Distribution Dynamics of States in Post Reform India

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\textbf{Abstract:} The literature on the regional convergence of the Indian states has generated a lot of debate. In the post reform period, most of the literature found divergence of per capita income among Indian states. These studies analysed the growth dynamics of these states using either a regression based approach or one based on some summary measure of inequality. As a result, these studies were unable to reveal what happened to the entire cross-section of the Indian states in the context of convergence. Following Quah (1997), we use the distribution dynamics approach to analyse growth dynamics in different groups of Indian states for the period of 1993 to 2005, and study the evolution of the entire distribution over time. We use the stochastic kernel, and its 3-dimentional surface plots and 2-dimentional contour plots to study the dynamics. Our result shows that in post reform period per capita income distribution shows a tendency towards bimodality and polarization.

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1. Introduction

For most of its post independence history, the Indian economy adopted inward looking policies based on the ‘import substitution’ framework. The balance of payment crisis at the beginning of 90’s however, forced the policy makers to change course and open up the economy, and integrate it with the international markets by implementing a series of policy reforms. Since then, the growth rates have been raising and are likely to continue at these high levels in the future. Clearly, it is important to analyse the nature and causes of this growth performance, as future developmental policies could be fine-tuned on the basis of such analysis. One way of doing this is to study what is happening to the growth at the regional (state) level i.e., whether the growth performance is largely restricted to certain states of the country. From the data it is evident that, while some of the states are performing well, others have consistently lagged behind. For example, while, Delhi, Maharastra, Haryana and Punjab are situated in the top bracket (per capita income almost double of national average per capita income), Bihar, Orissa, Uttar Pradesh, Rajasthan and Madhya Pradesh have consistently lagged behind (per capita income almost 50 % of national average). In between these two extremes, there are many states showing constant upward or downward movements, as an evidence of mobility in the post reform period. Over the years, Tamil Nadu and Karnataka have shown upward movement in terms of their relative position while Jammu & Kashmir and Madhya Pradesh have been losing its relative position consistently. In order to study these growth dynamics, we have to adopt a framework for our analysis that allows us to capture the formation of convergence clubs and tendency towards polarization. As we shall show in a subsequent section, a regression based approach does not throw sufficient light on club formation. We use, instead, a non-parametric approach based on the estimation of a stochastic kernel that helps us to through light on the distribution dynamics of states in post-reform India.

A study of convergence for the post-reform period is important, since increasing inequality can create social tension in a country which is extremely varied in culture,
language and religion. There exists, a large number of studies on the probable convergence among Indian states. Most of these studies used growth regressions or some measure of inequality (Cashin and Sahay, 1996, Aiyar, 1999 and Nagraj et al, 1997). Analyzing regional growth in post reform India, one of the authors (Kar and Sakthivel, 2007) have in an earlier study shown evidence of divergence among Indian states. The distribution dynamics approach has evolved from the vast literature on convergence. Bandyapadhaya (2006), using this approach for 17 major Indian states and covering largely the pre-reform period (1965 to 1997), has shown the emergence of “twin peak” i.e., bimodality of the distribution. The present study uses this framework to understand the nature of the Indian growth dynamics, particularly for the post reform period.

The study attempts to throw light on a number of related issues. Firstly, it studies the dynamics of regional growth as a result of the reforms process and tries to identify trends towards polarization and the formation of convergence clubs. Secondly, we try to identify the approximate time when the distribution shows clear transition towards bimodality. Thirdly, we identify the states that play important roles in the process of transition and the formation of the convergence club. The rest of the paper is arranged as follows. Section 2 deals with the theoretical background of convergence analysis. In section 3, theories of convergence club formation and polarisation have been discussed. Section 4 surveys some of the important regional convergence studies in India. Section 5 describes the distribution dynamics approach as a technique of convergence analysis. Its estimation using kernel density estimator is presented in section 6. In section 7, we report our findings. Section 8 concludes.

