Signaling and Disclosure of Product Quality with Price Competition.

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December 3, 2010

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1 Introduction

In a large number of markets sellers are more informed than potential buyers about the "quality" attributes of their product (and/or production process) that buyers care about. Such quality attributes include satisfaction from consuming the product, durability, safety, potential health hazards and environmental damage. The nature of market outcomes in such situations depends significantly on the extent and manner of communication of private information about product quality to buyers. There are two important channels for communication of such private information about quality. The first is voluntary disclosure of quality when such disclosure is credible and verifiable. The second is signaling through other actions chosen by the firm such as prices, output, advertising, warranties etc. that allow consumers to infer the private information of firms. The possibility of disclosure as a means of communication does not, however, preclude signaling of information. Indeed, it is plausible that when disclosure does not occur, firms may still convey their product quality through prices, advertising or other actions so that the alternative to disclosure is not non-revelation. Disclosure affects market outcomes by altering the information structure in which firms and consumers interact. Signaling not only alters the information structure but also requires modification of the actions chosen by a firm (relative to that under full information). While a market with full disclosure essentially attains the full information outcome, the same is often not true for a market where information is fully revealed through signaling. Further, as signals of product quality (such as prices or advertising) are also instruments of market competition between firms, the outcome of competition may be radically altered by the choice between disclosure and signaling, and vice-versa. This paper studies the endogenous choice between voluntary disclosure and signaling of pure private information about product quality in a duopoly where firms engage in strategic price competition and prices may signal quality.

The existing theoretical literature on voluntary disclosure and signaling of product quality appear to be largely separated. Models of voluntary disclosure often assume that the effective marginal cost of supplying the product is independent of quality. This assumption rules out the possibility of signaling because sellers with lower quality would have an incentive to imitate any action that would signal higher quality. On the other hand, signaling models typically assume away the possibility of voluntary disclosure. Daughety and Reinganum (2008b) were the first to systematically study the choice between disclosure and signaling of private information about product quality. Their analysis is carried in the

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1 While credible and verifiable disclosure of quality may not be possible or prohibitively costly in some markets, there are many markets where such disclosures may be possible, and are often facilitated by institutions such as independent auditing with public announcement of findings, other credible third party certification including labelling by reputed trade associations or public agencies and government regulation that penalizes false disclosure (such as "truth in advertising").

2 Bernhardt and Leblanc (1995) also touch on this tradeoff between disclosure and signaling in a somewhat different context. Fishman and Hagerty (2003) incorporate both disclosure and signaling but do not model them as substitutes; signaling occurs "along with" rather than "instead of" disclosure.
framework of a single seller (monopoly) whose marginal cost of production is increasing in quality. The seller may signal his quality through price. Signaling requires a seller with higher product quality to distort the price he charges (relative to what he would charge under full information) in order to convince consumers that it would never be in his interest to charge this price if he had lower product quality. This, in turn, implies that signaling profit is often lower than full information profit. This, in turn, creates an incentive for voluntary disclosure. When higher quality is supplied at higher marginal cost, the distortion in profit due to signaling and therefore, the incentive for voluntary disclosure, is increasing in product quality. Unless the cost of voluntary disclosure is very large, the firm voluntarily discloses if its product quality is above a certain level and chooses to signal through price if quality is below that level. In particular, disclosure occurs with probability one as disclosure cost converges to zero.3

This paper studies the effect of strategic competition between sellers on the choice between signaling and disclosure. In particular, we consider a symmetric duopoly where firms engage in price competition. The products of the firms may differ only in quality; there is no other form of product differentiation. Unit cost of production depends on product quality which may be either high or low. Consumers are identical and have unit demand with valuation depending on quality. Firms have pure private information about their product quality i.e., a firm’s product quality is unknown to its rival as well as to buyers.4 We view product quality disclosure as a relatively long term decision that takes place prior to price competition. Firms, having observed their own product quality, simultaneously decide whether or not to disclose their product quality. After observing the outcome of voluntary disclosure, firms choose prices. If quality is not disclosed, consumers may make inferences about the product quality of a firm on the basis of the price they observe.

The main contribution of this paper is to argue that strategic competition between firms reduces the incentive for voluntary disclosure (relative to the monopoly benchmark), and increases the likelihood that the market outcome corresponds to the prediction of a signaling model. In particular, we show that when the low quality product generates higher social surplus than the high quality product, the unique symmetric equilibrium is one where neither firm discloses product quality no matter how small the cost of disclosure; the market outcome corresponds to a pure signaling outcome. When the high quality product generates higher surplus (than the low quality product), and unit cost of production is increasing in quality, there is a symmetric equilibrium with full disclosure if the disclosure cost is

3 Daughety and Reinganum (2008c) study a version of the model with two quality types (safe and unsafe) where the high quality product may be supplied at lower unit cost (because of lower expected liability); while there are certain differences in the results, the primary incentive for disclosure rather than signaling is also based on signaling distortion of monopoly profit.

4 Caldieraro, Shin and Stivers (2008) study a model of signaling and disclosure in a duopoly where firms know each others product quality (while consumers are uninformed); in particular, the product qualities of the two firms are perfectly negatively correlated. The strategic incentives for disclosure and signaling as well as equilibrium outcomes in this framework are very different from the pure private information case that we address.
small enough; however, for a large subset of the parameter space in this region, there is another symmetric equilibrium with non-disclosure. In addition, this non-disclosure equilibrium may be Pareto dominant in terms of expected profits of both types. If unit cost of production is decreasing in quality (for instance, due to high expected future liability associated with low quality product), then under certain conditions, the unique symmetric equilibrium involves non-disclosure with probability one by both firms no matter how small the cost of disclosure; firms may also randomize between disclosure and non-disclosure in this region of the parameter space.

As in Daughety and Reinganum (2008b), in the monopoly version of our specific model the seller always chooses to communicate through disclosure if the cost of disclosure is small enough. That in our duopoly model firms often choose not to disclose, even though the cost of disclosure is arbitrarily close to zero, is clearly an effect of strategic competition. Our analysis uncovers several strategic incentives for firms not to disclose their quality voluntarily in the presence of price competition. If neither firm discloses information, the continuation game is essentially one of signaling through prices (specifically analyzed in Janssen and Roy, 2010) and the unique symmetric D1 signaling equilibrium is one where the high quality firm charges a relatively high deterministic price and low quality firms randomize over a lower interval of prices; consumers are indifferent between buying the high quality product and buying the low quality product at the highest possible price charged by a low quality seller. In this outcome, low quality types earn rent that prevents them from imitating the high quality price and this rent, in turn, is based on the stochastic market power that the low quality seller in the state where its rival is of high quality and charges higher price. Sustaining sufficient rent for the low quality type in the signaling outcome requires that the high quality price be large enough and, under certain conditions, that the high quality firms earn sufficient rent. As a result, the signaling equilibrium may generate higher expected profit for both low and high quality sellers compared to the one induced by full disclosure (i.e., full information outcome) even if we ignore disclosure costs.5

