four years since CSO has started releasing quarterly data[10], thereby limiting the 'number' of revised data to only four. Out of that, it was noticed that first release of quarterly estimates were revised extraordinarily. Disregarding first two revisions, this leaves us with only the penultimate year on which sensitivity of results to data revisions can be tested. This is a problem because ideally one would want to see revisions over a sufficient length of time to be able to notice the sensitivity of the potential output series to data revisions.

Table 4 below lists the standard deviation in the changes in the series when compared with data of the penultimate vintage. Smoothed estimates from *UCMs* have been used for comparison. Potential output series estimated using the two vintage of data for all the models are presented in *Figure 2.* Estimates from *UCM-2* look to be most sensitive, and *HP/HPA* least, but as argued above, these comparisons are at best illustrative. For the period prior to 1996 only a single estimate exist, and not the quick, advanced and revised estimates of quarterly output which now CSO makes available since 1999 .

Table 4
Comparison of Standard Deviation in Changes in Potential Output on Data Revisions

Method	*HP*	*HPA*	*UCM-1*	*UCM-2*	*UCM-3*
Standard Deviation	0.0039	0.0026	0.0019	0.0091	0.0047

Out-of-sample Performance of the UCMs

For the out-of-sample performance of the *UCMs*, 'smoothed' predictions of output and inflation from the *UCMs* are compared against actual. In-sample comparison is presented in *Figure 3.* Clearly, the Kalman Filter does a good job of using the information contained in the sample.