2. Neoclassical Growth Theory and Convergence

The neoclassical growth literature provides a theoretical framework to analyse the issue of convergence of per capita income. Let us assume that the economy produce a homogenous good using capital (K) and labour (L) as inputs.
Therefore, the transition equation, using per capita effective amount of capital and labour, can be written as

\[
\frac{\dot{k}}{k} = s f(k) - n \mu - \delta
\]

(1)

Equation (1) shows that the countries with lower stock of initial physical capital will grow rapidly compared to the capital advance countries in the transition period. According to Barro and Sala-i-Martin (1991, 1992), from the above equation using Taylor’s series expansion one gets the equation for cross-section dynamics around the steady state as

\[
\log y(t) = [\log y(0) - (\log \bar{y}^* + \log A(0))]e^{-\beta t} + [\log \bar{y}^* + \log A(0) + \mu t]
\]

(2)

Here \( \beta \) describes the speed of convergence towards the steady state. Depending on the sign of \( \beta \) one can infer about the possibility of convergence. A positive sign for \( \beta \) implies \( \beta \) convergence i.e., initially poorer countries will grow more rapidly compared to the initially richer ones. This proposition can be empirically tested by running a regression on per capita growth, with initial per capita income and other structural variables as the explanatory variables. In such a framework, the sign of the coefficient of initial income has to be negative in order to ensure \( \beta \) convergence.

According to Friedman (1992), Quah (1993) among many others, \( \beta \) convergence suffered from what is termed as Galton’s fallacy. According to them, a negative coefficient on the initial income is not sufficient for diminishing cross-section dispersion over time. A positive \( \beta \) (implies negative coefficient of initial income) is very much compatible with constant or even diverging cross-section distribution of per capita income.

Accepting this shortcoming of \( \beta \) convergence approach, Barro and Sala-i-Martin (1995, 1996) proposed \( \sigma \) convergence (dispersion of log per capita income diminishing over time) to compliment the idea of \( \beta \) convergence. According to them, \( \beta \) convergence is a
necessary but not a sufficient condition for catch up. However, the concept of $\sigma$ convergence also has problematic implications. To understand this, we have to look into the underlying implications of the neoclassical growth models. $\sigma$ convergence in these models is based on the assumption that there is a one time shock to the economy in the initial period and consequently, the economy reaches its own steady state following a monotonic smooth path. Thus, the cross-section distribution diminishes over time. Countering this assumption, Quah (1993) has shown that the shocks to the economy are continuous rather than only at the initial period. As a result, the cross-section dispersion may remain constant ($\sigma$ constant) over time. Quah (1996) and Darlouf and Quah (1999), shows that various types of dynamics of per capita income distributions are possible with a constant $\sigma$. They have shown that even if $\sigma$ is constant rather than diminishing, there are the possibilities of leap-frogging, criss-crossing, persistent inequality and even poverty traps. Therefore, to study the possibility of convergence more rigorously, one needs a framework which focuses on the entire distribution and its dynamics rather than on a summary measure of the distribution. The objective of such an analysis is to study convergence in terms of convergence club formation and polarization. The Distribution Dynamics approach is used to fulfill this objective.

3. Convergence Club and Polarisation

There has been some attempt in the theoretical literature to construct models that give rise to convergence clubs. An alternative approach has involved the construction of models that lead to polarization of the distribution (of, say, per capita income). Both of these approaches lead to the generation of multiple equilibriums. In this section we discuss these theories in some details.

Azariadis and Drazen (1990) is one of the pioneering contributions in the literature on multiple equilibria. They demonstrate the existence of multiple equilibria in the neoclassical growth framework of Solow (1956) and Diamond (1965). They augment the neoclassical model of economic growth with a feature that is sufficient to produce
multiple, locally stable balanced growth paths in equilibrium. This feature is technological externalities with a "threshold" property that permits returns to scale to rise very rapidly whenever economic state variables, such as the quality of labor, take on values in a relatively narrow "critical mass" range”. The model is based on threshold externalities (increasing returns to scale) in human capital accumulation, which becomes pronounced after the state variables crosses a critical mass. These threshold properties produce multiple and locally stable balanced growth paths in the neoclassical growth framework. The idea behind the model is that if the stock of human capital has a positive externality in the growth process, there will be two types of equilibria. If the initial stock of human capital is too low (lower than a threshold level), the opportunity cost of investing in human capital will be lower than investing in physical capital and the stock of human capital will not be increasing in the economy. As a result, there will be a stationary equilibrium without growth. In the other case, if the initial stock of human capital is higher than a threshold level, incentive for investing in human capital generation will be high enough to sustain a continuous investment in this sector and the economy will grow. So, this neoclassical framework shows that multiple equilibria is consistent with the neoclassical model of growth and it can explain the observed stylized facts of persistent inequality and ‘poverty traps’ in the process of growth in a cross-section of economies.

