When money matters: liquidity shocks with real effects

John Driffill and Marcus Miller
Birkbeck and University of Warwick
July 2009, revised December 2010

Abstract
How and why do financial conditions matter for real outcomes? The ‘workhorse model of money and liquidity’ of Kiyotaki and Moore (2008) shows how a ‘Big Bang’ enhancement of liquidity can stimulate investment and future aggregate supply, with full employment maintained by flexible prices. But an unexpected contraction of liquidity - though temporary – can lead to Keynesian demand failure if the flex-price assumption is dropped for goods and labour markets in excess supply. Optimistic expectations may speed recovery, but simulation results suggest that prompt liquidity infusion by the central bank is the key to checking sharp recession.

Keywords: Credit Constraints; Temporary Equilibrium; Liquidity Shocks.

JEL codes: B22, E12, E20, E30, E44.

Acknowledgements
While retaining responsibility for the views expressed, we are most grateful for comments by seminar participants at CESifo in Munich, at the IDB and the Federal Reserve Board in Washington; and for discussion with Marco Del Negro, Gauti Eggertsson, Andrea Ferrero, Peter Hammond, Anton Korinek, Andrew Powell, Neil Rankin, Alessandro Rebucci, Peter Sinclair and Jouko Vilmunen. We thank Han Hao Li, Giovanni Melina, and Federico di Pace for able research assistance, funded by World Economy and Finance programme of the ESRC.
‘Surely, the Second Great Contraction – the financial crisis of the late 2000s – will have a profound impact on economics, particularly the study of the linkages between the financial markets and the real economy.’


Introduction

The history of market economies is one of repeated credit booms and busts: and learning from the past is crucial in handling them, as Reinhart and Rogoff (2009) testify. Fortunately, at the level of policy:

In the current crisis, central banks and treasuries around the world, drawing to some degree on the lessons learned during the Great Depression, have reacted with an unprecedented series of moves to inject gigantic amounts of liquidity into the credit market and provide capital to banks. Without these measures, there is little doubt that the world’s financial system would have collapsed as dramatically as it did in the 1930’s.

Liaquat Ahamed (2009, p.500)

What about macroeconomic theory? The New-Keynesian economic paradigm which suited conditions during the Great Moderation, has been faulted for neglecting the role of financial markets and the danger of shocks emanating from them, Blanchard et al. (2009). The need to bring in financial factors has, of course, has been recognized in the light of recent events. Curdia and Woodford (2008) have, for example, introduced financial frictions in a setting with heterogeneous consumers; and, in his Schumpeter Lecture to the European Economic Association, the Deputy Governor of the Bank of England has called for further research on financial factors as an urgent priority, Bean (2009).

The question addressed in this paper is whether the macroeconomic framework developed by Kiyotaki and Moore (2008), where heterogeneous investors face limited recourse to outside finance and resale constraints on financial assets they hold, could be useful in this context. Could it provide a ‘work horse’ macro model which
incorporates the missing financial factors, and also acts as a bridge between RBC and Keynesian economics?

Researchers at the Federal Reserve Bank of New York, working together with Kiyotaki, report that when such restrictions are added to a DSGE model including Calvo contracts changes in credit conditions can have substantial real effects. In an exercise where this framework is calibrated to match the US economy, Del Negro et al (2010), an unanticipated tightening of credit constraints expected to last only 8 quarters leads to a serious recession. Specifically, a temporary shock which reduces the re-saleability of equity by about three-quarters, and reduces Tobin’s q by 10 percent would, in the absence of intervention, lead to a roughly proportional cut in investment, consumption and output, as shown by the dashed lines in Figure 1.

![Figure 1. Effect of a liquidity shock that is expected to last for eight quarters](source: del Negro et al (2010))

The effects of intervention, where the government exchanges highly liquid government liabilities for less liquid private assets, are shown by the solid lines in the figure: the drop in output is limited to 6 percent, with investment falling by 7 percent and consumption by 5 percent.
The focus of the FRBNY study is on the quantitative evaluation of how policy response - in the form of cutting interest rates and injecting liquidity of $1 trillion - can substantially avert these real effects. Rather than examining the details of monetary policy using a large-scale calibrated model, we focus on the analytical properties of the approach to financial frictions recently proposed by Kiyotaki and Moore. To this end, we adopt a two-regime approach, with prices and wages rising in response to an unanticipated loosening of credit, but not falling in response to a sudden contraction. The former matches the flex-price analysis adopted by the authors themselves: the latter is much closer to neo-Keynesian approach of the applied study by the FRBNY. Impulse responses are calculated using the parameters from the latter study: but the structure is kept simple enough so that phase diagrams can be used to illustrate the key results.

We analyse how favourable expectations of a prompt reversal can check the real effects of a liquidity squeeze - whether these expectations are attributable to specific policy actions or to a spontaneous recovery of market sentiment. But, given the contractionary effect of even a short liquidity squeeze, our simulations support the case for dealing with the problem at source: by an asset swap which puts liquidity promptly back into the system.

Our results echo the contention of Eggertson (2008) - that it was Roosevelt’s willingness to challenge the precepts of a balanced budget and a fixed price of gold that helped the US recover from the Great Depression. Paul Tucker (2009) has argued that, in response to the current crisis, Central Banks had to go far beyond the normal parameters of action, acting not only as Lender of Last Resort but also as Market Makers and – in conjunction with the Treasury – as Suppliers of Capital too. Was this willingness of policy-makers to step outside the usual rules of the game to avert market failure the modern equivalent of FDR?
The paper is structured as follows. First, in Section 1, the key features of the flex-price model of Kiyotaki and Moore (2008), hereafter KM, are presented, together with a summary of the formal model. Then, after linearizing and imposing the parameters of the FRBNY, the framework is used to show the positive real effects of a sudden increase in liquidity - ‘Big Bang’ - in moving the economy towards the modified Golden Rule; the role of the Pigou effect in stabilising demand in the short run; and the loss of entrepreneurial ‘rents’ as liquidity constraints are eased. In Section 2, by contrast, we study a negative liquidity shock in a fix-price context, analogous to Del Negro et al. (2009). The effect of expectations is highlighted by contrasting a short versus a protracted liquidity squeeze. Section 3 picks up the theme of boom followed by bust emphasized by Reinhart and Rogoff (2009): as historical movements in the stock market are much larger than can be associated with fundamentals, we discuss whether a liquidity crisis might be triggered by an asset price correction. Finally, we qualify some of the policy conclusions; and suggest that the approach may indeed offer a useful bridge between differing macroeconomic perspectives.

