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Bertrand-Edgeworth equilibrium with a large number of firms

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Abstract

We examine a model of price competition where the firms simultaneously decide on both price and quantity, and are free to supply less than the quantity demanded. We demonstrate that if the tie-breaking rule is 'non-manipulable', then, for a large class of rationing rules, there is a unique equilibrium in pure strategies whenever the number of firms is large enough. We then show that the 'folk theorem' of perfect competition holds. Finally, we examine if the results go through when the firms are asymmetric, or produce to order.

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1 Introduction

Let us consider a Bertrand duopoly where the firms decide on both their price and output levels and the firms are free to supply less than the quantity demanded. Edgeworth (1897) argues that in such models equilibria in pure strategies may not exist.[1] In this paper we seek to establish that if the number of firms is large enough, then, for a 'large' class of residual demand functions, there exists a unique equilibrium in pure strategies. Moreover, this equilibrium exhibits some interesting limit properties.

We focus on the case where the firms make their price and output decisions simultaneously, though we also examine the model where the firms produce to order. We examine a class of residual demand function with a rationing rule that is satisfied by almost 'all' rationing rules (except the proportional one) and a 'non-manipulable' tie-breaking rule. Suppose that several firms are charging the same price. We say that the tie-breaking rule is 'non-manipulable' if, by increasing their output level, none of these firms can increase the residual demand coming to it.

In this paper we allow the price level to vary over a grid, where the size of the grid can be arbitrarily small. There are generally two problems associated with the existence of pure strategy equilibrium under price competition. The first reason is technical and has to do with the well known open-set problem. The second one has to do with the fact that the profit function of a firm may not be quasi-concave in its own price. The grid assumption allows us to side-step the open set problem, and solve, at least when the number of firms is large enough, what we believe is the essential Edgeworth paradox. This assumption can also be motivated by appealing to the practice of integer pricing, or to the fact that there are minimum currency denominations. Some other papers that model such discrete pricing

[1] See Dixon (1987), or Friedman (1988) for formal statements of the problem, often referred to in the literature as the Edgeworth paradox.

include, Dixon (1993), Harrington (1989), Maskin and Tirole (1988), Ray Chaudhuri (1995), Roy Chowdhury (1999) and Roy Chowdhury (2002).[2]

We demonstrate that if the number of firms is large enough, then a unique Nash equilibrium exists. We then discuss the limit properties of this equilibrium as one takes the number of firms to infinity. However, relative to market demand, firm size is kept constant.[3] We find that in the limit as the grid size becomes very small, and the number of firms becomes very large, the price level approaches the competitive one and the output level of each firm becomes vanishingly small.

This result is a vindication of the 'folk theorem' of perfect competition, which suggests that the perfectly competitive outcome can be interpreted as the limit of some oligopolistic equilibrium as the number of firms becomes large. While this issue has been thoroughly investigated in the context of Cournot competition,[4] in the Bertrand framework this question remains relatively unexplored. In our model the competitive price is obtained in the limit even though firms are price-setters, thereby providing a non-cooperative foundation for perfect competition in the context of price competition.

We then go on to argue that similar results hold even if the firms play a two stage game, where in stage 1 the firms decide on their price, and in stage 2 they decide on their output.

We next examine the case where the cost functions are asymmetric. The results for the symmetric case generalize in a natural fashion when the marginal cost at zero is the same for all firms. If the marginal cost at zero is different for different types, then the earlier results go through if it is the

[2]In models with discrete strategy spaces, Dasgupta and Maskin (1986a) discuss the sensitivity of equilibrium outcomes to the size of the grid.

[3]Other papers to employ this limiting procedure include Ruffin (1971) (in case of Cournot competition) and Tasnádi (1999a) (in case of price competition).

[4]See, for example, Novshek (1980) and Novshek and Sonnenschein (1983).

number of 'efficient' firms that is taken to infinity. Otherwise, an equilibrium may fail to exist.

We then relate our paper to the literature.

There are different ways of modelling a game of price competition. Under the production to stock (or PTS) approach, the firms simultaneously decide on both their price and output levels. One way to interpret this game is as one with advance production, so that firms must decide on their output levels before trading starts. Thus they make their price and output decisions without knowing the price and output decisions of the other firms. Retail markets are often characterized by such production conditions (see Mestelman et al. (1987)).

This framework has been examined, among others, by Dixon (1987), Dixon (1993) and Maskin (1986).[5] While Maskin (1986) proves existence in mixed strategies, Dixon (1987, 1993) look for equilibrium in pure strategies. Dixon (1987) introduces the notion of menu costs and demonstrates that in the presence of such costs there is an *epsilon*-Nash equilibrium in pure strategies if the economy is replicated. Dixon (1993) examines the existence of pure strategy equilibria when costs are convex and price varies discretely. There two papers, however, differ from our paper in several respects. To begin with the replication procedure is quite different. While under our approach the market demand is kept unchanged, Dixon (1987, 1993) replicate the market demand function as well, so that individual firms become small relative to market demand. Moreover, while Dixon (1993) examines a parallel residual demand function, our results apply to almost 'all' residual demand functions (except the proportional one), provided the tie-breaking rule is 'non-manipulable'. In terms of results, Dixon (1993) finds that equilibria are non-unique and may not exist for some parameter

[5] In fact, Shubik (1955) formulates a production to stock game, but does not analyze it, merely pointing out the difficulties in analyzing such a game. Of course, both Maskin (1986) and Shubik (1955) also examine other game forms.

values. Moreover, the highest equilibrium price could be arbitrarily far from the competitive price. In contrast, we find that for large markets, equilibrium is unique, always exists and the equilibrium price approximates the competitive price arbitrarily closely.[6]

Under the production to order (or PTO) approach, the firms first simultaneously decide on their price levels and then on their output levels. Papers in this framework include Dixon (1990), Maskin (1986), Yoshida (2002) etc. Maskin (1986) proves existence in mixed strategies, while Yoshida (2002) characterizes the symmetric mixed strategy equilibrium for a duopoly with identical, strictly convex costs. On the other hand, Dixon (1990) shows that if there are costs of turning away customers then a pure strategy equilibrium exists if the economy is replicated.