Quah (1996), another important contribution to this literature, constructed a model which stresses on the generation of ideas as a determinant of growth. The model is based on two opposite forces - consolidation and fragmentation - leading to the consolidation of coalition at different parts of the distribution of per capita income. These coalitions then behave like convergence clubs. We will explain briefly the model of formation of convergence clubs in this section.

The model assumes that there are some coalitions in the cross-section of economies. Each economy in the coalition is endowed with a stock of human capital. In the one hand this stock represent the potential for generating new ideas and on the other, it produces non-storable consumption good, jointly with other members of the coalition. Assuming exact
product exhaustion, each economy in the coalition gets according to its marginal product. Output of each economy is an increasing function of total output of the coalition and the stock of human capital of that economy. The compensation principle ensures that all the economics will like to be in a single grand coalition. This is the force for consolidation.

Let us discuss the force for fragmentation. Assume that an economy in the coalition generates the ideas of average quality equal to the stock of its human capital. First, it uses these ideas and then distributes among the economies of its own coalition. Ideas are fully mobile within own coalition but immobile beyond the boundary of that coalition. According to Quah (1996), “this might be because ideas or memes are like viruses and thus could be dangerous- members of different coalitions are not trusted. Or, members of a coalition are able to enforce intellectual property rights perfectly across coalitions”. If the coalition generates an average level of human capital, then it can be said that the evolution of human capital is an increasing function of the ratio of average human capital of the coalition and the stock of human capital of every economy in that coalition. Therefore, the economics in higher average human capital coalition have faster rates of growth and they will not agree to accumulate economies with lower average stock of human capital, since, this will lower the average human capital of the coalition and consequently the growth rate of all the economics in the coalition will be lower. Quah (1996) describes an equilibrium where the distribution of income across economics within a same coalition will converge to equality but different coalitions will diverge from each other. In the equilibrium the rich economics will converge and form rich convergence club. Similarly, the poor economics will converge among themselves and will remain poor. The middle class eventually will vanish. Economics in the middle part of the distribution will diverge from each other, instead of the fact that they have started almost from the same position. Very limited difference in human capital formation or generation of ideas in the initial stage may create a huge difference in the long run for these economies. In the long run, there will be two convergence clubs at the extreme parts of the cross-section distribution of economies. In contrast, within each convergence club the economies will reach to equality over time. Quah called these phenomena as
emerging twin peaks’. In reality there may exist multiple convergence clubs and it can be identified by the multiple modes in the cross-section distribution.

Another important theory in the context of convergence club formation is given by Estaben and Ray (1994). Though the model is in context of income distribution (polarization of the income distribution), it can throw sufficient light on the theory of convergence club formation and the consequences of polarization in the form of social tension in a diversified and stratified world like ours. The idea of polarization given by Estaben and Ray (1994) is a modification on the concept of inequality but the two concepts are thoroughly different. They axiomatically derived the idea of polarization introducing the concepts of ‘Identification’ and ‘Alienation’ among the individuals. According to them inta-group homogeneity accentuates polarization. An individual feels a sense of identification with another who has the same income as that individual in the distribution of individuals in the society. Therefore, the identification felt by that individual is an increasing function of the number of individuals having same level of income. If the number of individuals having a particular income of the hypothesized person is high in a society then identification will be higher.

Similarly, an individual’s alienation towards others depends on the distance of that individual from others. This concept of alienation is perfectly symmetric, in the sense that the amount of alienation felt by the poor towards the rich is perfectly equal in the case of rich towards the poor.

Putting together these two concepts Estaben and Ray (1994) tried to find out “effective antagonism”, an individual feel towards the others. Therefore, effective antagonism is equal to the amount of alienation an individual felt towards the others added with the amount of identification of that individual with the others. The reason for inclusion of the sense of identification is important due to the fact that the sense of identification separates the concept of polarization from the concept of inequality. The total polarization in the society is equal to the sum of all effective antagonisms in the society.
4. Convergence in India

There are a large number of studies on regional growth and convergence in India. The studies varied in their methodology and findings. The study of Cashin and Sahay (1996) examines the growth experience of 20 states of India during the period of 1966 to 1991. They used cross-sectional estimation of Barro and Sala-i-Martin and the analytical framework of Solow-Swan neoclassical growth models. They find evidence of weak absolute convergence (speed of convergence parameter ($\beta$) is 1.5) and widening dispersion of real per capita state domestic products in the above mentioned period. According to them, transfers from central government to the states are responsible for this absolute convergence and widening per capita dispersion over time. Nagraj et al (1998) find no evidence of absolute convergence during the period of 1970 to 1994. But they have shown the evidence of conditional convergence. The coefficient of variation decreased in the 60’s, then go up sharply in the 70’s, stabilize in the 80’s and increased rapidly there after. They have shown that share of agriculture, infrastructure, political and institutional factors affect convergence in this period. Aiyar (2001) confirms the above findings of conditional convergence. According to him, infrastructure, private investment and non-measured institutional factors are responsible.