Section 1 The model of KM (2008) in outline and effects of a ‘Big Bang’ in liquidity

Although the original KM approach is substantially modified by the assumption of wage and price stickiness in the application by the FRBNY and in Section 2 below, we start here with the original flex-price approach. Specifically, we use a somewhat simplified version to look at the effects of an unexpected relaxation of liquidity constraints - which, by analogy with the freeing-up of UK capital markets during Mrs. Thatcher’s first administration, we refer to as a ‘Big Bang’. (We also consider how the impact will be magnified if a future round of financial easing is expected to ensue.) Impulse responses, calculated using parameters from the FRBNY study, are reported in the Appendix, but the simplifications we have adopted allow us to illustrate the outcomes using standard phase diagrams.
Key features of the KM framework: a brief overview

As an alternative to the representative agent assumption of DSGE models, the key idea is that investors are *ex ante* identical, but only a fraction actually turn out to have ideas that will generate investment in the current period. This is rather like the specification of Diamond and Dybvig (1983) in their classic paper on banking, where agents identical *ex ante* turn out to have patient or impatient consumer preferences. Here the application is to investment not consumption: but, as for Diamond and Dybvig, there is no insurance market to handle the risk (of needing cash in a hurry).

Rational expectations prevail in the stock market; but credit markets are far from perfect. Workers choose not to hold financial assets and they cannot borrow: so households are income constrained, with all wages are spent on consumption. Entrepreneurs optimise over time but they face limits in terms of new equity finance available and of re-selling existing shares to finance investment - and there are no banks to supply loans.

These constraints on inter-temporal arbitrage lead to a Hicksian type of temporary equilibrium, with a precautionary demand for money by entrepreneurs to ensure that investment opportunities are not wasted. As the reformulated relations do include inter-temporal optimising behaviour by entrepreneurs, the KM approach could be described as Dynamic Stochastic Temporary Equilibrium, DSTE. In sharp contrast to the fix-price Hicksian economics, however, prices and wages are perfectly flexible and there is continuous market clearing with full employment due to the operation of a Pigou effect. Conditional on the current capital stock, the clearing of goods and money market determines the aggregate price level and the real price of equity: and the investment equation determines the evolution of the capital stock.

Formal structure of KM model

*Entrepreneurs:*
KM take an economy consisting of entrepreneurs and workers. Entrepreneurs, who own capital and financial assets, are responsible for organising production and for all real investment. Their objective function is to maximise the expected present discounted utility value of current and future consumption, i.e.
\[ E \sum_{t=0}^{\infty} \beta^{t+1} \log(c_t) \]  

with \( \beta (0 < \beta < 1) \) the discount factor. They can employ labour \((l_t)\) and capital \((k_t)\) to produce general output \((y_t)\), using a constant-returns-to-scale Cobb-Douglas production function with capital share \( \gamma \) and productivity parameter \( A_t \)

\[ y_t = A_t k_t^{\gamma} l_t^{1-\gamma} \]  

Entrepreneurs can also invest, i.e. convert general output into capital goods, but are only able to do so when they have ‘an idea’ for an investment project. These arrive randomly, with probability \( \pi \) each period. Given large numbers, it may be assumed that a given fraction \( \pi \) of entrepreneurs receive an idea each period, and the remaining \((1-\pi)\) does not.

Entrepreneurs can finance investment by issuing equity claims to the future returns from newly produced capital; but, owing to limited commitment, they can only do this against a fraction \( \theta \) of the new capital investment they undertake. Because of this ‘borrowing constraint’, entrepreneurs can use their own money holdings, which are perfectly liquid and can be spent immediately, and/or sell the shares they own in existing firms to finance real investment. But access to financial markets is also restricted by a ‘resaleability constraint’- only a fraction \( \varphi \) of these holdings can be sold each period- representing the illiquidity of equity in the model. (As a simplification, KM assume that after one period, the equity held by an entrepreneur in his own firm is just as liquid as the equity in other firms.)

As a result of this, an entrepreneur who enters the period with holdings of equity \( n_t \) and holdings of money \( m_t \), and who has an investment idea, can invest an amount \( i_t \), which must satisfy the constraints that at least a fraction \( (1-\varphi) \) of initial equity (after allowing for depreciation at rate \( \lambda \)) is retained and at least an amount of new equity \((1-\theta)i_t\) in the new capital is retained. Therefore the entrepreneur holds equity \( n_{t+1} \) at the start of the next period satisfying

\[ n_{t+1} \geq (1-\theta)i_t + (1-\varphi)\lambda n_t \]  

and money balances

\[ m_{t+1} \geq 0 \]

The spending of the entrepreneur on consumption \( c_t \) and investment \( i_t \), together with acquisition of new money balances and equity, satisfies the budget constraint
\[ c_t + i_t + q_t(n_{t+1} - i_t - \lambda n_t) + p_t(m_{t+1} - m_t) = r_t n_t \]  

In this equation, \( q_t \) denotes the price of a unit of equity, and \( p_t \) the price (in terms of goods) of one unit of money; and \( r_t \) is the rate of return on capital.

Workers:

The role of workers, who do not have investment opportunities cannot borrow against future labour income, is much more straightforward. They supply labour and consume goods. In principle they may hold money and equity to smooth consumption and labour supply over time: but they choose not to do so, as the rates of return they earn on these assets are less than their rate of time preference. Workers supply labour as an increasing function of the real wage \( w_t \):

\[ I^* = \left( \frac{w_t}{\omega} \right)^{(1/\nu)} \]  

where \( \omega \) and \( \nu \) are preference parameters.