One interesting class of models assumes that cost functions are linear and capacity constrained. Firms compete over prices and, given prices, are willing to supply till capacity.[7] Papers in this framework include Allen and Hellwig (1986, 1993), Dasgupta and Maskin (1986b), Osborne and Pitchik (1986), Vives (1986) etc.[8] These papers solve for equilibria in mixed strategies, sometimes using the fixed point theorems for discontinuous games developed by Dasgupta and Maskin (1986a).

In an interesting paper Mestelman et al. (1987) use laboratory experiments to compare the PTS approach with the PTO one. They find that under the PTS approach, market prices tend to be lower compared to the PTO approach. Moreover, with repeated play, the market price under the

[6] Dixon (1992) examines a model where firms announces a price and the maximum quantity it is willing to supply at that price. In such models even two firms are sufficient to generate the competitive outcome.

[7] In fact, in some of these papers the cost of production is assumed to be zero. In that case this class of models can be interpreted as one of price competition with a given stock of output (see Dasgupta and Maskin (1986b)).

[8] Tasnádi (1999a) examines a PTS framework with linear and capacity constrained cost functions.

PTS approach converges quickly to the competitive one.

Finally, Davidson and Deneckere (1986) and Kreps and Scheinkman (1983) assume that firms first decide on their capacity levels and then on prices. This approach differs from the PTS approach in that the firms know the capacity level of the other firms when they make their pricing decisions.

The rest of the paper is organized as follows. Section 2 introduces and analyzes the basic model. Section 3 considers the production to order game. Section 4 extends the analysis to the asymmetric case. Section 5 concludes. Some of the technical details can be found in Appendix 1 and 2.

2 The Model

There are n identical firms, all producing the same homogeneous good.[9] The market demand function is $q = d(p)$ and the common cost function of all the firms is $c(q)$.[10]

Throughout we maintain the following assumptions on the demand and the cost functions.

Assumption 1. $d(p)$ is negatively sloped and intersects the price axis at some price $p^{\max}$, where $0 < p^{\max} < \infty$.

Assumption 2. The cost function $c(q)$ is twice differentiable, increasing and strictly convex. Moreover, $p^{\max} > c'(0)$.

We assume that prices vary over a grid. Define the set of feasible prices $F = \{p_0, p_1, \cdots\}$, where $p_0 = 0$, and $p_i = p_{i-1} + \alpha$, $\forall i \in \{1, 2, \cdots\}$, where $\alpha > 0$.

[9] Another strand of the literature examines price competition with differentiated products, e.g. Benassy (1989), Friedman (1988), Simon (1987) etc.

[10] Like most of the literature, this paper is set in a partial equilibrium framework. Papers that do analyze price competition in a general equilibrium framework include Dubey (1982) and Simon (1984).

The i-th firm's strategy consists of simultaneously choosing both a price $p_i \in F$ and an output $q_i \in [0, \infty)$.[11] All firms move simultaneously. We solve for the pure strategy Nash equilibrium of this game.

We then specify the residual (or the contingent) demand function. Let $R_i(P, Q)$ denote the residual demand facing the i-th firm when the price and the quantity vectors are given by $P = \{p_1, \cdots, p_n\}$, and $Q = \{q_1, \cdots, q_n\}$. Define $\underline{p}$ to be the minimum element in P such that at least some of the firms charging this price has a strictly positive level of output. Then if the total production of all firms charging $\underline{p}$ is greater than $d(\underline{p})$, then we assume that all firms who charge a price greater than $\underline{p}$ obtain no demand, thus ensuring that $R_i(P, Q)$ is indeed a residual demand function. Moreover, for any price p, the sum of the residual demands facing all the firms charging this price p can be at most $d(p)$.

We then impose some more structure on the residual demand function. Assumption 3(i) is a restriction on the *rationing rule*, whereas Assumption 3(ii) is a restriction on the *tie-breaking rule.* (In Appendix 2 we provide an example of a residual demand function satisfying Assumption 3.)

Assumption 3. (i) Let $r_i(p_i, p, n)$ denote the residual demand facing the i-th firm if, firm i charges a price $p_i \geq p$, and the other $(n-1)$ firms charge p and produce $\frac{d(p)}{n}$. Then $r_i(p_i, p, n)$ is twice differentiable, decreasing and (weakly) concave in p_i. Moreover, $\forall p < p^{\max}$, $\lim_{n \to \infty} r_i'(p_i, p, n)|_{p_i = p} < 0$, where $r_i'(p_i, p, n) = \frac{\partial r_i(p_i, p, n)}{\partial p_i}$.

(ii) Consider a situation where m of the firms charge $\tilde{p}$, and all other firms either charge prices that are strictly greater than $\tilde{p}$, or charge prices that are strictly lower than $\tilde{p}$, but have an output level of zero. Then the residual demand facing all the firms charging $\tilde{p}$ is at least $\frac{d(\tilde{p})}{m}$.

(a) The residual demand facing the i-th firm charging $\tilde{p}$ is exactly $\frac{d(\tilde{p})}{m}$

[11] Grossman (1981) and Mandy (1993), among others, consider a model where firms use supply schedules as strategies.

whenever the other firms charging $\tilde{p}$ supply at least $\frac{d(\tilde{p})}{m}$.

(b) If k ($\leq m$) of the m firms charging $\tilde{p}$ supplies nothing, then the residual demand facing the other $m-k$ firms charging $\tilde{p}$ is at least $\frac{d(\tilde{p})}{m-k}$. The residual demand facing such a firm is exactly $\frac{d(\tilde{p})}{m-k}$ whenever the other firms charging $\tilde{p}$ and supplying a positive amount, supply at least $\frac{d(\tilde{p})}{m-k}$.

We next relate Assumption 3 to the literature.

To begin with we claim that Assumption 3(i) is satisfied by 'all' rationing rules, except the proportional one. Using the combined rationing rule introduced by Tasnádi (1999b), the residual demand $r_i(p_i, p, n)$ can be expressed as $\max\{d(p_i) - \frac{n-1}{n}d(p)[(1-\lambda)\frac{d(p_i)}{d(p)} + \lambda],\ 0\}$, where $\lambda \in [0,1]$. Note that for $\lambda = 1$ we have the efficient rationing rule, whereas for $\lambda = 0$ we have the proportional rationing rule. For intermediate values of λ other rationing rules emerge.[12] Clearly, if $d(p_i)$ is concave then $r_i(p_i, p, n)$ is decreasing and concave in p_i.[13] Moreover, notice that $\lim_{n\to\infty} r_i'(p_i, p, n)|_{p_i=p} = \lambda d'(p)$. Hence $\forall\lambda > 0$, $\lim_{n\to\infty} r_i'(p_i, p, n)|_{p_i=p} < 0$. Thus Assumption 3(i) is satisfied by 'all' rationing rules barring the proportional one.