In contrast to the above studies, Rao et al (1999) find the existence of absolute and conditional divergence during 1965 to 1995 among 14 major states. They have identified unequal private investment as contributing factor for this divergence. Sachs et at (2002) confirm the above findings for the period of 1990 to 1998. Their study reflects some social and geographical variables as responsible for this diverging trend. Ahaluwalia (2000, 2002) using population weighted Gini Coefficient confirms the earlier findings. Bandyopadhyay (2006) following Quah (1997) adopt the distribution dynamics approach covering the period of 1965 to 1997 and has shown the convergence in 1960’s and emergence of ‘twin peaks’ and ‘polarisation’ in the early 90’s among 17 major Indian states. Comparing with the panel data regression approach, the study establishes the superiority of the distribution dynamics approach in the Indian context, and identified infrastructural inequality as the main factor responsible for the emerging ‘twin peaks’.
Though most of the studies covered the period before the nineties, some recent studies have looked into the post reform period. Since various reform measures in the economy have been implemented in the early 1990’s and have continued throughout the decade in different phases, studies of the trend in per capita state domestic product in post reform period is important to understand the impact of reform on regional inequality and convergence. Among the Inequality based studies, Ahaluwalia (2000, 2002) have studied the whole 1990’s using population weighted Gini-coefficient, and have shown that inequality in real per capita GSDP has tended to increase from 0.175 in 1991-92 to 0.233 in the 1998-99 among 14 major states. Sachs et al (2002) using different measures of convergence find no sign of absolute as well as conditional convergence in post reform India. The above studies have taken only major Indian states due to data limitations for union territories and small states. Shetty (2003) depart from the earlier trend and have taken all the Indian states and union territories, and confirmed the trend of divergence among Indian states. According to Nagraj (1998), coefficient of variation of per capita GSDP increased rapidly in 1990’s after a stabilization in the 80’s. Rao et al (1999) also find the evidence of divergence in the early 90’s in a similar study. Bhattacharya and Sakthivel (2004) find that inequality in per capita GSDP have increased in post reform period compared to 80’s. The coefficient of variation of per capita GSDP (measure of $\sigma$-convergence) found to be increased from 0.22 percent per annum to 0.43 per cent per annum in 1990’s. In a more recent study, Kar and Sakthivel (2007) using “new geography” framework analyse the impact of reforms on per capita GSDP. They also have studied the impact of reforms on the contributions from different sectors and confirmed the evidence of post reform divergence in India. The above studies in summary have shown that regional inequality remain stable till 1986-87 and increase thereafter.

There exist some other studies which found no clear evidence of convergence or divergence in post reform India. Singh et al (2003) find no uniform trend of divergence in post reform period. Dholakia (2003) has also shown that there is no significant trend of divergence and confirm the results of Singh et al (2003).
5. Distribution Dynamics Approach

Quah (1993a and b, 1996a and b, 1997) made path breaking contributions to the convergence literature by introducing the distribution dynamics framework as an alternative to that based on growth regressions. This approach studies the evolution of the cross-section distribution of per capita income of regions over time. The framework uses stochastic kernel to study the shape of intra-distribution dynamics of per capita income, and can be used to give possible explanations to the formation of convergence club in terms of characteristics of the club members. Convergence is analysed by studying the shape of a three-dimensional plots of stochastic kernel and its corresponding contour plots. We discuss this framework in some details below.

Let us assume that at time t, the cross-section distribution of per capita income can be represented by $F_t$ and an associated probability measure is $\phi_t$. Following Quah (1993, 1996 and 1997), the simplest form of the dynamics of the stochastic process $\{\phi_t: t \geq 0\}$ is similar to the first order AR (1) process i.e., current value depends on the value of one period lag.

$$\phi_t = T^* (\phi_{t-1}, u_t) = T^*_{ut} (\phi_t), \quad t \geq 1,$$

(3)

where, $u_t$ the disturbance term, $T^*$ is an operator mapping Cartesian product of probability measures at time t and t-1, i.e., $T^*$ maps how the distribution evolves from time t-1 to t. $T^*_{ut}$ absorbs the disturbance into the definition of the operator. The only difference between equation (3) and a stochastic difference equation is that the values here are the income distributions.