The reason why profits earned in the signaling equilibrium are not dissipated through Bertrand price competition is related to the out-of-equilibrium beliefs that punish the high quality firm if it charges a slightly lower price. Not disclosing product quality precommits a high quality firm to being subject to the discipline imposed by out-of-equilibrium beliefs and thereby, to not undercutting its rival. This, in turn, softens competition and explains why in many situations, if the rival does not disclose, it is gainful for a firm to not disclose unilaterally even if it has a high quality product: by disclosing it knows it has an incentive in the pricing game to undercut the rival firm strengthening competition in the market. In addition to the above, there are other strategic incentives for non-disclosure that play a role in our framework. When a unit of the high quality product generates lower surplus than a low quality product, the high quality firm can never make positive profit if

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5Similar results are also contained in analysis of signaling of product quality with price competition between horizontally differentiated firms (Daughety and Reinganum, 2007, 2008)
it reveals its own type (disclosure can only make it a more precise target for expropriation by its rival). On the other hand, if the low quality product creates higher surplus and is produced at lower cost than the high quality product, the low quality type cannot gain from disclosing (it will only make its rival more aggressive and consumers would think no better of his product). Finally, if the low quality product is produced at higher unit cost than high quality, the seller with the high quality product always has an incentive to deviate from an equilibrium where high quality sellers disclose; it can thereby pretend to be a low quality seller (higher marginal cost) and deceive its rival, induce the rival to increase her price, and then undercut it sufficiently to reveal true quality to buyers.

Our analysis also indicates that there are situations where a high quality firm may disclose product quality to free itself from the constraint imposed by out-of-equilibrium beliefs. This happens, for instance, when the high quality product generates greater social surplus, produces at higher unit cost than the low quality product and the \textit{ex ante} probability that quality is high is low. If the high quality firm does not disclose, it cannot charge a price higher than its unit cost and sell with positive probability as the low quality type (of the same firm, whose equilibrium profit is zero) has a greater incentive to charge this price, and so D1 out-of-equilibrium beliefs assign probability one to the event that the firm charging such a price is of low quality. On the other hand, if it does disclose there is a high probability that its rival is of low quality and in that case it can sell at a strictly positive margin. In this situation, if disclosure cost is small, the high quality firm is better off disclosing. When the \textit{ex ante} probability that quality is high is higher, however, there are multiple equilibria where nondisclosure is one of the possible outcomes.

There is evidence that in many markets where credible disclosure may be possible, full voluntary disclosure does not occur\footnote{See, for instance, Mathios (2000).}; the latter is also evident in the fact that consumers do respond to mandatory disclosure laws in some of these markets\footnote{See, Zarkin et al, 1993, Mathios, 2000, Jin and Leslie, 2003.}. In contrast, the theoretical literature on voluntary disclosure (where signaling is assumed away) indicates that if disclosure is credible and costless, a monopolist will fully reveal his product quality\footnote{The argument is that as long as there is a range of product qualities that are not revealed, a seller whose product quality is better than the "average quality" in that range will always be better off disclosing his quality.}. Levin, Peck and Ye (2009) show that if product quality is, as in our model, unknown to rival sellers as well as consumers, then even in a duopoly full voluntary disclosure occurs as the cost of disclosure goes to zero\footnote{An earlier literature on strategic information sharing in oligopoly (see, for instance, Galor, 1986) has highlighted the fact that price competition tends to create disincentive for disclosing private information about production cost. In this literature, there is no issue of revealing product quality nor do the models allow for signaling.}. In this literature, explanations of insufficient disclosure are based on an assumed high cost of disclosure, insufficient understanding by
consumers of seller’s disclosure, 10, lack of information about disclosure11 or the desire to soften competition when sellers know each other’s quality i.e., quality is not pure private information.12 An important contribution of our paper is to argue that if firms can signal their quality through other means and the market is sufficiently competitive, then voluntary disclosure of product quality may not occur even when the cost of disclosure is arbitrarily small. When firms have pure private information about product quality, the combination of signaling and strategic competition can make firms opt out of voluntary disclosure. Further, the full disclosure outcome may be Pareto dominated in terms of profits of both low and high quality types and this provides an explanation of why many industries lobby against mandatory disclosure regulation13.

The rest of the paper is organized as follows. The next section presents the model. Section 3 briefly discusses the monopoly case to provide a benchmark for our duopoly results. Section 4 presents the analysis of the different pricing subgames as they may be of independent interest. Sections 5-7 then present the main results of this paper on the equilibrium disclosure decisions for the relevant different cases, depending on whether or not low quality generates more surplus than high quality and whether or not low quality comes with lower cost.

2 The Model.

The basic model is similar to that in Janssen and Roy (2010). There are two firms, \(i = 1, 2\), in the market. Each firm has private information about the quality of its product. In particular, a firm’s product quality may be either high (H) or low (L) and true product quality is known only by the firm (and not by its rival firm or by consumers). It is, however, common knowledge that the \(\text{ex ante}\) probability that a firm’s product is of high quality is \(\alpha \in (0, 1)\). The products of the firms are not differentiated in any dimension other than quality. There is a unit mass of identical consumers in the market; consumers have unit demand and each consumer’s valuation of a product of quality \(s\) is given by \(V_s, s = H, L\), where

\[V_H > V_L, V_s > c_s, s = L, H.\]

Firms produce at constant unit cost and the unit cost \(c_s\) of a firm depends only on its true quality \(s, s = H, L\). The unit cost subsumes both current production cost including costs of compliance with any form of prevalent regulation as well as expected future costs related to current sale of product such as those arising through liability, damages, legal costs, other costs associated with settlement of disputes and complaints as well as future regulations.

We will consider two kinds of cost regimes:

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12 See, Board (2009), Hotz and Xiao (2008).
13 See, for instance, Hotz and Xiao (2008).
(i) Regular Cost Configuration

\[ 0 \leq c_L < c_H \]  

(ii) Cost Reversal (high liability) where the effective cost of supplying the low quality product is higher than that of supplying the high quality product:

\[ c_L > c_H \geq 0 \]

Under cost reversal (where (1) holds) we have two possibilities:

(i.a) Quality premium (quality consciousness) is low: where the consumers willingness to pay for quality improvement satisfies

\[ V_H - V_L < c_H - c_L \]

and the production and consumption of the low quality good creates more social surplus than the high quality good.

(ii.a) Quality premium (quality consciousness) is high: where the consumers willingness to pay for quality improvement satisfies

\[ V_H - V_L < c_H - c_L \]

and the production and consumption of the high quality good creates more social surplus than the low quality good.