We then consider Assumption 3(ii). Observe that the firms charging $\tilde{p}$ cannot increase the residual demand coming to them by increasing their output level beyond $\frac{d(\tilde{p})}{m}$. Thus Assumption 3(ii) formalizes the notion that the residual demand function is 'non-manipulable'.

It is easy to see that Assumption 3(ii) is not inconsistent with Maskin (1986), which provides one of the most general formulations of the tie-breaking rule.[14] In fact, the second of the two examples of tie-breaking

[12] See Tasnádi (1999b) for an interpretation of the combined rationing rule.

[13] This follows since $r_i(p_i, p, n)$ can be re-written as $\max\{d(p_i)[1 - \frac{n-1}{n}(1-\lambda)] - \frac{\lambda(n-1)}{n}d(p)],\ 0\}$.

[14] In fact papers that solve for mixed strategy price equilibria often imposes very weak restrictions on the tie-breaking rule e.g. Allen and Hellwig (1986), Maskin (1986), Vives (1986) etc., though there are some exceptions e.g. Allen and Hellwig (1993), Dasgupta and Maskin (1986b), Osborne and Pitchik (1986), etc.

rules in Maskin (1986) is very similar to Assumption 3(ii). If $n = 2$, and both the firms charge $\tilde{p}$, then this tie-breaking rule specifies that the residual demand facing the i-th firm is $\alpha_i d(\tilde{p})$, where α_i is some exogenously given weight. Clearly, if the weights are symmetric then the residual demand is simply $\frac{d(\tilde{p})}{2}$.[15] In contrast to Assumption 3(ii), however, this formulation does not allow for the possibility that if one of the firms supplies less than $\frac{d(\tilde{p})}{2}$, then the unmet residual demand may spill-over to other firm, so that the residual demand facing the other firm may be greater than $\frac{d(\tilde{p})}{2}$.

Such spill-overs of unmet residual demand is, in fact, explicitly allowed for by Davidson and Deneckere (1986) and Kreps and Scheinkman (1983).[16] Both these papers consider a duopoly model. The residual demand facing firm i, when $p_1 = p_2$, is assumed to be $\max\{\frac{d(p_i)}{2}, d(p_i) - q_j\}$. Clearly, the residual demand facing firm i is independent of the amount produced by firm i, so that it is 'non-manipulable'. Moreover, if $q_j < \frac{d(p_i)}{2}$, then the residual demand facing firm i is strictly greater than $\frac{d(p_i)}{2}$. Thus the tie-breaking rule adopted in the present paper can be considered to be a generalization of the Davidson-Deneckere-Kreps-Scheinkman one.

While Assumption 3(ii) is consistent with much of the literature, it is, of course, a serious restriction. There are quite a few papers in the literature where the tie-breaking rule is 'manipulable', e.g. Allen and Hellwig (1993), Osborne and Pitchik (1986) and Tasnádi (1999b).[17]

The supply function of a firm charging a price p is given by $\min\{c'^{-1}(p), R_i(P,Q)\}$.[18] Thus we follow Edgeworth (1897) in assuming that firms are

[15]This formulation is, in fact, adopted by Dixon (1984), Levitan and Shubik (1972) and Yoshida (2002). It is also widely used in the literature on Bertrand-Chamberlin price competition where the firms are assumed to supply the whole of the demand coming to them, e.g. Dastidar (1995), Novshek and Roy Chowdhury (2003), Vives (1999), etc.

[16]This tie-breaking rule is also discussed in Vives (1999).

[17]Roy Chowdhury (2000) examines a Bertrand-Edgeworth model where the tie-breaking rule is 'manipulable'.

[18]Since the cost function is strictly convex, $c'^{-1}(p)$ is well defined.

free to supply less than the quantity demanded, rather than Chamberlin (1933), who assumes that firms meet the whole of the demand coming to them.

Next let p^* be the minimum $p \in F$ such that $p > c'(0)$.[19] Thus p^* is the minimum price on the grid which is strictly greater than $c'(0)$. Since $p^* \in F$, let $p^* = p_j$ for some integer j. We are going to argue that for n large, p^* can be sustained as the unique Nash equilibrium price of this game.

Moreover, let $q^* = c'^{-1}(p^*)$ and let n^* be the smallest possible integer such that $\forall N \geq n^*$,

$$\frac{d(p^*)}{N} < c'^{-1}(p^*) = q^*.$$

Thus for all N greater than n^*, if a firm charges p^* and sells $\frac{d(p^*)}{N}$, then the price p^* is strictly greater than marginal costs.

Next let $\hat{n}$ be the smallest possible integer such that $\forall N \geq \hat{n}$,

$$r_i'(p^*, p^*, N)[p^* - c'(\frac{d(p^*)}{N})] + \frac{d(p^*)}{N} < 0.^{20}$$

Definition. $N_1 = \max\{n^*, \hat{n}\}$.

We next define $\tilde{\pi}$ to be the profit of a firm that charges p^* and sells $\frac{d(p^*)}{n^*}$. Thus $\tilde{\pi} = \frac{p^* d(p^*)}{n^*} - c(\frac{d(p^*)}{n^*})$. Since $\frac{d(p^*)}{n^*} < q^*$, it follows that $\tilde{\pi} > -c(0)$, where $-c(0)$ denotes the profit of a firm which does not produce at all.

Now consider some $\overline{p}_i \in F$, such that $\overline{p}_i > p^*$. Let $\overline{q}_i$ satisfy $\overline{p}_i = c'(\overline{q}_i)$. Next consider a firm that charges $\overline{p}_i$ and sells $\frac{d(\overline{p}_i)}{k}$. Clearly the profit of such a firm is $\overline{p}_i \frac{d(\overline{p}_i)}{k} - c(\frac{d(\overline{p}_i)}{k})$.