Labour Markets:

The labour demand of entrepreneurs is determined by the marginal productivity of labour, and, when wages and prices are flexible so that we have labour market clearing, labour supply equals labour demand, and:

\[ \left( \frac{w_t}{\omega} \right)^{(1/\nu)} = K_t \left( 1 - \gamma \right) A_t / w_t \left[ (1/\gamma) \right] \]

This ties down the real wage rate and the marginal product of capital as functions of the capital stock:

\[ r_t = a_t \left( K_t \right)^{a-1} \]

with \( a_t = \gamma \left( \frac{1 - \gamma}{\omega} \right)^{1+\gamma} A_t^{1+\gamma} \) and \( \alpha = \frac{\gamma(1+\nu)}{\gamma+\nu} \), and \( K_t \) is the aggregate capital stock of the economy.

Real Investment:

When the value of capital \( q_t \) exceeds one, entrepreneurs who have an investment idea will issue as much equity as they can, and sell as much of their existing equity holdings as possible, given the credit limits given above, and they will use all their holdings of money to invest. Thus their flow of funds is:

\[ c_t + (1 - \theta q_t)i_t = (r_t + \lambda \phi q_t)n_t + p_t m_t \]
They carry no money forward to the next period. Taking account of the liquidity constraints, the equity held over to the next period satisfies:

\[ c^i_t + q^R_t n_{s+1} = r n_t + [\phi q_t + (1 - \phi) q^R_t] \lambda n_t + p m_t \]  

with \( q^R_t = \frac{1 - \omega q_t}{1 - \theta} \), where the right hand side of the equation denotes the entrepreneur’s net worth at the start of period \( t \). With log utility, these entrepreneurs are assumed to consume a fraction \( (1 - \beta) \) of this each period:

\[ c^i_t = (1 - \beta) \left\{ r n_t + [\phi q_t + (1 - \phi) q^R_t] \lambda n_t + p m_t \right\} \]  

and therefore they invest an amount:

\[ i_t = \frac{(r + \lambda \phi q_t) n_t + p m_t - c^i_t}{(1 - \theta q_t)} \]  

**Financial Assets:**

Things are different for entrepreneurs who do not have an investment idea. They accumulate money and equity to build up resources for use in future period if an investment opportunity comes along. Their flow-of-funds constraint is simply

\[ c^s_t + q n_{s+1} + p m_{s+1} = r n^s_t + q_\lambda n^s_t + p m^s_t \]  

showing the value of net worth on the right-hand side. The superscript, \( s \), against their holdings of money and equity and consumption in equation (13) distinguishes these as variables referring to non-investing entrepreneurs. Optimal consumption for these entrepreneurs is once again a fraction \( (1 - \beta) \) of net worth:

\[ c^s_t = (1 - \beta) \left\{ r n_t + q_\lambda n_t + p m_t \right\} \]  

The non-investing entrepreneur has to decide what fraction of assets to put into money and how much into equity. The marginal utility of consumption in period \( t \) has to equal the discounted expected marginal utility of holding additional units of money into period \( t+1 \) and consuming them then. Also, it must equal the expected discounted utility of holding additional equity into period \( t+1 \). Thus we have KM’s equation (21) for portfolio balance:

\[ u'(c_t) = E_t \left\{ \frac{P_{t+1}}{P_t} [(1 - \pi) u'(c^s_{t+1}) + \pi u'(c^i_{t+1})] \right\} \]

\[ = (1 - \pi) E_t \left\{ \frac{r_{t+1} + \lambda q_{t+1}}{q_t} u'(c^s_{t+1}) \right\} \]

\[ + \pi E_t \left\{ \frac{r_{t+1} + \lambda \phi_{t+1} q_{t+1} + \lambda (1 - \phi_{t+1}) q^R_{t+1}}{q_t} u'(c^i_{t+1}) \right\} \]  

\[ (15) \]
Aggregate relationships:

The above analysis describes the behaviour of individual entrepreneurs. It is necessary to aggregate across all entrepreneurs to find how the economy as a whole evolves. The expressions for consumption and investment of each type of entrepreneur are linear in start-of-period holdings of equity and money, which simplifies matters considerably. As KM note, a fraction $\pi$ of aggregate capital $K_t$ and money $M_t$ is held by investing entrepreneurs, therefore aggregate investment is:

$$(1 - \theta q_t)I_t = \pi \{ \theta [r_t + \lambda \phi q_t]K_t + p_t M_t - (1 - \beta)(1 - \phi t)\lambda q_t^R K_t \}$$

(16)

The aggregate demand equation, balancing the net output of goods with the demand for investment plus consumption from the two types of entrepreneurs implies:

$$r_t K_t = a_t K_t^S = I_t + (1 - \beta) \bullet \left[ r_t + (1 - \pi + \pi \phi_t)\lambda q_t + \pi(1 - \phi_t)\lambda q_t^R \right] K_t + p_t M_t.$$  

(17)

The aggregate portfolio balance equation is obtained by aggregating over the wealth of the non-investing entrepreneurs. They buy equity in the amount $\theta I_t$ from the investing entrepreneurs, and a fraction $\phi$ of their depreciated equity $\pi \lambda K_t$. The non-investors also retain the depreciated equity they carried over from the preceding period. Therefore their equity holdings at the start of period $t+1$ are $N_{t+1}$, defined as:

$$\theta l_t + \phi \pi \lambda K_t + (1 - \pi) \lambda K_t \equiv N_{t+1}^S.$$  

(18)

The non-investing entrepreneurs also hold all the money stock $M_t$. As utility is logarithmic, marginal utility is the reciprocal of consumption. The portfolio balance equation, (15) above, then becomes, at the aggregate level:

$$(1 - \pi)E_t \left[ (r_{t+1} + \lambda q_{t+1})/q_t - p_{t+1}/p_t \right] / (r_{t+1} + \lambda q_{t+1})N_{t+1} + p_t M_t = \pi E_t \left[ p_{t+1}/p_t - \left[ r_{t+1} + \phi_t t + (1 - \phi_t)\lambda q_{t+1}^R \right]/q_t \right] / (r_{t+1} + \phi_t t + (1 - \phi_t)\lambda q_{t+1}^R)N_{t+1} + p_{t+1} M_t.$$  

(19)

Finally the aggregate capital stock evolves as:

$$K_{t+1} = \lambda K_t + I_t,$$  

(20)

where $(1 - \lambda)$ is the rate of depreciation.
Reducing the model to a first order system in K and q