The game proceeds in four stages. First, nature draws independently the type (or quality) \( \tau_i \) of each firm \( i \) from a distribution that assigns probabilities \( \mu, 1 - \mu \) to \( H, L \) respectively; the realization of \( \tau_i \) is observed only by firm \( i \). Next, both firms (having observed their own types), simultaneously decide whether or not to disclose or reveal their type publicly by incurring a cost \( d > 0 \). Disclosure is assumed to be truthful. In the third stage, firms choose their prices simultaneously. Finally, consumers decide whether to buy and if so, from which firm. The payoff of each firm is its expected profit net of any disclosure cost. The payoff of each consumer is her expected net surplus. The solution concept used is that of perfect Bayesian equilibrium (PBE) where the out of equilibrium beliefs satisfy the D1 refinement.

Before analyzing the competitive model, we discuss the benchmark monopoly version of our model.

### 3 Benchmark (Monopoly)

To be able to discuss the role of competition in disclosing information, we first briefly consider as a benchmark the case where a monopoly firm has the possibility to disclose private information about cost. To consider the incentives of a monopolist firm to disclose
information, let us first consider the case where the monopolist does not disclose information. In this case, there cannot be an equilibrium where the high valuation monopolist enjoys full monopoly profits and sets \( p_H = V_H \). This can be seen as follows. Consider first a separating equilibrium where \( p_L < p_H = V_H \) and where the consumers buy at price \( p_H \) with probability \( \gamma \), with \( 0 \leq \gamma \leq 1 \) and at price \( p_L \) with probability \( \beta \), with \( 0 \leq \beta \leq 1 \). For this to be an equilibrium it should be the case that the low quality monopolist does not have an incentive to mimic the high quality type, i.e., \( \gamma (V_H - c_L) \leq \beta ((p_H - c_L)). \) As \( (V_H - c_L) > (p_H - c_L) \), it is clear that this can only be the case if \( \gamma < \beta \) and thus that the high quality monopolist will sell with a probability strictly smaller than 1. Therefore, in such a separating equilibrium, the high valuation monopolist cannot enjoy full monopoly profits.

A similar reasoning applies to a possible separating equilibrium with \( p_H < p_L = V_L \). In this case there is an additional effect, however, that as \( p_H < V_L \) and \( V_L < V_H \), the high quality firm will sell at a price below the high quality price under full information. Next, consider a pooling equilibrium. In a pooling equilibrium, the maximal possible price both types can set equals \( \alpha V_H + (1 - \alpha) V_L < V_H \). Thus, also in a pooling equilibrium the high valuation monopolist cannot enjoy full monopoly profits.

On the other hand, if the high type monopolist can disclose its private information at a small cost of \( d \), then it will always be able to make a profit of \( V_H - c_H - d \), which (for small enough values of \( d \)) is always strictly larger than the equilibrium profits if it does not disclose. Thus, due to the signaling distortion in the monopoly case, price signaling is not a substitute for disclosure and high quality will always want to disclose.

### 4 Pricing Subgames (Duopoly)

Before analyzing the equilibrium outcomes of the entire game, we first consider certain price setting games that potentially arise following the actions of the two firms at the disclosure stage and outline the perfect Bayesian equilibrium outcomes of such games that satisfy the D1 criterion.

To begin, consider the situation where types of both firms are fully revealed after the disclosure stage so that the continuation game is one of complete information. This could be the case because both firms have revealed their information, or if firms follow a separating equilibrium where one cost type reveals information and the other does not. In the pricing game when both firms are revealed to be of identical type \( \tau \), denoted by \( (\tau, \tau) \), the unique Nash equilibrium is clearly one where both firms charge \( c_\tau \) and their profit (gross of any disclosure cost) is 0. In the pricing game when one firm is revealed to be of type \( H \) and the other of type \( L \), denoted by \( (H, L) \) or \( (L, H) \), the equilibrium outcome is one where the firm that generates most surplus sells to all consumers with probability 1. That is, if \( V_H - c_H > V_L - c_L \), the \( H \) type firm charges a price equal to \( c_L + V_H - V_L \) giving a profit equal to \( (c_L + V_H - V_L - c_H) \); the other firm (of \( L \) type) either sells zero while charging \( c_L \) or randomizes above \( c_L \) in a manner so that its rival has no incentive to charge a price
above \( c_L + V_H - V_L \). Similarly, if \( V_H - c_H < V_L - c_L \), the \( L \) type firm charges a price equal to \( c_H - (V_H - V_L) \) giving a profit equal to \(-(c_L + V_H - V_L - c_H)\); the \( H \) type either does not sell while charging \( c_H \) or randomizes above \( c_H \) in a manner so that its rival has no incentive to charge a price above \( c_H - (V_H - V_L) \). It follows that a situation where firms disclose information no matter which quality they produce cannot be an equilibrium, as the firm that generates least surplus always makes zero operating profit in the market and therefore cannot recover the cost of disclosure. Thus, if there is a disclosure equilibrium it is because there is a separating equilibrium where one type discloses and the quality of the other type is then indirectly disclosed by the fact it has not disclosed.

Next, consider the pricing game where only one firm’s (say firm 1) type is fully revealed after the disclosure stage. These asymmetric pricing games are important to analyze to determine the pay-offs after a deviation from a symmetric situation in the disclosure stage. Let the updated posterior belief about the type of the other firm (firm 2) be one that assigns probability \( x \) to type \( H \) and \( 1 - x \) to type \( L \) where \( x \in (0,1) \). For later reference, we denote this pricing game by \((\tau, HL)\) when firm 1’s type is \( \tau \) and firm 2 could be either type. In this section we only consider asymmetric pricing games with \( c_L < c_H \). As the logic underlying the asymmetric pricing games with \( c_L > c_H \) is somewhat different, we postpone that discussion to the section on cost reversal.

First, consider the case where consumers have low quality consciousness so that \( V_H - c_H < V_L - c_L \). In this case competition between firms drives prices down such that the high quality firm of the nondisclosing firm does not make any profits, while the low quality of the nondisclosing firm does make profit. The next Proposition formalizes this result.

**Proposition 1** Suppose \( V_H - c_H < V_L - c_L \). If only one firm discloses and the quality of the other firm is unknown, then there exist a unique D1 equilibrium in the pricing game where the nondisclosing firm of type \( H \) sets a price equal to \( c_H \). If the disclosing firm is of high quality it sets a price \( c_H \), while type \( L \) of the nondisclosing firm sets a price equal to \( c_H - (V_H - V_L) \). If the disclosing firm is of low quality, both low quality types choose a mixed strategy over the support \([\alpha(c_H - (V_H - V_L)) + (1 - \alpha)c_L, c_H - (V_H - V_L)]\) and the disclosing firm chooses price \( c_H - (V_H - V_L) \) with probability \( \alpha \).