We then define n_i to be the smallest possible integer such that $\forall k \geq n_i$, $\frac{d(\overline{p}_i)}{k} < \overline{q}_i$ and

$$\overline{p}_i \frac{d(\overline{p}_i)}{k} - c(\frac{d(\overline{p}_i)}{k}) < \tilde{\pi}.^{21}$$

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[19] We assume that α is not too large in the sense that $p^* < p^{\max}$.

[20] Notice that $\lim_{n\to\infty}[r_i'(p^*, p^*, n)\{p^* - c'(\frac{d(p^*)}{n})\} + \frac{d(p^*)}{n}] = \lim_{n\to\infty} r_i'(p^*, p^*, n)[p^* - c'(0)]$. Since, $p^* > c'(0)$ and $\lim_{n\to\infty} r_i'(p^*, p^*, n) < 0$ (Assumption 3(i)), this term is negative.

[21] Clearly the left hand side of this inequality is decreasing in k. Moreover, as k goes to

Suppose that in any equilibrium the number of firms charging $\overline{p}_i$, say $\tilde{m}$, is greater than or equal to n_i. Then at least one of these firms would have a residual demand that is less than or equal to $\frac{d(\overline{p}_i)}{\tilde{m}}$. Since $\frac{d(\overline{p}_i)}{\tilde{m}} < c'^{-1}(\overline{p}_i)$, this firm would sell at most $\frac{d(\overline{p}_i)}{\tilde{m}}$ and have a profit less than $\tilde{\pi}$.

Let p_k be the largest price belonging to F such that $p_k \leq p^{\max}$.

Definition. $N_2 = \sum_{i=j+1,\cdots,k} n_i + n^* - 1.$[22]

Proposition 1 below provides a resolution of the Edgeworth paradox.

Proposition 1. *Let $n \geq \max\{N_1, N_2\}$. Then the unique equilibrium involves all the firms charging a price of p^*, and producing $\frac{d(p^*)}{n}$.*

Proof. Existence. From the definition of p^* undercutting is not profitable. We then argue that for the i-th firm, charging a higher price, p_i, is not profitable either.

Notice that since $n \geq n^*$, $\frac{d(p^*)}{n} < c'^{-1}(p^*)$. Hence for any $p_i \geq p^*$,

$$c'^{-1}(p_i) \geq c'^{-1}(p^*) > \frac{d(p^*)}{n} = r_i(p^*, p^*, n) \geq r_i(p_i, p^*, n), \tag{1}$$

where the equality follows from Assumption 3(ii)(a) and the last inequality follows from Assumption 3(i). Since $c'^{-1}(p_i) > r_i(p_i, p^*, n)$, for any $p_i \geq p^*$, the deviant firm always supplies the whole of the residual demand coming to it. Hence the profit of a firm which charges a price p_i $(\geq p^*)$

$$\pi(p_i, r_i(p_i, p^*, n)) = p_i r_i(p_i, p^*, n) - c(r_i(p_i, p^*, n)). \tag{2}$$

Clearly

$$\frac{\partial \pi(p_i, r_i(p_i, p^*, n))}{\partial p_i} = r_i'(p_i, p^*, n)[p_i - c'(r_i(p_i, p^*, n))] + r_i(p_i, p^*, n). \tag{3}$$

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infinity, this term goes to $-c(0) \leq 0$. Thus n_i is well defined.

[22] Notice that the assumption that the demand function intersects the price axis is required for this definition, i.e. while proving uniqueness. It is not required in any of the existence proofs.

Next from equation (1) it follows that $\forall p_i \geq p^*,\ p_i > c'(r_i(p_i, p^*, n))$. Hence from the concavity of $r_i(p_i, p^*, n)$ it follows that $\pi(p_i, r_i(p_i, p^*, n))$ is concave in p_i.[23] Moreover,

$$\frac{\partial \pi(p_i, r_i(p_i, p^*, n))}{\partial p_i}|_{p_i=p^*} = r_i'(p^*, p^*, n)[p^* - c'(\frac{d(p^*)}{n})] + \frac{d(p^*)}{n}. \quad (4)$$

This follows since from Assumption 3(ii)(a) we know that $r_i(p^*, p^*, n) = \frac{d(p^*)}{n}$. Since $n \geq \hat{n}$, we have that $\frac{\partial \pi(p_i, r_i(p_i, p^*, n))}{\partial p_i}|_{p_i=p^*} < 0$. Next, from the concavity of $\pi(p_i, r_i(p_i, p^*, n))$ it follows that $\forall p_i \geq p^*$, the profit of any deviant firm is decreasing in p_i.

Finally, given that all firms supply $\frac{d(p^*)}{n}$, the residual demand facing all firms is exactly $\frac{d(p^*)}{n}$ (Assumption 3(ii)(a)). Given that $\frac{d(p^*)}{n} < c'^{-1}(q^*)$, it is optimal for all the firms to produce exactly $\frac{d(p^*)}{n}$.

Uniqueness. The proof is in several steps.

Step 1. We first claim that there cannot be an equilibrium where the output level of some of the firms is zero. This follows since these firms can always charge p^* and obtain a residual demand of at least $\frac{d(p^*)}{n}$ (Assumption 3(ii)). Since $p^* > c'(0)$, producing a small enough positive output would increase their profit from $-c(0)$.

Step 2. We then argue that there cannot be some p_i $(\in F) > p^*$, such that some of the firms charge p_i and supply a positive amount.

Suppose to the contrary that such a price exists.

This implies that the total number of firms charging p^*, say $\tilde{n}$, can be at most $n^* - 1$. Otherwise the residual demand facing these firms would be exactly $\frac{d(p^*)}{\tilde{n}}$.[24] Since $\frac{d(p^*)}{\tilde{n}} < c'^{-1}(p^*)$, all such firms would supply $\frac{d(p^*)}{\tilde{n}}$ and

[23]This follows since

$$\begin{aligned}\frac{\partial^2 \pi(p_i, r_i(p_i, p^*, n))}{\partial p_i^2} = r_i''(p_i, p^*, n)[p_i - c'(r_i(p_i, p^*, n))] \quad &+ \quad 2r_i'(p_i, p^*, n) \\ &- \quad c''(r_i(p_i, p^*, n))r_i'^2(p_i, p^*, n).\end{aligned}$$

[24]Given that $\frac{d(p^*)}{\tilde{n}} < c'^{-1}(p^*)$, all firms must be supplying at least $\frac{d(p^*)}{\tilde{n}}$. The assertion

the residual demand at any higher price, p_i, would be zero.