These four equations (17) - (20) define the dynamic system, whether in the flexible-price or fixed-price demand-deficient mode. Before looking at the dynamics, we first compute the steady state, assuming that the liquidity constraints are such that precautionary holding of money is justified. These equations can be reduced to three relationships in the steady state, written as follows:

\[ \pi \beta r + \pi \beta \frac{bM}{K} = 1 - \lambda + \pi \lambda (1 - \beta) \left( \frac{1 - \phi}{1 - \theta} \right) (1 - \theta q) - \pi \beta \lambda \phi q \]  \hspace{1cm} (21)

\[ \beta r - (1 - \beta) \frac{bM}{K} = 1 - \lambda + \pi \lambda (1 - \beta) \left( \frac{1 - \phi}{1 - \theta} \right) + (1 - \beta) \left( 1 - \pi \left( \frac{1 - \phi}{1 - \theta} \right) \right) \lambda q \]  \hspace{1cm} (22)

\[ r - (1 - \lambda) q = \pi \lambda \left( \frac{1 - \phi}{1 - \theta} \right) (q - 1) \frac{q + pM / (\chi K)}{r + \lambda \left( \frac{1 - \phi}{1 - \theta} \right) + \lambda (\phi - \theta) + \frac{pM}{\chi K}} \]  \hspace{1cm} (23)

These three equations (corresponding to KM’s equations 26-28) determine three unknowns: \( pM/K, r, \) and \( q \). The first two can be solved for \( pM/K \) and \( r \) as linear functions of \( q \). When these solutions are substituted into (23), this can be solved as a quadratic in \( q \), and we select the economically meaningful of the two solutions.

Having found the stationary state, we take linear approximations around it, and reduce the model to a system of two first-order, linear difference equations, one in \( K \) and one in \( q \). The analysis may be compared with the canonical model of investment under adjustment costs, also known as the ‘q-theory of investment’, as described in Acemoglu (2009, page 269 ff): here, however, the adjustment costs of changing capital stock are effectively replaced by liquidity constraints.

The flexible-price solution

In flexible price mode, the investment equation, (16) above, and the aggregate demand equation, (17), determine \( p_t \) and \( I_t \) as functions of \( K_t, q_t, \varphi_t \), and the other parameters of the model \( (M, \pi, \theta, \lambda, \beta) \). We treat \( \varphi_t \) as a constant \( (\varphi_t = \varphi \text{ for all } t) \), and add it to the list of fixed parameters. The return on capital \( r_t \) is a function of the capital stock \( K_t \) and various parameters of the model. These functions can then be substituted into the portfolio balance equation, in place of \( r_t, p_t, \varphi_t, \) and \( I_t \). The portfolio
balance equation is then reduced to an equation in \( q_{t+1}, K_{t+1}, K_t, \) and \( q_t \). We then have a first order dynamic system in two variables, \( K_t \) and \( q_t \).

The two equations that describe the evolution of \( K_t \) and \( q_t \) are the capital accumulation equation, (20) above, and the portfolio balance equation, where \( p \) is ‘solved out’ using the aggregate demand equation. This is a non-linear system, which we solve by linearizing around steady state values for \( K \) and \( q \). How does it work?

**A ‘Big Bang’ in financial development**

We start by considering an unanticipated loosening of these credit constraints – which we interpret as a ‘Big Bang’ in financial development. What we find is that an increases in the ‘resaleability constraint’ \( (\phi) \) increases corporate investment and generates real economic improvement as the economy moves to higher long-run levels of capital, output and consumption. In the short-run, however, consumption by entrepreneurs is crowded out by the increase in their investments - the mechanism for doing this being a jump in price level which cuts the value of real balances, the Pigou effect. Even in the long run, the level of entrepreneurial consumption falls.

Entrepreneurs choose to invest more when liquidity is made more available, because they take return on capital as parametric in making investment decisions; but the latter, in aggregate, cause the rate of return to fall.

In Figure 1, E and E’ are the steady state equilibria associated with *permanently* low and *permanently* high levels of liquidity, and the corresponding stable and unstable eigenvectors indicate saddle point dynamics. In equilibrium, higher liquidity leads to more investment, capital and aggregate consumption,\(^1\) so E’ lies to the right of E. Assuming for convenience that the system begins at E, the impact of an unexpected liquidity increase is shown by the jump in asset prices to B, followed by a convergence to E’ along the new stable eigenvector\(^2\). Full employment prevails throughout, by assumption, with the potential excess demand checked by the jump in prices and reduction in real balances, i.e. there is a Pigou effect at work. (Due to the impact of price increases on the real balances held by entrepreneurs, their

---


\(^2\) In a discrete time model, there will be an impact effects on \( K \) as well as \( q \); while these are not shown explicitly in the diagram, they appear in the simulation results.
consumption will fall immediately as $\varphi$ is increased: for entrepreneurs who take the price of money as parametric, the Pigou effect is a negative externality.)

Figure 1. A ‘Big Bang’ in liquidity: an unexpected increase in $\Phi$.

The impulse responses reported in Appendix 1 confirm that reducing financial frictions can have substantial real effects even with flexible prices and wages. The added investment leads to an increase of the capital stock and an increase in output; but the rate of return on capital declines. A 20% increase in the resaleability constraint (so 15.6% of shares can be sold per period rather than 13%) increases the equilibrium capital stock by over one percentage point.³

What if the current improvement of liquidity leads agents to anticipate (correctly) a further round of liquidity enhancement, to be implemented $T$ periods later? While the first improvement comes as a surprise, the second will be fully anticipated, so there

³ These effects will be more pronounced than those described in the KM paper, where $\varphi$ follows a two-state stochastic process, so a reversal is anticipated.
should be no jump in the nominal price of equity, \( p_e \), when it occurs\(^4\). The implications may be seen from Figure 2 where the path for the real share price and for capital accumulation is indicated as EBICE'. The initial jump in the real price of equity is larger than before - in anticipation of future liquidity expansion. The real equity price will, however, jump down when the latter occurs. The no-arbitrage condition at time T is captured by the schedule AA, constructed so that the fall in the real asset price will match the rise in the nominal price level when re-saleability constraint is relaxed for the second time. Thus, asset prices will reach I at the end of period T-1, falling to C on the stable eigenvector leading to E' when \( \phi \) increases.

\[ q, p_e \]

\[ K*, K**, K \]

\[ q \]

\[ A, B, C, E, E', N \]

\[ S_{F, m}, S_{F, h}, S_{F, i} \]

**Figure 2. ‘Big Bang’ - with anticipated encore**

Despite the relaxation of liquidity constraints, we have been assuming money has value for precautionary reasons. If the constraints were relaxed sufficiently, however, the economy would move to a non-monetary equilibrium\(^5\), such as N in the Figure 2,

\(^4\) This is the no-arbitrage condition that KM apply in discussing deterministic productivity changes.