**Proof.** Let us first consider the pricing game \((H, HL)\) where the disclosing firm is of type \( H \) for sure. It is clear that the low type of the nondisclosing firm should make positive profits in any equilibrium. From this it follows that if the high type of the nondisclosing firm makes zero profit by setting a price larger than \( c_H \) in equilibrium, the D1 out-of-equilibrium beliefs always assign consumers believing a deviation to a price \( p > c_H \) comes from a high type of the nondisclosing firm. Thus, for any equilibrium price of the disclosing firm larger than \( c_H \), the high quality type of the nondisclosing firm would have an incentive to deviate. Thus, the disclosing firm has to set \( p_1 = c_H \) in any equilibrium and in order to prevent him from deviating the high type of the nondisclosing firm should do likewise. In order to prevent the low quality type to imitate its high type price, the disclosing firm
has a "disclosure premium" (which equals 0 as the price equals marginal cost) of selling to all consumers if the two firms both set a price equal to \( c_H \). Given that the disclosing firm sets \( c_H \), it is clear that the low type of the nondisclosing firm should set a price equal to \( c_H - (V_H - V_L) \).

Next, consider the case where the disclosing firm is of low quality. We first argue that in any D1 equilibrium of the pricing subgame the high quality type of firm 2 should set a price \( p^*_H = c_H \). The reason is the following. It is clear that the equilibrium must be such that firm 1 makes some positive ex ante expected operating profit as it can set a price just below \( c_H - (V_H - V_L) \) and sell in case firm 2 turns out to be of high quality. In addition, the equilibrium profit firm 1 makes cannot stem only from the situation where firm 2 happens to be of low quality. If that would be the case, firm 1 and the low quality type of firm 2 would engage in Bertrand competition resulting in an operating profits of 0.

Let us denote by \( \overline{p}_1 \) the upper bound of the support of the possibly mixed price strategy firm 1 chooses. It is clear that the low quality type of firm 2 will not set a price \( p > \overline{p}_1 \) and will only consider choosing \( p = \overline{p}_1 \) if \( \overline{p}_1 \) is not chosen with a strictly positive probability mass. Therefore, by setting \( p = \overline{p}_1 \) firm 1 will only sell in case firm 2 happens to be of high quality. Therefore, \( \overline{p}_1 = p^*_H - (V_H - V_L) \). Moreover, as firm 1 will not set prices equal to \( c_L \) the low quality type of firm 2 will also be able to make strictly positive profits in any equilibrium. Suppose now that \( p^*_H > c_H \). The high type of firm 2 will not make any profit, while the low type will. If a type of firm 2 will deviate by charging a price in the interval \((\overline{p}_1, p^*_H)\) the D1 requirement says that consumers should evaluate their out-of-equilibrium beliefs in such a way that the type that has the most reason to deviate is believed to be the deviator. It is clear that this is the high type firm. Therefore, consumers will buy with positive probability after observing such a price and as long as \( p^*_H > c_H \), this gives the high quality type of firm 2 an incentive to deviate and set a (slightly) lower price. The remaining of the equilibrium characterization in this subgame is as follows. Firm 1 has market power if firm 2 is of high quality and sets price \( c_H \). Its equilibrium pay-offs are then

\[
\pi_1(L) = [\alpha + (1 - \alpha)(1 - F_L(p))] (p_1 - c_L)
\]

for all \( p_1 \leq c_H - (V_H - V_L) \), where \( F_L(p) \) is the mixed strategy chosen by the low type of firm 2. Setting this expression equal to \( \alpha [c_H - c_L - (V_H - V_L)] \) -which is the profit firm 1 gets if it sets a price (just below) \( c_H - (V_H - V_L) \) -gives the mixed strategy distribution of firm 2 with support \( \{\alpha (c_H - (V_H - V_L)) \} \). A similar consideration for the low type of firm 2 makes clear that firm 1 should choose a similar mixed strategy, but with a mass point at a price \( c_H - (V_H - V_L) \) of probability \( \alpha \). Consumers buy from firm 1 if firm 2 is of high quality and buy at the lowest price if both firms are of low quality.

With low quality consciousness, the asymmetric pricing equilibrium is as competitive as the full disclosure equilibrium in case high quality is disclosed. When the disclosing firm is of low quality, the resulting asymmetric pricing equilibrium is also more competitive as in the nondisclosure case.
Next, consider the case of high quality consciousness so that \( V_H - c_H \geq V_L - c_L \) and \( c_H > c_L \). The next Proposition argues that the low quality type of firm 2 sets its price equal to \( c_L \) and that the high quality type of firm 2 sets a price equal to \( c_L + V_H - V_L \), while and all consumers buy from firm 1. Which price the disclosing firm sets depends on the information it discloses: effectively it always sets the same price as the same type of the nondisclosing firm. The reason firm 2 of type \( H \) cannot undercut is that consumers would believe quality to be low after such a deviation.

**Proposition 2** Suppose \( V_H - V_L \geq c_H - c_L \) and \( c_H > c_L \). If only one firm discloses and the quality of the other firm is unknown, then the equilibrium pay-offs in a D1 equilibrium of the pricing game are uniquely defined. The nondisclosing firm of type \( H \) sets a price equal to \( c_L + V_H - V_L \), while its type \( L \) sets a price equal to \( c_L \). If the disclosing firm is of type \( H \), it sets a price equal to \( c_L + V_H - V_L \), while if it is of type \( L \) it sets a price equal to \( c_L \). In both cases all consumers buy from the disclosing firm.

**Proof.** We first consider the pricing subgame where the disclosing firm is of high quality. It is clear that the equilibrium must be such that firm 1 makes some positive ex ante expected operating profit as it can set a price just below \( c_L + (V_H - V_L) \) and sell in case firm 2 turns out to be of low quality. Therefore, the lowest price firm 1 will set in an equilibrium, denoted by \( p^*_1 \), is such that \( p^*_1 > c_H \). We will next show that both the high and low type of firm 2 must make an ex ante expected D1 equilibrium profit, denoted by \( \pi^*_2(H) \) and \( \pi^*_2(L) \), equal to 0. To see this, we rule out -case by case- all other possible cases. First, consider the situation where \( \pi^*_2(H) = 0 < \pi^*_2(L) \). If there would be such an equilibrium, the high quality type of firm 2 could deviate, set an out-of-equilibrium\(^{14}\) price \( p \) such that \( c_H < p < p^*_1 \) and make a positive profit. The reason is that if \( \pi^*_2(H) = 0 < \pi^*_2(L) \), the high type profitably deviates to such a price no matter how small the probability is that consumers will buy, whereas the low type will benefit from such a deviation only if consumers buy with sufficiently high probability. According to the D1 refinement, the high type has therefore more incentives to deviate than the low types and consumers then have to believe that this price is set by a high quality firm, and therefore they buy.