Now consider some $p_i > p^*$. Clearly, the number of firms charging p_i is less than n_i. Otherwise, some of these firms would have a profit less than $\tilde{\pi}$. Hence such a firm would have an incentive to deviate to p^*, when it can supply at least $\frac{d(p^*)}{n^*}$ and earn $\tilde{\pi}$. Thus the total number of firms producing a strictly positive amount is less than N_2, thereby contradicting step 1. ∎

The idea behind the existence result is quite simple. Consider a market price of p^*. If the number of firms is large then the residual demand coming to every firm is very small, so that it is residual demand rather than marginal cost which determines firm supply. In that case price would not equal marginal cost, and firms may no longer have an incentive to increase its price level. Assumption 3 specifies a set of conditions under which this is indeed true.

We next examine the limit properties of the equilibrium as the grid size becomes small and the number of firms becomes large. To begin with notice that $\lim_{\alpha \to 0} \; p^*(\alpha) = c'(0)$ and $\lim_{n \to \infty} \; \frac{d(p^*(\alpha))}{n} = 0$. Thus in the limit the equilibrium price approaches the competitive one and the output of each firm becomes vanishingly small.

Hence our results provide a non-cooperative foundation for the theory of perfect competition. In fact, given α, it is sufficient to take $n \geq \max\{N_1(\alpha), N_2(\alpha)\}$ in order to ensure that the equilibrium price is p^*, so that p^* is sustainable for a finite number of firms.[25] This is similar in spirit to the well known result that with linear cost functions the competitive price is obtained whenever $n \geq 2$.

Allen and Hellwig (1986) demonstrate that if the firms are capacity con-

now follows from Assumption 3(ii)(a).

[25] However, under some simplifying assumptions it is easy to show that $\lim_{\alpha \to 0} N_1(\alpha) = \infty$. Assume that $\inf_{p \in [0, p^{\max}]} \; [\lim_{n \to \infty} r_i'(p,p,n)] \geq -B$, where $B > 0$. (Note that this condition is satisfied for the parallel residual demand function). It is now straightforward to show that $\lim_{\alpha \to 0} \hat{n}(\alpha) = \infty$.

strained and the residual demand function is proportional, then, in the limit, the mixed strategy equilibrium converges in distribution to the competitive price. Vives (1986) proves that in case the residual demand is parallel, the convergence is in support as well. For the parallel rationing rule Börgers (1992) shows that iterated elimination of dominated strategies yields prices close to the competitive price.

Dixon (1987, 1990, 1993) also study the limit properties of pure strategy Bertrand equilibria when the economy is replicated. Dixon (1987) shows that any *epsilon*-Nash equilibrium will be approximately competitive if *epsilon* is small enough and the industry is large enough. Dixon (1990) demonstrates that if the industry is large enough then the competitive price will be an equilibrium. Moreover, if costs of turning away consumers are small, then all equilibria will be close to the competitive one. Dixon (1993) provides an example where the highest equilibrium prices could be arbitrarily far from the competitive price.

Note that in the papers discussed above the limiting procedure involves taking firm size, relative to market demand, to zero. In Allen and Hellwig (1986) and Vives (1986) this is done by taking the capacity level of the firms to zero, while in Dixon (1987, 1990, 1993) this is done by replicating the market demand function. Under our approach, however, firm size is kept unchanged.

3 A Two-stage Model

We then examine the case where the firms produce to order. Thus the firms play a two stage game where, in stage 1, the firms simultaneously announce their prices, and in stage 2, they simultaneously decide on their output levels.

In Proposition 2 below we solve for the subgame perfect Nash equilibrium of this game (see Appendix 1 for the proof).

Proposition 2. Assume that $n \geq \max\{\hat{n}, n^* + 1, N_2\}$. Then the following strategies constitute the unique subgame perfect Nash equilibrium of this game:

Stage 1. All firms charge a price of p^*.

Stage 2.

Case (i). Suppose that in stage 1 all the firms charge p^*. Then, in stage 2, all the firms produce $\frac{d(p^*)}{n}$.

Case (ii). Next suppose that in stage 1, $(n-1)$ of the firms charge p^*, while one of the firms charges a price strictly greater than p^*. Then, in stage 2, the firms charging p^* produce $\frac{d(p^*)}{n-1}$, while the output level of the other firm is zero.[26]

In this case also $\lim_{\alpha \to 0} p^*(\alpha) = c'(0)$ and $\lim_{n \to \infty} \frac{d(p^*(\alpha))}{n} = 0$, so that the 'folk theorem' goes through under this formulation as well.

4 Asymmetric Costs

In this section we examine the case where firms are asymmetric.

Deneckere and Kovenock (1996) is one of the very few papers that examine Bertrand-Edgeworth competition in an asymmetric framework. They

[26]Notice that in Proposition 2 we describe the equilibrium strategies in stage 2 for two classes of histories only. Under some simplifying assumptions it is easy to write down the equilibrium strategies in all possible subgames. Assume that the residual demand function is symmetric, i.e. $r_i(p_i, p, m) = r(p_i, p, m)$, $\forall i$. Moreover, let the residual demand at any price p_i only depend on quantities produced by firms who charge prices less than p_i. Now consider the following algorithm.

Step 1. All firms that charge a price strictly less than p^* produce no output.

Step 2. Let the number of firms charging p^* be N^*. Then the equilibrium output level of all such firms is $\min\{c'^{-1}(p^*), \frac{d(p^*)}{N^*}\}$.

Step 3. Let the residual demand facing all firms who charge a price of $p_{j+1} = p^* + \alpha$ be at least R^{j+1}. Then the equilibrium output of all such firms is $\min\{c'^{-1}(p_{j+1}), R^{j+1}\}$. We can inductively write down the output level of the firms who charge higher prices.

explore a price-setting duopoly with the efficient rationing rule where the firms differ in terms of both their unit costs and capacities. They characterize the set of equilibria and then, as an application, re-examine the Kreps and Scheinkman (1983) model with asymmetric costs, demonstrating that the Cournot equilibrium capacity levels need not emerge in equilibrium. In keeping with our approach, however, in this section we shall be interested in the case where the number of firms is large.