From equation (3) we get

$$\phi_{t+1} = M' \phi_t, \quad t \geq 1$$

$$\phi_{t+s} = (M^s)' \phi_t, \quad \text{for all } s \geq 1$$
as \( s \to \infty \), the long-run distribution of income becomes

\[
\phi_{\infty} = M' \phi_{\infty}, \tag{4}
\]

\( \phi_{\infty} \) is the long-run limit of the distribution of income across economies. Convergence can be obtained if the distribution after \( s \) periods \((\phi_{t+s})\) and/or the Ergodic (long-run) distribution \((\phi_{\infty})\) show tendency towards a point mass. Alternatively, if \( \phi_{t+s} \) or \( \phi_{\infty} \) shows tendency towards bimodality then it can be identified as polarisation. If more than two modes are identified, then it is the evidence of stratification.

In order to operationalize these concepts, we now show that the operator \( T^*_{ut} \) can be represented in a continuous income space by a stochastic kernel (Quah, 1996 and 1997). Let us assume that \( \{(y_1, z_1), \ldots, (y_n, z_n)\} \) represents the set of a pair of per capita relative income of different regions in the cross-section and \( n \) represents number of regions. Here, \( y \) and \( z \) denote the initial income and the income after \( s \) years (Gross state domestic products (GSDP), \( y \) and \( z \), have been normalized as average of the total net state domestic product) respectively. If the cross-section distribution of income is represented by the density functions \( f_t(y) \) and \( f_{t+s}(z) \) at time \( t \) and \( t + s \) respectively, then stochastic kernel is defined as the equation

\[
f_{t+s}(z) = \int_{0}^{\infty} g_s(z|y) f_t(y) dy, \tag{5}
\]

where \( g_s(z|y) \) is the conditional distribution after time \( s \)

and the Ergodic (long-run) distribution is given by

\[
f_{\infty}(z) = \int_{0}^{\infty} g_{\infty}(z|y) f_{\infty}(y) dy \tag{6}
\]

The concept of convergence as an evolution of the entire distribution of per capita income over time can be obtained from directly analysing the shape of three dimensional plot of
the stochastic kernel, and its two dimensional contour plot. The next step is the estimation of the stochastic kernel using kernel density function.


Let \( Y \) and \( Z \) denote vectors of per capita income of different regions of a country at period \( t \) and \( t + s \) (\( s > 0 \)) respectively, and the observations are \( \{ (y_1, z_1), \ldots, (y_n, z_n) \} \).

If \( f_{t, t+s}(y, z) \) denote joint density of \( (Y, Z) \) and \( f_t(y) \) the marginal density of \( Y \), then by definition, the conditional density of \( Z \) given \( Y \), can be represented as

\[
g_s(z|y) = \frac{f_{t, t+s}(y, z)}{f_t(y)}
\]

(7)

where, \( f_{t, t+s}(y, z) \) is the joint density of \( y \) and \( z \) and \( f_t(y) \) is the marginal density of \( y \).

Hence, \( g_s(z|y) \) can be estimated as

\[
\hat{g}_s(z|y) = \frac{\hat{f}_{t, t+s}(y, z)}{\hat{f}_t(y)}
\]

Next is to estimate the stochastic kernel (the conditional density function) using kernel density estimator. The accuracy of the kernel density estimation depends on the choice of the kernel function and the bandwidth matrix used to estimate the density. There is a large literature which shows that kernel density estimation is not very sensitive to the particular kernel function, but very responsive to the choice of the bandwidth of the kernel (Silverman, 1986; Wand and Jones, 1995). In this paper we use the bivariate Gaussian kernel function which is of the form

\[
k(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}
\]
The next important step is the selection of the bandwidth. There are actually three issues in choosing bandwidth for the stochastic kernel estimation.

1. Bandwidths should be chosen according to some global error criteria.
2. One should choose different bandwidths for different dimensions.
3. For each dimension bandwidths should vary according to some rescaling rule of the fixed bandwidths.