Next, consider the case where \( \pi^*_2(H) > 0 = \pi^*_2(L) \). This cannot be an equilibrium as the low type will have an incentive to imitate (one of) the equilibrium price(s) of the high type firm. Consider then the case where \( \pi^*_2(H) > 0 \) and \( \pi^*_2(L) > 0 \). It cannot be the case that firm 1 will charge a pure strategy as both types of firm 1 will then be able to undercut and take the whole market. The same argument applies in case firm 1 chooses a certain price with strictly positive probability. So, suppose that firm 1 will choose a mixed strategy over the support \([p^*_1, p_1]\). In order to get strictly positive profits when charging a

\(^{14}\)It is clear that such a price should be an out-of-equilibrium price as if such a price would be set in equilibrium by the high type of firm 2 only, then consumers update beliefs, figure out this is a price set by a high quality firm and buy. On the other hand, it could also not be a pooling price as at this price it cannot be the case that \( \pi^*_2(H) = 0 < \pi^*_2(L) \).
price close to $\bar{p}_1$ the pricing behavior of at least one of the two types of firm 2 should be such that there is a strictly positive probability that consumers buy with strictly positive probability if firm 1 charges such a price. But this would imply that with some strictly positive probability one of the types of firm 2 sets prices at which it does not make any sales, which is inconsistent with the assumption that $\pi^*_2(H) > 0$ and $\pi^*_2(L) > 0$.

Thus, it must be that in a D1 equilibrium of the continuation game, $\pi^*_2(H) = 0 = \pi^*_2(L)$. The maximum price, firm 1 could set in such an equilibrium is $c_L + (V_H - V_L)$ as otherwise the low quality will want to choose a price above $c_L$ and get some demand with positive probability. Thus, the maximum profit firm 1 could get in the continuation game is $c_L + (V_H - V_L) - c_H$. Firm 1 will get exactly this profit if players follow the following strategies. Firm 1 chooses $p^*_1 = (V_H - V_L) + c_L$. Firm 2 chooses $p^*_2(H) = (V_H - V_L) + c_L$ and $p^*_2(L) = c_L$. At these prices, consumers always buy from firm 1 (who discloses it is of high quality). If firm 2 chooses to set different prices, consumers believe these prices are set by a low quality type and therefore they will not buy (given the price set by firm 1). These beliefs are consistent with the D1 criterion in this case where $\pi^*_2(H) = 0 = \pi^*_2(L)$. If firm 1 deviates, consumers buy from firm 2.

Next consider the case where the disclosing firm 1 is of low quality. using the same logic as above in this proof, it can be argued that in any D1 equilibrium, both types of firm 2 should make zero profit. But then it follows that firm 1 should set a price equal to $p^*_1 = c_L$ in this subgame (as otherwise the low quality of firm 2 has an incentive to undercut). Thus, all firms make zero profits in any D1 equilibrium. One way to achieve this is to specify the prices of the firms as in the Proposition and have the consumers buy from firm 1.

With high quality consciousness, the asymmetric pricing equilibrium is as competitive as the full disclosure equilibrium in case low quality is disclosed. When the disclosing firm is of high quality, the resulting asymmetric pricing equilibrium is very similar. The main difference is that the high quality type of the nondisclosing firm suffers from the fact that it will be mimicked by the low quality type if it would make positive profit in equilibrium. The disclosing high quality firm in this case just acts as if it only competes with a low quality rival.

Given the above discussion on the pricing games we are now ready to discuss the equilibrium properties of the entire game. The asymmetric pricing games for the cost reversal case are discussed in that section.

5 Low Quality Consciousness

We start the discussion of the incentives of firms to voluntarily disclose information about the quality they produce, by considering the case where

$$V_H - c_H < V_L - c_L.$$
We will argue that when $V_H - c_H < V_L - c_L$ the unique symmetric pure strategy equilibrium in the disclosure game has no firm disclosing any information. The reason why a no-disclosure policy can be an equilibrium outcome for both low and high quality firms is as follows. With low quality consciousness, a high quality firm cannot make profit in an equilibrium of the pricing game and therefore is better off not disclosing. If low quality discloses, it lowers its profits for two reasons. First, and most important, it has the effect that the high quality type of the nondisclosing firm becomes more aggressive in the pricing game and sets price equal to its marginal cost. This will also lower the prices that low quality firms charge (compared to the nondisclosure pricing game). Second, the firm has to pay a disclosure fee. Nondisclosure is, however not only an equilibrium, it is the unique symmetric equilibrium of the game. It is clear that with low quality consciousness, there cannot be an equilibrium where high quality discloses. The reason why there cannot be an equilibrium where only low quality discloses is that a low-quality firm has an incentive to deviate and not disclose: this does not have an effect on the profits it gets in case the other firm is of high quality, but in case the competitor is of low quality, it deceives this competitor by not disclosing, pretending therefore to be of high quality and then undercut the price the low quality competitor sets in case it expects the other firm to be of high quality. Moreover, by deviating the low quality firm economizes on the disclosure cost. Disclosure thus only brings about costs and no benefits.

The next Proposition formalizes this result.

**Proposition 3** For any $d > 0$, the unique symmetric pure strategy equilibrium has no firm disclosing any information if $V_H - c_H < V_L - c_L$.

**Proof.** The proof proceeds by considering the three possible symmetric pure strategy equilibrium scenarios that remain after we know from the previous section that a symmetric equilibrium with firms always disclosing no matter what their type is, does not exist.

(i) Consider first the candidate equilibrium where a firm discloses information if, and only if, it is of high quality. As the strategies in the disclosure stage are fully revealing, we have argued in the previous section that in the pricing game low quality undercuts high quality sufficiently by setting a price equal to $c_H - (V_H - V_L)$ and all consumers buy low quality. High quality makes a negative overall profit of $-d$. The high quality firm is therefore better off deviating and not disclosing information guaranteeing itself a pay-off of at least $0$.

(ii) Consider next the candidate equilibrium where a firm discloses information if, and only if, it is of low quality. Again, the strategies in the disclosure stage are fully revealing so that along the equilibrium path, the pricing game results in the same outcomes as under (i) implying the high quality type getting a profit of $0$ and the low quality type getting an expected pay-off of $\alpha (c_H - (V_H - V_L) - c_L) - d$. A low quality type can actually do better then this by not revealing and setting a price just below $c_H - (V_H - V_L)$. By deviating and not disclosing information, the other competitor believes that the firm is a high quality.
type. The competitor will thus set a price equal to $c_H$ if it is of high quality and a price equal to $c_H - (V_H - V_L)$ if it is of low quality. By subsequently, after the deviation setting a price just below $c_H - (V_H - V_L)$ the low quality is therefore able to win the competition whatever the type of the competitor, making an overall profit of $(c_H - (V_H - V_L) - c_L)$, which is clearly higher than the candidate equilibrium profits.