\(^5\) See KM(2008), Claim 3.
where $q$ is equal to 1 and the marginal productivity of capital equals the rate of depreciation plus the rate of time preference$^6$. The excess of profit over the needs for replacement investment constitutes the flow of entrepreneurial consumption in this modified Golden Rule equilibrium; and we find that entrepreneurial consumption is less than in the liquidity constrained equilibria studied above$^7$. The easing of financial constraints evidently dissipates the rents entrepreneurs enjoy from liquidity shortage.

Missing from this highly schematic account is an explicit role for monetary policy: a commitment to inflation targeting – as in Del Negro at al. (2009) study - would, of course, involve a tightening monetary policy to offset the temporary inflationary effect of the liquidity enhancement.

**Section 2: Negative liquidity shocks and recession**

We come now to the main focus of this paper, the impact of negative liquidity shocks on the real economy using the framework of KM (2008) but assuming that prices and wages are not flexible downwards. The fit of the original Phillips curve to UK data 1861 -1913 provides empirical evidence for sticky prices under the Gold Standard for example$^8$. More recently, New Keynesian macro-economists have chosen to capture temporary wage-price stickiness by the analytical device of Calvo contracts, which allow for gradual revision in response to expected future events, e.g. Woodford (2003), Del Negro et al. (2009). In such specifications, however, there is no asymmetric response to positive and negative news. While one could impose an asymmetry in Calvo framework to capture the downward rigidity of wages and prices, Bewley (2002), we adopt instead a two-regime approach, switching to a fixed price regime in situations where there is excess supply.

With fixed prices, there will be no Pigou effect to stabilise aggregate demand in the face of a fall of investment, so a contraction of liquidity may lead to failure of market

---

$^6$ For the parameters used, the modified golden rule capital stock is about 10% higher than the base case.

$^7$ Relative to the baseline case in appendix 1, entrepreneurial consumption in the modified Golden Rule equilibrium falls by approximately 10%.

$^8$ As the breakdown of the Phillips Curve in the 1980s suggests, however, such price stability is regime dependent.
clearing in goods and labour markets, as in the fix-price macroeconomics of the 1970s. Assuming that the real wage is determined by bargaining, as in Layard and Nickell (1987) and Manning (1990), workers will be laid off as demand contracts. For convenience, we assume that at full employment the bargaining wage is close to the market clearing wage; and that this real wage is maintained even when the demand for labour falls.

**The fixed-price solution**

In fixed-price mode, assuming that there is excess supply of labour and goods, the same equations determine the dynamics of the system around steady state. However, some things change.

We take prices and wages as fixed: they are treated as fixed parameters in the analysis. As in the flexible-price case, we also treat \( \varphi_t \) as a constant (\( \varphi_t = \varphi \) for all \( t \)). In this case, in Keynesian fashion, aggregate demand from entrepreneurs for consumption and investment determines their income \( r_tK_t \); and the rate of return, \( r_t \), is no longer a simple function of the capital stock \( K_t \). Equations (16) and (17) now determine \( r_t \) and \( I_t \) as functions of \( K_t \) and \( q_t \) and the other parameters of the model (\( M, \pi, \theta, \lambda, \beta \) and \( \varphi \)) – and now we add \( p = p_t = p_{t+1} \) to the list of fixed parameters.

Having substituted these functions for \( r_t, r_{t+1} \), and \( I_t \), and taking \( p \) is fixed, the portfolio balance equation gives a relation between \( q_{t+1}, K_{t+1}, K_t, \) and \( q_t \). Together with investment equation, this yields a non-linear first-order difference equation system in the same two variables as in the flexible price case, namely \( K_t \) and \( q_t \).

**Aggregate demand for net output\(^9\) and goods market equilibrium**

Before turning to impulse responses for the complete model in the fixed price case, we describe briefly how a liquidity contraction can affect entrepreneurial income and national product for a given \( K \) and \( q \), i.e. we solve for the rate of return on capital conditional on \( K \) and \( q \).

\(^{9}\) i.e. output less what is consumed by employees, \( y_t - wI_t = x_t \).
First, we note that for a firm with the production function described by equation (2), which adjusts output by varying employment at a constant real wage $w$, the residual income available to entrepreneurs ($x$), the excess of production over the wage bill, varies with production $y_t$ as follows:

$$x(y_t; w_t, k_t) = y_t - w_l = y_t - w \left( \frac{y_t}{A_t} \right)^{\frac{1}{\gamma}} k_t^{-\frac{\gamma}{1-\gamma}}$$

(24)

Expressed as a rate of return on the (constant) capital employed, this may be written for brevity as:

$$r \equiv \frac{x_t}{k_t} = r(y_t) = \frac{y_t}{k_t} - w \left( \frac{y_t}{A_t k_t} \right)^{\frac{1}{1-\gamma}}$$

(25)

which is increasing in $y_t$ in the range from 0 to the point where the marginal product of labour equals the real wage. Where $y_t$ is demand determined, the relation between the rate of return on capital and the quantity of capital implied by equation (8), no longer applies: it is replaced by equation (25).

Since the price level is fixed, there will be no Pigou effect to ensure full employment. The level of output (and hence the return on capital) adjusts to bring supply and demand into balance. ‘While workers spend what they earn, entrepreneurs earn what they spend’, as Kalecki put it.

Turning to aggregates, we note that, from equation (16), other things being equal, the marginal effect of an increase in $r_t$, as defined in equation (25), on investment demand is:

$$dI_t = \frac{\pi \beta K_t dr_t}{1 - \theta q_t}$$

and on entrepreneurial consumption is:

$$dC_t = (1 - \beta) K_t dr_t .$$

Hence the total effect of an increase in $r_t$ on entrepreneurial income is:

$$\left( \frac{\pi \beta}{1 - \theta q_t} + (1 - \beta) \right) K_t dr_t$$
For stability at an interior solution (with excess supply of labour), we need

\[ \mu = \left( \frac{\pi \beta}{1 - \theta q_t} + (1 - \beta) \right) < 1 \quad \text{or} \quad \frac{\pi}{1 - \theta q_t} < 1, \]

where \( \mu \) denotes the marginal propensity to spend out of entrepreneurial income.