Let there be m types of firms with the cost function of the l-th type being $c_l(q)$. The number of type l firms is denoted by n^l, where $\sum_l n^l = n$.

Next let p_l^* denote the minimum $p \in F$ such that $p > c_l'(0)$. Let $r_{il}(p_i, p, n)$ denote the residual demand function facing the i-th firm of type l, when it charges p_i, and all other firms charge p ($\leq p_i$) and supply $\frac{d(p)}{n}$. The residual demand function satisfies an appropriately modified version of Assumption 3.[27] Next define n_l^*, $\hat{n}_l$ and N_1^l in a manner analogous to that of n^*, $\hat{n}$ and N_1 respectively, only taking care to use the cost function of the l-th type, $c_l(q)$, instead of $c(q)$ in the definitions.

Definition. $\tilde{N}_1 = \max_l N_1^l = \max\{n_1^*, \cdots, n_m^*, \hat{n}_1, \cdots, \hat{n}_m\}$.

We require some further notations. Let

$$\tilde{\pi}_l \;=\; \frac{p_l^* d(p_l^*)}{\max_q \; n_q^*} - c_l(\frac{d(p_l^*)}{\max_q \; n_q^*}).$$

Next consider some $p_x \in F$, such that $p_x > p_l^*$. Let q_{lx} satisfy $p_x = c_l'(q_{lx})$. Clearly if a type l firm charges p_x and sells $\frac{d(p_x)}{r}$, then the profit of such a firm is $p_x \frac{d(p_x)}{r} - c_l(\frac{d(p_x)}{r})$.

We then define n_{lx} to be the smallest possible integer such that $\forall r \geq n_{lx}$, $\frac{d(p_x)}{r} < q_{lx}$ and

$$p_x \frac{d(p_x)}{r} - c_l(\frac{d(p_x)}{r}) < \tilde{\pi}_l.$$

[27] Assumption 3(i) should be modified so that $r_{il}(p_i, p, n)$ is twice differentiable, decreasing and (weakly) concave in p_i. Moreover, $\forall p < p^{\max}$, $\lim_{n\to\infty} \; r_{il}'(p_i, p, n)|_{p_i = p} < 0$, where $r_{il}'(p_i, p, n) = \frac{\partial r_{il}(p_i,p,n)}{\partial p_i}$. Assumption 3(ii) requires no modification.

Suppose that in any equilibrium the number of firms charging p_x, say $\tilde{m}$, is greater than or equal to $\max_q n_{qx}$. Then at least one of these firms, say of type l, would have a residual demand that is less than or equal to $\frac{d(p_x)}{\tilde{m}}$. Since $\frac{d(p_x)}{\tilde{m}} < c_l'^{-1}(p_x)$, this firm would supply at most $\frac{d(p_x)}{\tilde{m}}$ and have a profit less than $\tilde{\pi}_l$.

For ease of exposition we shall focus on two cases.

Case (i). $c_1'(0) = c_2'(0) = \cdots = c_m'(0)$.

Note that if, at a given price, any firm finds it profitable to produce a strictly positive amount, then so will all other firms. For this case let us redefine $p^* = p_1^* = \cdots = p_m^*$.

Definition. $\tilde{N}_2 = \sum_{x=j+1,\cdots,k} \max_l n_{lx} + \max_l n_l^* - 1$.

We can now state our next proposition (see Appendix 1 for the proof).

Proposition 3. *Let $c_1'(0) = c_2'(0) = \cdots = c_m'(0)$. If $n \geq \max\{\tilde{N}_1, \tilde{N}_2\}$, then the unique equilibrium involves all the firms charging a price of p^*, and producing $\frac{d(p^*)}{n}$.*

It is easy to see that the 'folk theorem' goes through in this case.

Case (ii). $c_1'(0) < c_2'(0) < \cdots < c_m'(0)$.

Consider any p such that $c_1'(0) < p < c_2'(0)$. While at this price producing a small enough positive level of output is profitable for type 1 firms, firms of other types will not find it profitable to supply a positive level of output. Hence type 1 firms are, in some sense, the most efficient.

Let $p_1^* = p_h$ (say).

Definition. $\hat{N}_2 = \sum_{x=h+1,\cdots,k} n_{1x} + n_1^* - 1$.

Proposition 4 below solves for the equilibrium when the number of type 1 firms is large. The proof, which is quite simple, can be found in the

Appendix 1.

Proposition 4. *Let* $c_1'(0) < c_2'(0) < \cdots < c_m'(0)$. *Assume that* $\alpha < c_2'(0) - c_1'(0)$ *and* $n^1 \geq \max\{N_1^1, \hat{N}_2\}$. *Then the 'unique' equilibrium involves all firms of type 1 charging* p_1^* *and producing* $\frac{d(p_1^*)}{n^1}$. *Firms of all other types have an output level of zero.*

Interpreting $c_1'(0)$ as the competitive price, the 'folk theorem' goes through in this case as well.

Next suppose that $c_1'(0) = c_2'(0) = \cdots = c_j'(0) < c_{j+1}'(0) \leq \cdots \leq c_m'(0)$. Combining Propositions 3 and 4, it is easy to see that if the number of firms of type 1 to j are large enough, then there is a unique equilibrium where all firms of type 1 to j charge $c_1'(0)$, and all other firms have an output of zero.

Finally, consider the case when there are a large number of 'inefficient' firms and the 'efficient' firms are relatively few in number. Unfortunately, no equilibrium may exist even if the number of inefficient firms is very large. The following example illustrates the problems involved.

Example. Let there be two types of firms with $c_1(q) = q^2$ and $c_2(q) = q + q^2$, so that $c_1'(0) < c_2'(0)$. There are 2 firms of type 1 and n^2 firms of type 2. The demand function is $q = 4 - p$, and the residual demand function satisfies an appropriately modified version of Assumption 3. In fact, we assume that the rationing rule is efficient. Let $\alpha = 0.01$, so that $p_1^* = 0.01$ and $p_2^* = 1.01$.