(iii) Finally, consider the candidate equilibrium where firms never disclose information. In this case, the model analyzed in Janssen and Roy (2010) applies and they show that the unique D1 equilibrium in the pricing game is one where a high quality firm sets a price equal to $\theta_0$, where

$$\theta_0 = \max\{c_H, c_L + \frac{V_H - V_L}{1 - 1/2}\} = \max\{c_H, c_L + 2(V_H - V_L)\}.$$ 

and a low quality firm randomizes over the interval $[\alpha(\theta_0 - (V_H - V_L)) + (1 - \alpha)c_L, \theta_0 - (V_H - V_L)]$. Consumers in this equilibrium buy at the lowest low quality price if there is such a price and randomizes over the two high quality prices if both firms turn out to be high quality types. Both firms make an expected profit of $\alpha(\theta_0 - (V_H - V_L) - c_L)$ if they are of low quality and of $\alpha(\theta_0 - c_H)/2$ if they are of high quality.

Let us now see whether anyone of the two types has an incentive to deviate and disclose private information. If the high quality type deviates, we enter a subgame where the quality of one firm, say firm 1, is known to the consumers and to the competitor, firm 2, to be high quality and firm 2’s quality remains private information. As we have seen in the previous section, the only equilibrium price $p^*_1$ that can survive in this subgame is one where $p^*_1 = c_H$. But in this case the deviating firm makes a negative profit of $-d$ by disclosing. This deviation is therefore not profitable.

Next consider a deviation by the low quality of firm 1 and suppose that it discloses its private information. In this case we are in a subgame where the quality of one firm, say firm 1, is known to the consumers and to the competitor, firm 2, to be low quality and firm 2’s quality remains private information. We have seen in the previous section that in any D1 equilibrium of this subgame the high quality type of firm 2 should set a price $p^*_H = c_H$. It is clear then that the equilibrium pay-offs of the deviating firm in this subgame are given by $\alpha\{c_H - (V_H - V_L) - c_L\}$ and the overall profit of the deviation is $\alpha\{c_H - (V_H - V_L) - c_L\} - d$, which is clearly smaller than $\alpha(\theta_0 - (V_H - V_L) - c_L)$ as the operating profits are not larger and the firm has to pay the disclosure cost $d$.

6 High Quality Consciousness.

We next turn to the regular cost case ($c_H > c_L$), where consumers value high quality, i.e., where

$$V_H - c_H > V_L - c_L.$$
Note that now, unlike the low quality consciousness case, the high quality firm can make some *ex ante* positive profit under complete information, namely if the other firm turns out to be of low quality. Therefore, under this parameter configuration there is more hope that the high quality firm does have an incentive to disclose information. The next result shows that this is indeed partially true: when the disclosure costs are not too high, there is an equilibrium where high quality firms disclose information. However, it remains true that no disclosure is an equilibrium outcome if the ex ante probability of quality being high is large enough. The intuition behind this condition on $\alpha$ is twofold. First, in the no disclosure equilibrium, high quality firms make positive profit only when the competitor is also of high quality. Moreover, the ex ante profit is increasing in the probability that the competitor is of high quality and is close to zero when $\alpha$ is close to zero. By disclosing information, a high type can always make some ex ante profit as consumers are willing to spend a premium of $V_H - V_L$ for high quality and thus are willing to buy at a price above $c_H$ if they know this price is set by a high quality firm and the other firm produces low quality. Second, if $\alpha$ is relatively high, the benefit of disclosing is relatively low as the *ex ante* probability that there is competition between two products that are both of high quality is high. In this case the ex ante expected profit of disclosing high quality are close to low. Thus, $\alpha$ has to be above a critical threshold for no disclosure to be an equilibrium as the benefits to deviating to disclosure if you produce high quality are then relatively low.

The next Proposition formalizes this result. The precise statement of the Proposition is complicated by the fact that in case of a no disclosure, the equilibrium of the pricing subgame depends on whether the condition $V_H - V_L \geq \frac{1}{2}$ holds or not. In case this inequality does not hold, the pricing game has a D1 equilibrium where nondisclosing high quality firms set prices equal to the willingness to pay for consumers and some consumers do not buy the product. Nondisclosure in this case leads thus to an inefficient market outcome.

**Proposition 4** Let $V_H - c_H > V_L - c_L$, $c_H > c_L$ and $d_1, d_2$ and $d_3$ be defined as follows

\[
\begin{align*}
    d_1 &= (1 - \alpha)(V_H - V_L) - (c_H - c_L), \\
    d_2 &= (1 - \alpha)(V_H - V_L) - (1 - \alpha/2)(c_H - c_L) = d_1 - \alpha(c_H - c_L)/2, \\
    d_3 &= (V_H - V_L) - (c_H - c_L) - \alpha \left( \frac{(V_L - c_L)(V_H - c_H)}{V_H - c_L} \right).
\end{align*}
\]

If $d < d_1$, then there exists a symmetric equilibrium in which a firm discloses its private information if, and only if, it is of high quality. If $\frac{V_H - c_H}{V_H - c_L} \geq \frac{1}{2}$ and $d \geq \max(0, d_2)$, then there exists a symmetric equilibrium where no firm discloses any information and the high quality type chooses a price $p^*_H < V_H$, and if $\frac{V_H - c_H}{V_H - c_L} \leq \frac{1}{2}$ and $d \geq \max(0, d_3)$, then there exists a symmetric equilibrium where no firm discloses any information and the high quality type chooses a price $p^*_H = V_H$.

Thus, if (i) $\frac{V_H - c_H}{V_H - c_L} \geq \frac{1}{2}$ there always exists at least one symmetric pure strategy equilibrium and if $\max(0, d_2) \leq d < d_1$, then there exist multiple symmetric pure strategy
equilibria. If, $\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2}$ and $\frac{1}{V_H - c_L} + \frac{1}{V_H - c_H} < \frac{1}{V_L - c_L}$, then the situation is qualitatively similar and there also always exists at least one symmetric pure strategy equilibrium, with multiple symmetric pure strategy equilibria existing if $\max(0, d_3) \leq d < d_1$. If, on the other hand,

$$\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2} \text{ and } \frac{1}{V_H - c_L} + \frac{1}{V_H - c_H} \geq \frac{1}{V_L - c_L},$$

then there exists $d_1 < d < d_3$ where no symmetric pure strategy equilibria exists. In this case, there is semi-separating equilibrium where a high quality firm discloses with probability $q$, with $0 < q < 1$ and low quality chooses not to disclose.

**Proof.** The proof proceeds by considering the three possible symmetric pure strategy equilibrium scenarios.

(i) Consider first the candidate equilibrium where a firm discloses information if, and only if, it is of low quality. As the strategies in the disclosure stage are fully revealing, in the pricing game the low quality firm has to set price equal to its marginal cost and is thus better off deviating and not disclosing information guaranteeing itself a pay-off of at least 0.