As we are assuming \( q_t > 1 \), a necessary condition is that \( \pi + \theta < 1 \); i.e. there is a stability restriction on ‘induced investment’ such that the fraction of entrepreneurs who have new ideas plus the fraction of new investment they can fund via new equity issues must be less than 1.

The ‘fix-price macroeconomic’ framework we use here is much as described earlier in the writings of Benassy (1975) and Malinvaud (1977) for example, as is illustrated in Figure 3. The bottom panel on the left shows how the wage bill varies with employment at the fixed real wage. The bottom right panel shows how profits, \( X \), the residual income available to entrepreneurs, fall away as employment contracts. So too does demand by entrepreneurs as shown in the top panel, where the marginal propensity to spend is \( \mu \). Note that, for convenience, demand is shown at a constant real share price and constant \( K \).
Figure 3. Short-run determination of entrepreneurial income, X, and gross output, Y.

The figure illustrates how a fall in investment demand due to a fall in liquidity - represented by the downward shift in D(X) in the top panel - will lead to a greater contraction of entrepreneurial income, X, as equilibrium shifts from E to D. The impact on employment is even more pronounced as shown by the shift from E* to D* in the lower right panel.

Phase Diagram in K and q

As before, our approach is to solve the model by simulation, and illustrate the results using phase diagrams with K predetermined and q a jump variable. Figure 4 shows

---

10 To limit the impact on employment in the simulations below, it is assumed that the initial equilibrium is one where the marginal product of labour is five percent above the real wage.
how the capital and real price of equity evolve, assuming that the model remains in the fixed-price regime throughout. On the schedule labelled IB for Investment Balance in Figure 4, all investment is for replacement so the capital stock will be stationary: and the parameters of the model confirm that IB slopes upwards.

![Figure 4. Capital accumulation and stock market](image)

Likewise, stationary values for the stock market price define the asset market equilibrium given by the downward sloping schedule labelled PB in the figure. Stationary equilibrium is at E where they both intersect. Given the saddle point dynamics, the stable path to equilibrium will slope downwards, see SS in the figure. (The unstable eigenvector will have a positive slope). Also shown are integral curves that asymptotically approach SS and UU. This is the ‘workhorse model’ used to discuss the effects of a negative liquidity shock, with detail given by impulse responses.

---

11 In fact, as discussed a companion paper, there may be a regime change as recovery takes place: the switch of regime occurring when the economy reaches its capacity constraint.
A temporary decline in liquidity

What are the effects of a negative liquidity shock? Assume that the economy starts at its high employment, steady-state equilibrium $E$, as depicted in Figure 5, but the shock throws it into a demand-deficient regime. Since it tightens the financial constraints on firms who want to invest and since workers are income-constrained (with no Pigou effect to stimulate consumption of entrepreneurs), the impact will be to reduce entrepreneurial income (as discussed above) and the asset price. As this reduces the incentive to invest and also the attractiveness of equity, this will shift both schedules $IB$ and $PB$ to the left, so equilibrium moves left from $E$ to $E'$, as shown in Figure 5.

![Figure 5. A liquidity shock – capital decumulation and the stock market](image)

The immediate impact of the liquidity squeeze on the stock market will depend on how long the shock is expected to last. If the shock was expected to be permanent, the market would fall to $P$ on the new stable eigenvector $S_{\phi_1}^K S_{\phi_1}^K$, before converging to $E'$.

---

12 For present purposes, we assume that the economy remains in a demand deficient regime even when liquidity is restored.
The trajectory EDILE applies when the liquidity squeeze is expected to remain in force until \( T \) periods have elapsed.

This trajectory is constructed on the assumption that there will be a regime switch when the liquidity is restored, i.e. the relevant stable eigen-vector, \( S_{\phi_n}^F S_{\phi_n}^F \), is that which applies in the flex-price regime. As the regime switch will be fully anticipated, the no arbitrage equation at time \( t \) is represented by the schedule AA, constructed (as in Figure 2) such that fall in the real asset price will match the rise in the price level when the resaleability constraint is restored to the previous level. Thus the real price of equity will reach I at the just before time \( T \), with no jump in the nominal price of equities as the path joins the new stable eigen-vector at \( L \).

The impulse responses for a 20% cut in \( \phi \) expected to last for 2 and 8 years are given in Appendix 2. The effects on the economy are shown in Table 2 below, with numerical outcomes of the simulations illustrated in Figure 6.

The magnitude of initial jumps depend on the expected length of liquidity squeeze, whether short, long and permanent, as follows:

<table>
<thead>
<tr>
<th></th>
<th>Short (2 years)</th>
<th>Long (8 years)</th>
<th>Permanent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>-1.25%</td>
<td>-2.86%</td>
<td>-3.57%</td>
</tr>
<tr>
<td>( r )</td>
<td>-10.90%</td>
<td>-12.23%</td>
<td>-12.50%</td>
</tr>
<tr>
<td>( X )</td>
<td>-10.27%</td>
<td>-11.48%</td>
<td>-11.73%</td>
</tr>
<tr>
<td>( y )</td>
<td>-18.65%</td>
<td>-20.54%</td>
<td>-20.92%</td>
</tr>
</tbody>
</table>

*Table 2. Impact effects of a 20% cut in \( \phi \) for different lengths of time*
Figure 6. Numerical Results from simulation.

It turns out that the pattern of events is similar whether the squeeze is expected to last for a long time or not: all variables except for $K$ fall sharply in the first period then recover as the end of the liquidity squeeze is anticipated. The asset price recovers and ‘over-shoots’ before returning to equilibrium. The capital stock remains unchanged in period 1, but then keeps contracting until liquidity is restored. The initial impact on output is surprisingly large, and does not differ greatly for the different horizons – for the parameters used by FRBNY, we find that output falls by more or less the same percentage as the liquidity squeeze (see the bottom line of table 2).