According to Wand and Jones (1995), bandwidth should be chosen according to some global error criteria. The common practice is to minimise mean integrated square error (MISE) between estimated density and actual density.

\[ \text{MISE} = \mathbb{E} \left\{ \left( \hat{f}_x - f_x \right)^2 \right\} \]

However, MISE depends on the bandwidth in a very complicated way, and it is very difficult to interpret it in terms of bias and variance. Wand and Jones (1995) recommended using its asymptotic approximation (AMISE), which is related to the bandwidth in very simple way. As a result, AMISE can give greater insight into the relationship of the bandwidth with the kernel density estimator. AMISE can be calculated using first two terms of the Taylor’s series expansion of the MISE. Then, the AMISE of the kernel estimator can be defined as

\[ \text{AMISE} = (nh)^{-1} R(K) + \frac{1}{4} h^4 \sigma^4 k R(f') \]  

(8)

where, \( R(K) = \int K(x)^2 \, dx \) is a measure of roughness of the kernel function (K).

The first term in the above equation is the integrated variance, which is proportional to \( ((nh)^{-1}) \). The second term is the integrated square bias and proportional to \( h^4 \). As can be seen from the above AMISE equation, there is a trade-off between the bias and the variance of the kernel density estimation. A small value of \( h \) leads to the increase in the value of the variance and so the increase in the spurious nature of the estimation. In
contrast, a large value of \( h \) means there will be higher bias and as a result, the essential
details of the distribution may not appear. Therefore, the choice of the bandwidth should
try to establish global compromise between the bias and the variance.

The second step is to choose a form of the bandwidth matrix. There are two different
dimensions of the data in our analysis viz., per capita income for the year 1993 and 2005.
It will not be a very good idea to choose a single bandwidth for both the dimensions,
since the observations in both the periods are different (so as the distributions). So, it is
better to use two different bandwidths for two different dimensions. Hence, we are using
diagonal bandwidth matrix \( H = \text{diag}(h_x, h_y) \), where two different bandwidths will
look after the smoothing in different dimensions. Following Wand and Jones (1995) we
use diagonal bandwidth matrix and adopt product kernel for our analysis.

It is clear that different amount of smoothing is applied to different dimensions but same
amount of smoothing along each dimension. As a result the bias-variance trade-off
appears again along each dimension of the data. Variable bandwidth selector has been
used in this paper to deal with the problem, as this bandwidth varies according to the
density of the observations. When data is sparse bandwidth is wide and vise-versa.

Therefore, the estimates of joint and the marginal densities in equation (9) becomes

\[
\hat{f}_{t,t+s}(y,z) = \frac{1}{n h_y h_z} \sum_{i=1}^{n} k\left( \frac{\|y - y_i\|}{h_y} \right) k\left( \frac{\|z - z_i\|}{h_z} \right)
\]

and

\[
\hat{f}_{t}(y) = \frac{1}{n h_y} \sum_{i=1}^{n} k\left( \frac{\|y - y_i\|}{h_y} \right)
\]

respectively, where \( \|y - y_i\| \) and \( \|z - z_i\| \) are the Euclidian distance matrices. \( h_x \) and \( h_z \)
represents the bandwidths. Here the kernel is the product of two kernels. Each estimated
using the bandwidths $h_x$ and $h_z$ respectively. The ergodic (long-run) distribution has been calculated following Jonson (2000 and 2005). All the calculations have been done using matlab routine developed and used by Magrini (2007).

Clearly, the estimated conditional density using kernel density function is the stochastic kernel. Figure 5, for example, is a representation of such a stochastic kernel. Using this stochastic kernel, convergence can be analysed from the shape of the three-dimensional plot. The main diagonal of this diagram is of importance, as this helps in confirming the presence or absence of persistence. If most of the probability mass concentrates around this line, then one can conclude that there is high level of persistence, i.e., elements of the cross-section distribution remain where they had started. If most of the mass concentrate along the 1-value in the axis for the terminal year and parallel to the axis for the initial year (figure 5 and 6), it indicates convergence towards equality. A counter clock wise movement of the mass from the diagonal indicates mobility of high income classes to the lower income groups, while a clock wise movement of the same from the 45-degree line is symbolic of better performance by the higher income strata.