(ii) Consider next the candidate equilibrium where a firm discloses information if, and only if, it is of high quality. As the equilibrium is fully revealing, in the symmetric case where both are of the same type, the firms make no profits. In case the firms are of different types, the high quality firm wins the competition and sets a price equal to $c_L + (V_H - V_L)$, which is larger than $c_H$ under our assumption that $V_H - c_H > V_L - c_L$. The ex ante equilibrium profit of a high type firm in this case is therefore equal to

$$(1 - \alpha)(c_L + (V_H - V_L) - c_H) - d,$$

whereas the low type firm makes no profits.

It is clear that the low type does not have any incentive to deviate and disclose its information at a cost $d$ as the behaviour in the pricing game is not affected by disclosing information (as by not revealing, the information is already implicitly revealed in the separating equilibrium). Let us then consider a possible deviation by the high quality type of say firm 1. If it does not disclose information, its competitor (firm 2) thinks it is of low quality, and sets a price equal to $c_L$ if it is itself of low quality, and of $c_L + (V_H - V_L)$ if it is of high quality. To determine the optimal pricing strategy of firm 1 in the pricing game, the out-of-equilibrium beliefs of consumers are important for any price $p > c_H$. (For any other price it is in any case not optimal to deviate). As the equilibrium profit of a low quality firm are equal to 0, this type has an incentive to deviate in the pricing game for any positive probability that the consumer will buy. Therefore, D1 requires that consumers believe the deviating price is set by a low quality type and the consumer will therefore not buy. Thus, the high quality firm does not have an incentive to deviate and not disclose its information and the symmetric candidate equilibrium where firms choose
"disclose information if, and only if, it is of high quality" is an equilibrium as long as the equilibrium pay-off is nonnegative, i.e., \( d \leq (1 - \alpha)(c_L + (V_H - V_L) - c_L) = d_1 \).

(iii) Consider then the candidate equilibrium where firms never disclose information. Janssen and Roy (2010) show that depending on the parameter values, there is one of two types of equilibria. If

\[
\frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2},
\]

then there exists a unique D1 equilibrium where the high quality firm sets a price \( p^*_H = c_L + 2(V_H - V_L) \), the low quality firms choose a mixed pricing strategy over the interval \([c_L + \alpha(V_H - V_L), c_L + (V_H - V_L)]\) and the consumers buy at the lowest low quality price if at least one of the firms is of low quality and otherwise randomly buys at one of the two shops. If, on the other hand,

\[
\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2},
\]

then there exists a unique D1 equilibrium where the high quality firm sets a price \( p^*_H = V_H \), the low quality firms choose a mixed pricing strategy over the interval \([c_L + \alpha(V_H - c_L), V_L]\) and the consumers buy at the lowest low quality price if at least one of the firms is of low quality and otherwise buys with a probability \( \eta = \frac{2V_L - c_L}{V_H - c_L} \) and if she buys, she randomly does so at one of the two shops. The equilibrium profits of the high quality firm in the two respective cases are given by

\[
\frac{\alpha}{2} \left[ 2(V_H - V_L) - (c_H - c_L) \right] \quad \text{and} \quad \alpha \left[ (V_H - c_H)(V_L - c_L) \right] \frac{V_H - c_L}{V_H - c_L},
\]

respectively.

We will now show that under the conditions specified in the Proposition, no type has an incentive to deviate. Suppose first that the low quality type deviates. In this case, we enter a subgame where the quality of one firm, say firm 1, is known to the consumers and to the competitor, firm 2, to be low quality and firm 2’s quality remains private information. In the previous section we have argued that the only equilibrium price \( p^*_1 \) that can survive in this subgame is one where \( p^*_1 = c_L \). But in this case the deviating firm makes a negative profit of \(-d\) by disclosing. A deviation of a low quality type is therefore not profitable.

Next consider the case where the high quality type of firm 1 deviates and discloses information. In this case we are in a subgame where firm 1 is known to be of high quality and firm 2’s quality remains private information. Comparing the profits in the candidate equilibrium under the two different parameter constellations with the best possible deviation pay-off of \( c_L + (V_H - V_L) - c_H - d \) gives for the case where \( \frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2} \)

\[
\frac{\alpha}{2} \left[ 2(V_H - V_L) - (c_H - c_L) \right] \geq c_L + (V_H - V_L) - c_H - d,
\]

\[15\]Note that if \( \frac{V_L - c_L}{V_H - c_L} = \frac{1}{2}, c_L + 2(V_H - V_L) = V_H \).
or
\[ d \geq (1 - \alpha)(V_H - V_L) - (1 - \alpha/2)(c_H - c_L) = d_2, \]
and for the case where \( \frac{V_L - c_L}{V_H - c_L} < \frac{1}{2} \)
\[ \alpha \left[ \frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right] \geq c_L + (V_H - V_L) - c_H - d, \]
or
\[ d \geq (V_H - V_L) - (c_H - c_L) - \alpha \left[ \frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right] = d_3. \]
which are the conditions in the Proposition.

As it is easy to see that \( d_2 < d_1 \), the statements on when there is a unique equilibrium and when there are multiple equilibria in case \( \frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2} \), immediately follow. Moreover, it is easily seen that \( d_2 \) can be negative. If \( \frac{V_L - c_L}{V_H - c_L} < \frac{1}{2} \), similar considerations apply if \( d_3 < d_1 \). On the other hand, in this case it may be that \( d_3 > d_1 \) and that there is a region of disclosure cost \( d \) such that no pure strategy equilibrium exist. From the definitions of \( d_3 \) and \( d_1 \) it is easy to see that this is the case if \( \frac{1}{V_H - c_L} + \frac{1}{V_H - c_H} \geq \frac{1}{V_L - c_L} \). The proof concludes by showing that in this case there is a mixed strategy equilibrium where the high quality firm randomizes between disclosing and not disclosing and the low quality type chooses not to disclose.