First note that for n^2 large enough, any possible equilibrium must involve all firms of type 2 charging the price p_2^* and supplying the whole of the residual demand coming to them.[28] Given this, the only possible equilibrium must involve both the type 1 firms charging $c_2'(0) = 1$ and supplying $c_1'^{-1}(1) = 0.5$ when they have a profit of 0.25 each.[29] Moreover, since both

[28] The argument essentially mimics the uniqueness part of Proposition 1.

[29] Given that all type 2 firms are charging p_2^*, in equilibrium the type 1 firms cannot be

the type 1 firms supply $c_1'^{-1}(1) = 0.5$, the total amount supplied by the type 2 firms will be 1.99 $(= 4 - p_2^* - 1)$. Next suppose that a type 1 firms deviates to 1.02 $(= p_2^* + \alpha)$. Given that the rationing rule is efficient, it can supply the residual demand 0.49 $(= 4 - p_2^* - \alpha - 1.99 - 0.5)$ and increase its profit level to 0.2597. Hence no equilibrium exists.[30]

5 Conclusion

In this paper we re-examine the non-existence problem associated with pure strategy Nash equilibrium under price competition.

Comparison with Dixon (1993) suggests some interesting conclusions regarding the impact of the replication procedure on the equilibrium outcomes. If one replicates firms but not demand, then the present paper shows that for a sufficiently large market there is a unique Nash equilibrium. Moreover, in the limit, as the grid size goes to zero, and the number of firms becomes large, the equilibrium price converges to the competitive one, i.e. the 'folk theorem' of perfect competition holds. Whereas if one replicates both demand and firms, then Dixon (1993) shows that the Nash equilibrium is non-unique and it exists for a large parameter class whenever the industry is large enough. Moreover, the 'folk theorem' fails, at least in some cases. Thus the results are sensitive to the choice of the replication procedure.

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charging a price strictly greater than p_2^*, since in that case the type 1 firms will have no demand. Whereas if they charge a price strictly lower than $c_2'(0)$, then their profit will be lower compared to what they obtain from charging $c_2'(0)$. For n^2 large enough, their profit from charging $c_2'(0)$ is also lower, since they will have to share the demand at price $c_2'(0)$ with the type 2 firms.

[30]The example extends in a straightforward manner to all α of the form 10^{-I}, where I is some positive integer greater than 1.

6 Appendix 1

Proof of Proposition 2. Existence. We first argue that the quantity decisions are optimal. Suppose that in stage 1 all the firms charge p^*. Then given that all other firms produce $\frac{d(p^*)}{n}$, the residual demand facing firm i is also $\frac{d(p^*)}{n}$ (Assumption 3(ii)(a)). Moreover, it is optimal for the i-th firm to produce this amount since $\frac{d(p^*)}{n} < c'^{-1}(p^*)$.

Next consider the case where $(n-1)$ of the firms charge p^*. Clearly, given that all other firms produce $\frac{d(p^*)}{n-1}$, the residual demand facing the i-th firm charging p^* is also $\frac{d(p^*)}{n-1}$ (Assumption 3(ii)(a)). Since $n-1 \geq n^*$, it follows that $\frac{d(p^*)}{n-1} < c'^{-1}(p^*)$. Hence it is optimal for the firms charging p^* to produce $\frac{d(p^*)}{n-1}$.

The pricing decision is also optimal since if any of the firms increase its price then, in stage 2, the output level of the other firms are such that the deviant firm has zero residual demand.

Uniqueness. It is easy to see that we cannot have an equilibrium where the output level of some of the firms is zero, since it can always charge p^* in stage 1 and supply $\frac{d(p^*)}{n}$ in stage 2.

Next observe that the definitions of $\tilde{\pi}$, n_i and n^* are valid for this case also. Hence we can mimic step 2 of the uniqueness part of Proposition 1 to argue that the only price that is sustainable in equilibrium is p^*. ■

Proof of Proposition 3. Existence. Undercutting p^* is clearly not profitable. We then argue that for the i-th firm of type l, charging a higher price, p_i, is not profitable either.

Notice that since $n \geq n_l^*$, $\frac{d(p^*)}{n} < c_l'^{-1}(p^*)$. Hence for any $p_i \geq p^*$,

$$c_l'^{-1}(p_i) \geq c_l'^{-1}(p^*) > \frac{d(p^*)}{n} = r_{il}(p^*, p^*, n) \geq r_{il}(p_i, p^*, n). \qquad (5)$$

Since $c_l'^{-1}(p_i) > r_{il}(p_i, p^*, n)$, the deviant firm supplies the whole of the residual demand coming to it. Hence the profit of a firm which charges a price p_i $(\geq p^*)$

$$\pi_l(p_i, r_{il}(p_i, p^*, n)) = p_i r_{il}(p_i, p^*, n) - c_l(r_{il}(p_i, p^*, n)). \tag{6}$$

Clearly

$$\frac{\partial \pi_l(p_i, r_{il}(p_i, p^*, n))}{\partial p_i} = r'_{il}(p_i, p^*, n)[p_i - c'_l(r_{il}(p_i, p^*, n))] + r_{il}(p_i, p^*, n). \tag{7}$$

Next from equation (5) it follows that $\forall p_i \geq p^*, \quad p_i > c'_l(r_{il}(p_i, p^*, n))$. Hence from the concavity of $r_{il}(p_i, p^*, n)$ it follows that $\pi_l(p_i, r_{il}(p_i, p^*, n))$ is concave in p_i. Moreover,

$$\frac{\partial \pi_l(p_i, r_{il}(p_i, p^*, n))}{\partial p_i}|_{p_i = p^*} = r'_{il}(p^*, p^*, n)[p^* - c'_l(\frac{d(p^*)}{n})] + \frac{d(p^*)}{n}. \tag{8}$$

This follows since from an analogue of Assumption 3(ii)(a) we know that $r_{il}(p^*, p^*, n) = \frac{d(p^*)}{n}$. Since $n \geq \hat{n}_l$, we have that $\frac{\partial \pi_l(p_i, r_{il}(p_i, p^*, n))}{\partial p_i}|_{p_i = p^*} < 0$. Next, from the concavity of $\pi_l(p_i, r_{il}(p_i, p^*, n))$ it follows that $\forall p_i \geq p^*$, the profit of any deviant firm is decreasing in p_i.