7. Distribution Dynamics in Post Reform India

In this section, we track the evolution of the distribution of real per capita income in post reform India and study the possible formation of convergence clubs, polarisation or stratification during this period. It is based on per capita Gross State Domestic Product (GSDP), from 1993 to 2005 (base year 1999-2000), compiled by the Central Statistical Organisation (CSO). The per capita GSDP of each state is normalized by national average GSDP for the corresponding years to get the relative per capita GSDP. These relative per capita incomes are then used to estimate kernel density and the stochastic kernel for the period 1993 to 2005. Using relative per capita GSDP ensures that the aggregate growth effect is controlled and only the state specific effects are analysed.
The exercise is repeated for different groups of states in order to study the robustness of the results. The first group consists of 21 major Indian states, which comprises more than 98.5 of the population of India. The second group consists of 18 states, with three newly formed states have been merged with their original counterpart. This exercise enables us to compare the results with the earlier studies which was based on the undivided Indian states. A third group is similar to the second group except that it leaves out Delhi. Delhi has been dropped due to the fact that it is an artificial city state and its growth dynamics are not similar to that of other Indian states.

First we study the group of 21 major Indian states. The univariate kernel density plot (figure1) shows that in 1993 the distribution was unimodal, with more than 60 percent of the observations concentrated in the range of 75 percent to 125 percent of national per capita income. The mode of the distribution is slightly below the national average per capita income. In 2005, though the distribution is unimodal, some of the middle income states moved from its earlier position to both left and right of the 1993 mode. As a result of these movements during the post reform period, in the 2005 distribution, there is a tendency of another mode emerging at around 1.5 times the national per capita income by the end of the period. The highest point of the distribution has shifted slightly leftwards.
as a result of the mobility of some middle income states towards low income values compared to the 1993 distribution, as can be seen from figure 1. Therefore, in figure 1, there is a tendency from unimodality towards bimodality. This tendency is clear from the ergodic (long-run) distribution (figure 1) as defined in section 4. In the ergodic distribution, two modes exist very clearly. The lower mode is situated corresponding to 75 percent of national per capita income and the upper mode is at 150 percent of the national per capita income. The upper mode is larger than the lower mode, due to the fact that many of the middle income states (Gujarat, Kerala, Himachal Pradesh, Karnataka, Tamil Nadu, West Bengal etc.) have a tendency to gather around this mode and some of the richer states (like, Punjab and Maharashtra) also have a tendency to fall back to the same. From the above discussion it is clear that there is a tendency of ‘emerging twin peak’ in the distribution of per capita GSDP among the Indian states, in the post reform period.

Figure 3
Next, we have tried to identify the approximate time when the distribution changes from unimodality to bimodality. For the purpose we estimate the univariate kernel density for all the years between 1993 and 2005. Hence, for economy of space we present the kernel density plots for the years 1996, 2001 and 2005 along with 1993 kernel distribution. The estimated kernel density plots are presented in figure 3, 4 and 1, which shows the kernel density plots of the initial and the final years along with the ergodic distribution. The univariate kernel density and the ergodic distribution was unimodal in 1996 (Figure 3) and bimodality appears only from 2001. In 2005, the bimodality of the ergodic density becomes more prominent and the univariate kernel density also shows a tendency towards bimodality. Therefore, from these kernel density plots it is clear that the phenomenon of ‘emerging twin peak’ appears around 2001.

The bimodality of the distribution of the states is the outcome of their growth dynamics, which can be better understood in terms of persistence and mobility. The persistence or mobility of states, as defined in section 5, can be traced from the 3-dimensional plot of the stochastic kernel and the corresponding 2-dimensional contour plot represented by
figure 5 and figure 6. In figure 5, the X-axis represents the distribution of the per capita GSDP for the year 2005 and Y-axis represents the same for the year 1993. The Z-axis measures the transition probabilities i.e., the probability with which a part of the distribution of per capita GSDP corresponding to 1993, end up as another part of the distribution corresponding to 2005. The highest peaks of the stochastic kernel represent those parts of the distribution in 1993 and 2005 between which the probability of transition is highest.

The contour plot (figure 6) is the 2-dimensional counter-part of the 3-dimensional stochastic kernel. It is a set of lines, each of which connects all the points in the stochastic kernel with a particular transition probability i.e., the lines in the contour plot connects all the points of the stochastic kernel with a certain probability. In figure 6, since the mass of the distribution is situated around the 45-degree diagonal, the contour plot indicates overall persistence of the states. Two peaks (around 0.5 and 1.5 values in the X-axis) in figure 5 and the concentration of the equal probability lines (iso-probes) in figure 6 show that in these regions, the transition probabilities are the highest. The spread of the iso-probes in both the right and the left of the 45 degree diagonal is the largest around the 150 percent of national average income in figure 6. This shows the fall back of the high income states and increasing relative per capita income of the middle income states. Due to the movements of the states from both directions there exists a local convergence around this point. Similarly, around the 75 percent of national average per capita income, the spread of the iso-probes is higher compared to the middle and the lower part of the distribution. Clearly, in this part of the distribution reflects the fall back of some of the middle income states and the increasing relative per capita income of the low income states compared to their 1993 position leading to a local convergence. Therefore, figure 1 to 6, show the evidence of ‘emerging twin peaks’ and vanishing of middle income groups.
Figure 5