A prompt restoration of liquidity does, however, lead to a much faster recovery in the economy. It takes two years for the output loss to fall to 16% where liquidity to be restored promptly, but 6 years if not. The difference between these two trajectories is, of course, guided by the behaviour of $q$ – the forward-looking asset price – which falls much less and recovers more promptly when expectations are more optimistic, as indicated by the trajectory ERI'E shown in Figure 7.
Figure 7. Prompt liquidity easing shortens recession

Expectations of prompt restoration of liquidity do help, but if the simulations are to be believed, they do not prevent the economy from experiencing a serious recession. In the framework of this type of model, however, the authorities can take direct action to bring the markets back to life: the central bank, in particular, can purchase equity with money. As KM point out:

When the resaleability of equity falls with an arrival of liquidity shock, the central bank can do the open market purchase operation, increase the liquidity of an investing entrepreneurs, then the quantities and asset prices will be insulated from the liquidity shock… The unorthodox policy of the Federal Reserve Bank and the Bank of England, such Term Security Lending facility, is an attempt of increasing the liquidity by supplying more treasury bills against partially resalable securities, such mortgage backed securities.

Open Market Operations

What if policy makers do take ‘prompt corrective action’ to avert recession? This is the issue that Del Negro et al. (2009) examined in their model with sticky prices and credit constraints like those being considered here. As reported above, they find that – on the ‘conservative’ assumption that the expected duration of the credit crunch is only 8 quarters – an unanticipated tightening of both resale constraints ($\phi$) and borrowing constraints ($\theta$) leads to a 10 percent fall of output: but this contraction can be mitigated by an open market operation where the illiquid assets are purchased in substantial quantities by the central bank. Indeed, the claim made by the team from FRBNY is that, by injecting a trillion dollars into the financial markets in 2008-9, the Federal Reserve engineered a ‘Great Escape’ for the US economy.

It is clear from the structure of KM model - and their own discussion - that a swap of money for illiquid equity in the hands of would act directly to ease the liquidity constraint. To analyse this properly would involve adding more detail on the role of the government, something we do not do here. But the effect of the open market operation is pretty clear: it will reverse the leftward shifts of the PB and IB schedules induced by the liquidity squeeze as discussed earlier.

Eggertsson (2008) argues that it was Roosevelt’s willingness to take decisive fiscal action that helped the US escape from the Great Depression. Was the readiness of the central bank to take the extraordinary steps needed to preserve financial and economic stability the 21st century equivalent? While the Bank of England in its Quantitative Easing mainly purchased long-dated government securities, the Fed went a good deal further in lending against private sector ABSs and MBSs. As Sebastian Mallaby (2010) has noted:

‘…the Fed frequently ignored Walter Bagehot’s dictum that central banks should provide liquidity freely, but against good collateral and at high interest rates. The Fed’s borrowers included institutions such as Lehman and Citigroup, which were insolvent rather than illiquid. It accepted collateral that included toxic asset-backed securities, and it charged interest rates that were more palliative than punitive. Moreover, while the Fed took all these risks with US taxpayers’ money, a large chunk of its emergency lending went to foreign banks.’
So far, however, the analysis has focused on the central bank as ‘market maker of last resort’, Tucker (2009). What about fiscal stimulus? In models of this kind, where credit constraints are prevalent, the Ricardian equivalence theorems used to demonstrate the irrelevance of fiscal policy will not generally apply. So, active fiscal policy will help recovery. The multiplier effects of public spending will be regime dependent, however. Extra spending by public agencies, which can lift output and employment when the economy is in a demand-constrained recession, will lead to ‘crowding out’ in the flex-price fully employed economy. The policy implication is that the restoration of liquidity calls for a scaling back of fiscal stimulus.

Section 3. Bubbles and Irreversibility

While it allows for financial frictions, the model being used assumes that assets are correctly priced, so the variable q has limited volatility. Historical evidence on Tobin’s q, especially up to and during the Great Depression, paints a very different picture. As shown in Figure 8 below, prior to the stock market crash of 1929 the US enjoyed a substantial investment boom - with the real capital stock increasing by more than 3 percent a year since 1925: but the value of stock market, as measured by Tobin’s q, increased much faster, more than doubling over the same period.

Source: (US) Bureau Economic Analysis and Stephen Wright (2004): note that the capital stock is valued at 2005 replacement cost.
Then, in two short years, the stock market fell by more than 70% relative to replacement cost, and the capital stock began literally to contract. These were the years of the Great Depression, when the US banking system collapsed and unemployment grew to over 20% - leading Roosevelt to declare war on unemployment and Keynes to develop the theory of demand-determined GNP, published in 1936.

A run-up in asset prices can, of course, be captured in the KM model by looking at the integral curves that do not satisfy the transversality condition, as in Figure 9 where the integral curve above the stable manifold no longer correctly represents future fundamentals, but is simply a bubble.\(^\text{13}\)

\[q\]

\[K\]

\[U\]

\[E\]

\[E'\]

\[U'\]

\[B\]

\[K^*\]

\[K^{**}\]

\[S^K\]

\[S^F\]

\[S^F_{\phi_1}\]

\[S^F_{\phi_2}\]

Figure 9. Bursting of asset bubble leading to liquidity contraction

It was in fact, only after the substantial restructuring of the financial system – including the setting-up of FDIC, passage of the Glass-Steagall act, changes in

\(^{13}\) Abreu and Brunnermeier (2003) discuss how such mispricing may be sustained for some time by heterogeneous beliefs.
bankruptcy law and strengthening of securities regulation - that asset prices were able to recover.

Perhaps the historical data for the Great Depression could be interpreted as follows: the fall in the stock market after a prolonged and explosive bubble led – in the absence of prompt corrective action by the central bank – to a succession of liquidity shocks whose effects were only finally reversed by the restructuring of the financial system. In addition, the fall in asset prices could have led directly to a cut in aggregate demand.\(^{14}\)

Excessively overvalued \(q\) is one of the important features missing from the model, the other is the very low values that were observed in the Great Depression. Could this be attributable to the irreversibility of investment? Irreversibility increases the volatility of asset prices in theoretical models because investment is not undertaken until \(q\) exceeds one by a suitable margin, as firms exploit the option value of not investing. When \(q\) falls below one, firms cannot disinvest as fast as they might wish: they are limited to disinvesting at most at the rate at which capital depreciates. Meanwhile \(q\) can fall to low levels. In the KM model there is no explicit irreversibility, however, and \(q\) remains above 1 in the face of adverse liquidity shocks in our simulation exercises.