Finally, given that all other firms supply $\frac{d(p^*)}{n}$, the residual demand facing all firms is exactly $\frac{d(p^*)}{n}$. Since $\frac{d(p^*)}{n} < c_l'^{-1}(q^*)$, $\forall l$, it is optimal for all the firms to produce exactly $\frac{d(p^*)}{n}$.

Uniqueness. Step 1. We can first mimic the proof of Proposition 1 to argue that there cannot be an equilibrium where the output level of some of the firms is zero.

Step 2. We then demonstrate that there cannot be some p_y $(\in F) > p^*$, such that some of the firms charge p_y and supply a positive amount.

Suppose to the contrary that such a price exists.

This implies that the total number of firms charging p^*, say $\tilde{n}$, can be at most $\max_q n_q^* - 1$. Otherwise, $\tilde{n} \geq \max_q n_q^*$ and the residual demand facing all these firms would be exactly $\frac{d(p^*)}{\tilde{n}}$.[31] Since $\frac{d(p^*)}{\tilde{n}} < c_l'^{-1}(p^*)$, $\forall l$, all such

[31] Given that $\frac{d(p^*)}{\tilde{n}} < c_l'^{-1}(p^*)$, $\forall l$, all firms must be supplying at least $\frac{d(p^*)}{\tilde{n}}$. The assertion now follows from an analogue of Assumption 3(ii)(b).

firms would supply $\frac{d(p^*)}{\tilde{n}}$ and the residual demand at any higher price would be zero.

Now consider some $p_y > p^*$. Clearly, the number of firms charging p_y is less than $\max_q n_{qy}$. Otherwise, some of these firms, say of type l, would have a profit less than $\tilde{\pi}_l$. Hence such a firm would have an incentive to deviate to p^*, when it can supply at least $\frac{d(p^*)}{\max_q n_q^*}$ and earn $\tilde{\pi}_l$. Thus the total number of firms producing a strictly positive amount is less than $\tilde{N}_2$, thereby contradicting step 1. ■

Proof of Proposition 4. In the proposition the term unique is within quotes because the outcome is unique up to the strategies of type 1 firms. Firms of all other types can charge any price and supply an output of zero. This will not affect the outcome.

Existence. Notice that since $\alpha < c_2'(0) - c_1'(0)$, it follows that $\forall i \geq 2$, $p_1^* < c_i'(0)$. Thus no firm of type i, where $i \geq 2$ can profitably charge a price of p_1^* and produce a strictly positive output level. For type 1 firms we can simply mimic the proof in Proposition 1 to claim that they cannot have a profitable deviation.

Uniqueness. First note that there cannot be an equilibrium where the output level of some of the type 1 firms is zero.

We then argue that there cannot be some p_x $(\in F) > p_1^*$, such that some of the type 1 firms charge p_x and supply a positive amount. Suppose to the contrary that such a price exists.

This implies that the total number of type 1 firms charging p_1^*, say $\tilde{n}$, can be at most $n_1^* - 1$. Otherwise the residual demand facing these firms would be exactly $\frac{d(p_1^*)}{\tilde{n}}$.[32] Since $\tilde{n} \geq n_1^*$, we have that $\frac{d(p_1^*)}{\tilde{n}} < c_1'^{-1}(p_1^*)$. Hence all such firms would supply $\frac{d(p^*)}{\tilde{n}}$ and the residual demand at any higher price,

[32] First note that firms of type $j > 1$, even if they charge p_1^*, would have an output of zero. Thus the residual demand facing all firms of type 1 charging p_1^* is at least $\frac{d(p_1^*)}{\tilde{n}}$ (from Assumption 3(ii)). Given that $\frac{d(p_1^*)}{\tilde{n}} < c_1'^{-1}(p^*)$, all such firms of type 1 must be supplying at least $\frac{d(p_1^*)}{\tilde{n}}$. The assertion now follows from an analogue of Assumption 3(ii)(a).

p_x, would be zero.

Next consider some $p_x > p_1^*$. Clearly, the number of type 1 firms charging p_x is less than n_{1x}. Otherwise, one of the type 1 firms would have a residual demand that is less than or equal to $\frac{d(p_x)}{n_{1x}}$. Since $\frac{d(p_x)}{n_{1x}} < c_l'^{-1}(p_x)$, this firm would supply at most $\frac{d(p_x)}{n_{1x}}$ and have a profit less than $\tilde{\pi}_1$. Hence such a firm would have an incentive to deviate to p_1^*, when it can supply at least $\frac{d(p_1^*)}{n_1^*}$ and earn $\tilde{\pi}_1$. Thus the total number of firms producing a strictly positive amount is less than $\hat{N}_2$, a contradiction. ■

7 Appendix 2

In this appendix we provide an example of a residual demand function satisfying Assumption 3 when there are three firms, 1, 2 and 3.

If $\#\{k : p_k = p_1\} = 1$, then

$$R_1(P, Q) = \max[0, d(p_1) - \sum_{p_j < p_1} q_j].$$

If $\#\{k : p_k = p_1\} = 2$, then

$$R_1(P, Q) = \max[0, \frac{d(p_1) - \sum_{p_j < p_1} q_j}{2}, d(p_1) - \sum_{p_j < p_1} q_j - q_k|_{p_k = p_1,\, k \neq 1}].$$

If $\#\{k : p_k = p_1\} = 3$ and either $q_2, q_3 \leq \frac{d(p_1)}{3}$ or $q_2, q_3 > \frac{d(p_1)}{3}$, then

$$R_1(P, Q) = \max[\frac{d(p_1)}{3}, d(p_1) - q_2 - q_3].$$

If $\#\{k : p_k = p_1\} = 3$ and $q_j \leq \frac{d(p_1)}{3}$, $q_k > \frac{d(p_1)}{3}$, $j, k \neq 1$, then

$$R_1(P, Q) = \max[\frac{d(p_1) - q_j}{2}, d(p_1) - q_j - q_k].$$

It is clear that the associated rationing rule is efficient, while the associated tie-breaking rule is a generalization the Davidson-Deneckere-Kreps-Scheinkman one.

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