Figure 6
It is possible to throw more light on these trends by identifying the states that have contributed to them. The normalized per capita incomes of the states are presented in table 1. Comparing this table 1 with figure 5 & 6, it is clear that Assam, Jammu & Kashmir, Madhya Pradesh etc. are some of the states which were around the national average in 1993, become relatively poorer and shifted to the left in between 1993 to 2005. Similarly, Gujarat, Himachal Pradesh, Karnataka, Kerala, Tamil Nadu etc. are the states which were also situated around national average, become relatively richer and shifted to the right in the post reform period. Bihar was the poorest state in 1993 and remains the most backward state in 2005 also. Orissa, Rajasthan and Uttarakhand remain almost at their 1993 position and have shown the persistence over the time. As a result of these movements during the post reform period, in the 2005 distribution, there is a tendency of emerging modes at around 1.5 times and 0.75 times of average national per capita income. The highest point of 2005 distribution has shifted slightly leftwards from 1993 distribution as a result of the mobility of some middle income states towards low income values compared to the 1993 distribution, as can be seen from figure 5 and 6.

As explained in the beginning of this section, we have undertaken the same exercise with 18 states, merging the newly formed states of Jharkhand, Chattisghar and Uttarakhand
with their original counterpart (Bihar, Madhya Pradesh and Uttar Pradesh respectively). The univariate kernel density plot for these 18 states in 1993 (figure 7) was unimodal, as in the earlier case, with a long right hand tail showing the existence of a large number of states in the right side of the distribution compared to the left side. As with the earlier case, the distribution of income is unimodal in 1993 but becomes bimodal by 2005. The lower mode is at about 75 percent of national average income and the upper mode is around 150 percent of the same. Tamil Nadu, Gujarat, Kerala, Himachal Pradesh are the states that reached the higher income group around 150 percent of national average per capita income. Bihar and Uttar Pradesh performed badly and constituted the lower convergence club. The ergodic distribution in this case is showing a high peak around 150 percent of national average per capita income, indicating that in the long-run most of the middle income states will be at the higher peak. The 3-dimensional plot of the stochastic kernel (figure 8) and the corresponding 2-dimensional contour plot (figure 9) are also confirming the above tendency. These figures also showing a counter clockwise movement for the higher income states i.e., some of the higher income states like Delhi, Punjab and Maharastra, have performed relatively poorly during the post reform period.
Next, we drop Delhi from our sample of states. As already explained, this is due to the fact already mentioned that Delhi is an artificial city state and its growth dynamics is different from most other states in India.
Here, we now have only 17 states in our sample. In this case results again show a clear shift from unimodality to bimodality and the formation of ‘twin peaks’ i.e., two convergence clubs around 75 percent and 150 percent of national average income (figure 10, 11, 12). The ergodic distribution is showing a clear tendency of most middle income states moving to the higher income convergence club.

![Figure 11](image1)

![Figure 12](image2)
8. Conclusion

In this paper we have studied the issue of regional growth dynamics in post reform India using distribution dynamics approach. The results suggest overall persistence and emergence of ‘twin peaks’ in per capita real GSDP. Though, our study support the earlier findings of ‘twin peak’ by Bandyopadhyay (2006) among 17 major Indian states over the period of 1965 to 1997, we differ from the study in finding the time of the shift of the distribution from unimodality to bimodality is not the mid 90’s but the year 2001. This result is robust in the sense that it holds for different groups of Indian states over the post reform period. Our results are made stronger by merging the newly formed states and also dropping Delhi, which is a city state, from our sample. Our study also shows that some of the middle income states disperse from their 1993 position in two different directions to form two convergence clubs in 2005 at about 75% and 150% of national average. We then identified the club members (states) from their relative per capita values and our estimated stochastic kernel.
TABLE 1a (TWENTY ONE STATES)

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References
Kar, S., Sakthivel, S., 2007. Reforms and Regional Inequality in India, Economic and Political Weekly, November.


Sachs, J. D, Bajpai, N., Ramiah, A., 2002. Understanding Regional Economic Growth in India, Harvard University, CID, working Paper No. 88,