This brief discussion of the historical experience suggests that the fall in greatly over-valued stock prices could trigger a direct contraction of aggregate demand due to a wealth effect, as well as an indirect fall in \(q\) due to the liquidity contraction. In 2006-2008, it was house prices that fell, with disastrous effects on investment banking: Morgan Stanley and Goldman Sachs survived only by a timely switch to the status of bank holding companies, but the other major independent investment banks were either taken over or, in the case of Lehman Brothers, went into liquidation – leading to a ‘Great Panic’ marked by a liquidity squeeze.

\(^{14}\) How much would a financial collapse such as the Great Depression affect demand? Assuming the elasticity of entrepreneurial demand with respect to a change in the share price in the Kiyotaki and Moore model is around 0.8, a fall in \(q\) from 2 to 1 would imply a fall in entrepreneurial demand by 4% - roughly a fall of 1.33% of GDP.
Del Negro et al. (2010) also report the outcome of a more extreme scenario where the liquidity shock is expected to last for 8 years instead of 2 (i.e. be of similar duration as the shocks perturbing the Japanese economy during the Great Recession or the US during the Great Depression), and they conclude:

‘Without intervention the equilibrium is a disaster. Output collapses by about 20 percent and deflation reaches double digits. In short, the equilibrium outcome starts looking a bit like the Great Depression.’

Section 4. Conclusion

‘The global financial crisis at the late 2000s… stands as the most serious financial crisis since the Great Depression. The crisis has been a transformative moment in global economic history, whose ultimate resolution will likely reshape politics and economics for at least a generation.’ Reinhart and Rogoff (2009, page 208)

In his assessment of factors causing the Great Depression in the US, Milton Friedman emphasized financial factors - and he blamed the Federal Reserve for not acting to head off cumulative collapse of hundreds of banks; and the recent account of central bank mis-management provided in Ahamed’s ‘Lords of Finance’ adds weight to Friedman’s perspective. By way of contrast, believers in the Efficient Market Hypothesis and Real Business Cycle theory, argue that, in general, financial factors play no causal role in economic booms and slumps. With respect to the current financial crisis, for example, Fama has argued that financial factors simply reflecting prior deterioration in economic fundamentals.

The simple two-regime model used here endorses Friedman’s perspective and offers confirmation of the results obtained by the Federal Reserve Bank of New York using a much more complex DSGE model. The analytical tractability of the second-order system used here allows for qualitative analysis of liquidity shocks and of various expectational effects - including deviations from rational expectations.

The effect of financial accelerators in the credit market as discussed in Kiyotaki (1998), Bernanke et al. (1999) and in Miller and Stiglitz (2009) are not treated here, where the focus is on illiquidity. It has to be said, however, that the model includes no financial intermediation per se: the liquidity squeeze is a failure lending on a bilateral basis between one set of entrepreneurs and another. It would be preferable to include
intermediation explicitly, of course - and this would help link ‘irrational exuberance’ in asset markets to a subsequent liquidity crunch.¹⁵

So the results obtained in this paper for a liquidity squeeze can, perhaps, best be thought of as a reduced form of what happens after a collapse of financial intermediaries. The severe economic effects that follow - and the links with asset mispricing that precede - become much more plausible on this interpretation: but any policy conclusions must be treated with considerable care. If the crisis was due to moral hazard problems in intermediaries, for example, then liquidity injections which fix things in the short run may exacerbate problems in the long run – unless financial re-regulation were to follow.

As has been shown, with flexible prices - and the Pigou effect - a ‘Big-Bang’ can stimulate output and investment and leading in the limit to a modified golden rule equilibrium. But without flexible prices and Pigou effect, tightening credit constraints can convert a situation of efficient inter-temporal optimisation to one of inefficient temporary equilibrium. In this way, the KM approach, as analysed in this paper, can be seen as a bridge between real business cycle theorising on one hand, and Keynesian macro economics on the other. Will this be the impact of recent events – the development of integrated models of heterogeneous agents operating subject to financial constraints, which can encompass different macroeconomic views as special cases?

References


Ahamed, Liaquat (2009), Lords of Finance: The bankers who broke the world. London: William Heinemann


¹⁵ Before the Great Depression, banks lent heavily to those speculating on shares using the shares themselves as collateral; in the current ‘Sub-prime crisis’, shadow banks have performed a similar role in respect of real estate.


Blanchard, Olivier, Giovanni Dell’Ariccia, and Paolo Mauro (2010), ‘Rethinking Macroeconomic Policy’. IMF Staff Position Note, SPN/10/03, (February).


Woodford, Michael (2003), Interest and Prices, Princeton NJ: Princeton University Press

Appendix 1

Simulation results in the flex-price quarterly model

For an unexpected 20% increase in liquidity, q will immediately jump up by 0.30% percent, converging thereafter back to its original value; r, k and y, on the other hand, do not jump in the period when the ‘Big-Bang’ occurs, they will start to converge to their new equilibrium level monotonically in the next period, as shown below:
Memo items

Following Del Negro et al., the parameters for the linearized KM model used here are chosen as: $\varphi = 0.13; \beta = 0.99; \theta = 0.13; \pi = 0.075$ and $\lambda = 0.975$.

Initial (‘base case’) equilibrium values of the variables are then:
$q = 1.1175; r = 0.0374; K = 152.5056, y = 17.2644$ and $M_p/K = 0.1171$.
The eigen-values for the base case are 0.9837 and 1.1010.
Slope of stable eigen-vector: -0.0016.

Appendix 2

Simulation results in the fix-price quarterly model

What would be the effect of a 20% cut in the liquidity parameter $\varphi$ from 0.13 to 0.104, with no recovery until time $T$, starting from the stationary state? To complement the phase diagrams in the text, we present impulse responses for short and long squeeze on liquidity (2 and 8 years respectively):
Memo items

Parameters as specified in Appendix 1, with identical base case equilibrium values for the variables. The stable eigen value is 0.9969, and the slope of stable eigen-vector: -0.0012.