[10] First appearing in National Account Statistics, 1999

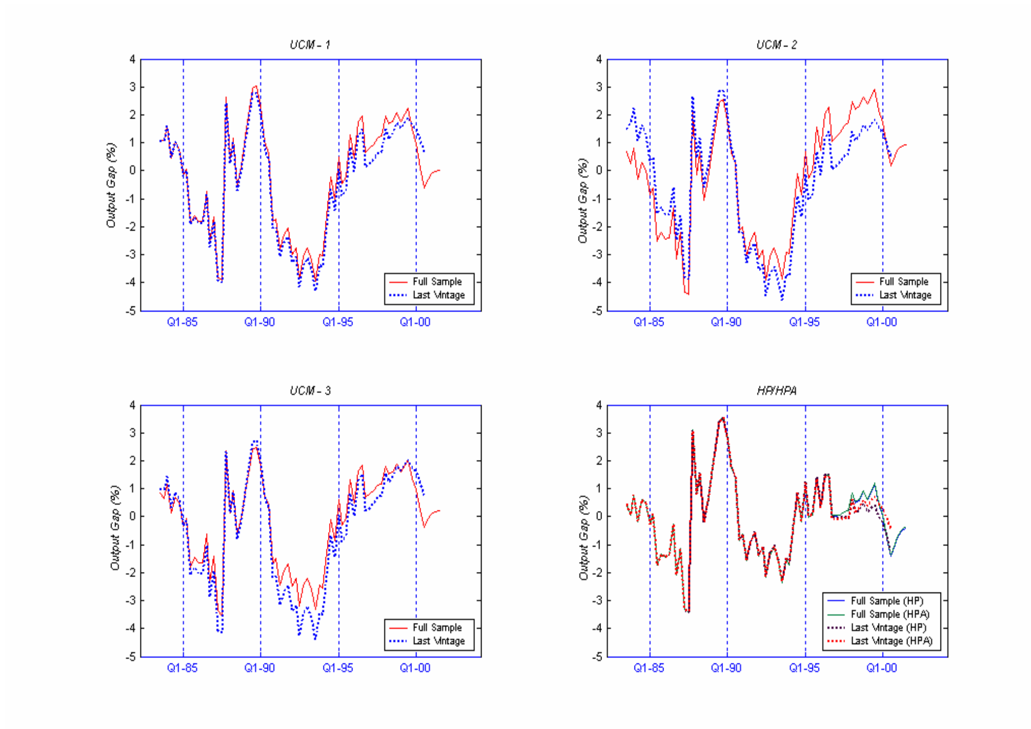

Figure 2

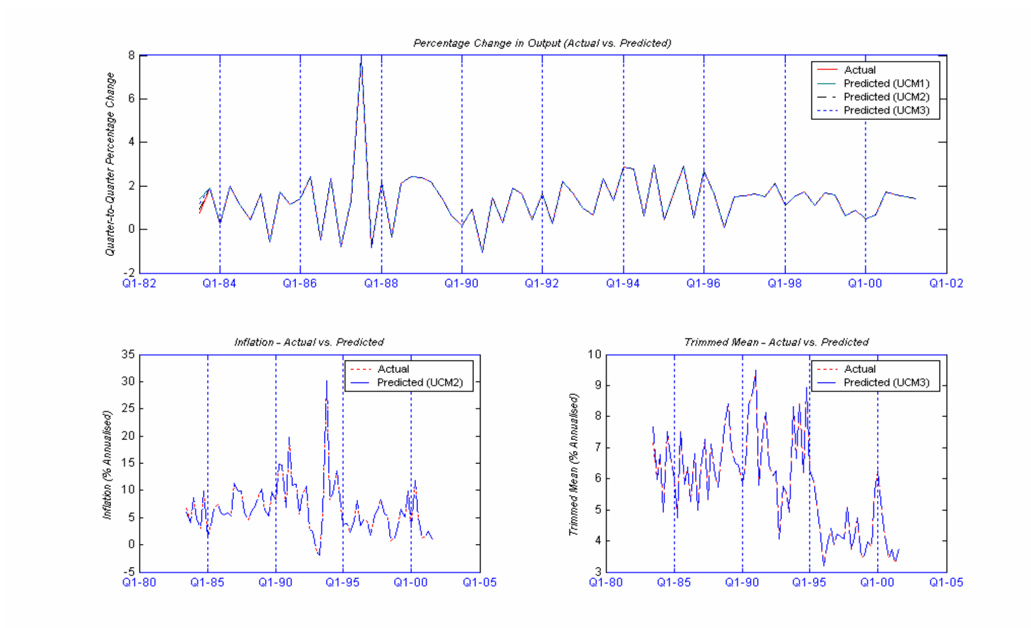

Figure 3

For out-of-sample performance of the *UCMs*, the Kalman Filter recursions are run to get the predicted values and the associated prediction *MSE*. Using the *MSEs*, density forecasts of quarter-to-quarter percentage growth in output for subsequent eight quarters (starting 2002Q1) are plotted in the top half of *Figure 4* below[11]. The density shown covers roughly 95% percent of the probability distribution (with increments of 0.5σ till $\pm 2.5\sigma$)[12] – with 'darker' bands indicating region of higher probability. Since as of now only one out-of-sample observation exists for output (CSO has only recently released 'first' estimates for GDP at factor cost for 2002Q1. From Virmani (2003), however, data on WPI at 1993-94 prices and the associated trimmed mean are available for four subsequent quarters, which are plotted as 'circles' in the lower half of *Figure 4*. Though not of much relevance in the context of measurement of potential output, such 'fan charts' are much in vogue with central banks across the world[13] to communicate their view of future inflation and output.

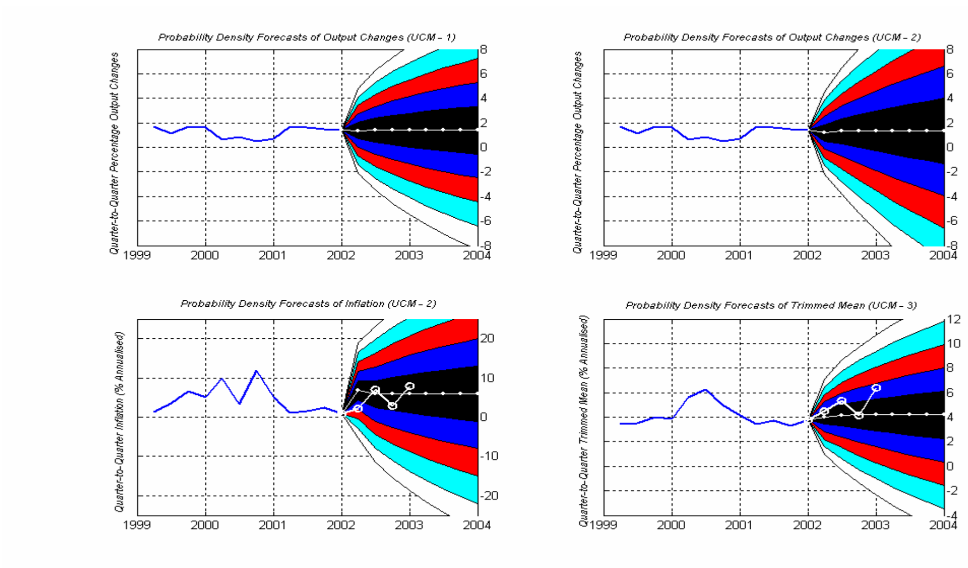

Figure 4

VI. Conclusion

Estimates of potential output/output gap for India have been provided using unobserved components model approach. Broadly results are in agreement. Though results from *UCM-1* and *HPA* are quantitatively superior, since *UCM-3* uses more information and is void of noise in high frequency inflation data, its use is recommended to estimate output gap. For a thorough validation of *UCM-3*, however, we must wait till we have more releases of data from the CSO.

[11] Performance of *UCM-2* and *UCM-3* was found to be quite similar, hence only one is shown
[12] here σ is square root of the MSE of the predicted values
[13] Such releases have become official statements with Sveriges Riksbank, Sweden and the Bank of England, U.K

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