So, suppose that \( d_1 < d < d_3 \) and that a high quality firm chooses to disclose with probability \( q \). In this case three possible pricing subgames can arise in equilibrium. First, if both firms disclose they are of high quality, there will be Bertrand competition resulting in no profits for either firm. Second, if one firm discloses it is of high quality and the other firm does not disclose, we are in the pricing equilibrium analyzed in Proposition 2 so that the high quality disclosing firm always sells and makes a profit equal to \( c_L + (V_H - V_L) - c_H \). Third, if no firm discloses we are in the pricing game analyzed in Janssen and Roy (2010) with the exception that now the firms believe their rival is of low quality with probability \( \alpha(1 - q)/(1 - \alpha q) \) because of Bayesian updating. The expected profit of a high quality firm in this case is therefore

\[ \pi_H^* = \frac{\alpha(1 - q)}{1 - \alpha q} \left[ \frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right]. \]

For a high quality type to be indifferent between disclosing and not-disclosing it therefore has to be the case that

\[ \alpha q \cdot 0 + (1 - \alpha q) \left[ c_L + (V_H - V_L) - c_H \right] - d = \alpha q \cdot 0 + \alpha(1 - q) \left[ \frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right], \]

where the two terms on both sides of the expression reflect the pay-off of disclosing (respectively not disclosing) in case the other firm discloses and does not disclose. This can
be rewritten as

\[(1 - \alpha q) [(V_H - c_H) - (V_L - c_L)] - \alpha (1 - q) \frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} - d = 0.\]

It is easy to see that the LHS of this equation is decreasing in \(q\), it equals \(d_3 - d > 0\) if \(q = 0\) and it equals \(d_1 - d < 0\) if \(q = 1\). Thus, there must be a unique value of \(q\) with \(0 < q < 1\) such that the indifference equation holds. As the argument showing that the low quality type does not have an incentive to deviate is identical as before, we conclude that in case \(\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2}\) and \(d_1 < d < d_3\) there exists a mixed strategy equilibrium where the high quality firm randomizes between disclosing and not disclosing. 

It is interesting to observe that the Proposition implies that for given other parameter values if consumer valuation for high quality is large enough, the unique equilibrium will always be one where high quality will disclose. The reason is simply that by disclosing the high quality firm can always make a profit in case the other firm is low quality and this profit becomes large if consumer valuation for high quality is large.

The previous proposition has analyzed the conditions on the parameter space such that each one of two possible equilibria exists. Naturally, the conditions for one of the nondisclosure equilibria to exist are easier to satisfy when \(d\) is relatively large. However, note that these conditions can also be satisfied when \(d = 0\), i.e., also in this case of high quality consciousness the possibility that nondisclosure arises in equilibrium does not depend on an assumption of disclosure cost being large. Note also that the no disclosure equilibria only exists when the ex ante probability of quality being high is large enough. It is also clear that the higher this probability, the lower the disclosure cost has to be for the disclosure equilibrium to exist.

In case both types of equilibria co-exist, one may wonder whether one can rank the equilibria according to which equilibrium is preferred by the firms. A few observations are in place. First, low quality firms will always prefer the non-disclosure equilibrium as they make positive profit in any such equilibrium, whereas their profits are equal to zero in the disclosure equilibrium (as these firms are outcompeted by high quality firms and if both firms do not disclose, the firms engage in Bertrand competition). The next Proposition shows that there is a reasonably large set of parameter values such that both type of firms are better off in the non-disclosure equilibrium.

**Proposition 5** Suppose \(V_H - c_H > V_L - c_L\), \(c_H > c_L\) and both nondisclosure and disclosure equilibria exist. Then there exist a critical value of \(a\), denoted by \(\alpha^*\) such that for all \(\alpha > \alpha^*\) the nondisclosure equilibrium gives both types of firms a higher pay-off. If \((V_L - c_L)/(V_H - c_L) \geq 1/2\), then \(\alpha^* < 1/2\).

**Proof.** We know that in the disclosure equilibrium the Low type makes zero profits, while the L type makes positive profits in both types of nondisclosure equilibria. We therefore
only need to check the profits of the high type firms in the different equilibria. In the disclosure equilibrium the H type makes an ex ante profit of \((1 - \alpha)(c_L + V_H - V_L - c_H) - d\). In the nondisclosure equilibrium where the high type charges a price strictly below \(V_H\) (if \((V_L - c_L)/(V_H - c_L) \geq 1/2\)), the H type makes a profit of \(\alpha(V_H - V_L) - \frac{3}{2}(c_H - c_L)\). Straightforward calculations show that the latter expression is larger than the former if

\[
\alpha > \frac{(V_H - V_L) - (c_H - c_L) - d}{2(V_H - V_L) - \frac{3}{2}(c_H - c_L)}.
\]

Denote the RHS of this inequality by \(\alpha^*\). It is easy to see that \(\alpha^* < \frac{(V_H - V_L) - (c_H - c_L)}{2(V_H - V_L) - \frac{3}{2}(c_H - c_L)} < \frac{1}{2}\).

In the nondisclosure equilibrium where the high type charges a price equal to \(V_H\), i.e., if \((V_L - c_L)/(V_H - c_L) < 1/2\), the H type makes a profit of \(\alpha \frac{(V_H - c_H(V_L - c_L)}{V_H - c_L}\). Straightforward calculations show that this profit expression is larger then the disclosure profit of high quality if

\[
\alpha > \frac{(V_H - V_L) - (c_H - c_L) - d}{(V_H - c_H(V_L - c_L)) + (V_H - V_L) - (c_H - c_L)}.
\]

Denote the RHS of this inequality by \(\alpha^*\) in case \((V_L - c_L)/(V_H - c_L) \leq 1/2\). As long as both equilibria exist, it is clear that there exists a critical value \(\alpha^* < 1\) such that for all \(\alpha > \alpha^*\) both types of firms make more profit in the no disclosure equilibrium. 

The above Proposition makes clear that it is not only the low quality firms that may have an interest to oppose mandatory disclosure rules. Even a high quality firm may make more expected profits in a no-disclosure equilibrium. Especially when it estimates the chances rival firms produce high quality are relatively large, high quality benefits from the remaining uncertainty in the market concerning product quality.

### 7 Cost Reversal

In this section, we analyze the situation where the effective marginal cost of supplying the low quality product exceeds that of supplying high quality:

\[
c_L \geq c_H
\]

(3)

This reflects situations where the low quality product may cause greater health, environmental or other hazards that makes the firm potentially liable for payment of compensatory and punitive damages in the future; the expected liability and therefore, the net expected marginal cost, may then be higher for the low quality product. Note that (3) implies that

\[
V_H - c_H > V_L - c_L
\]

(4)
Also, observe that the low quality firm has high competitive disadvantage in this market and indeed, whether or not its true type is revealed at the disclosure stage, such a firm cannot earn strictly positive profit in the pricing game. The next lemma summarizes the equilibrium outcome of price competition that follows certain outcomes of the disclosure stage.

**Lemma 6** Assume (3). Consider the game of price setting that follows the voluntary disclosure stage.

(i) Suppose that the types of both firms are fully revealed at the disclosure stage. Then, both firms charge price equal to marginal cost earning zero (gross) profit when they have identical types, and if their types differ, the L type firm charges its marginal cost selling zero output while the H type firm sells to the entire market charging price equal to $V_H - (V_L - c_L)$ and earning (gross) profit equal to $[V_H - c_H] - (V_L - c_L)$.

(ii) Suppose that the type of only one firm (say, firm 1) is fully revealed at the end of the disclosure stage. Let $x \in (0, 1)$ denote the probability that the other firm (firm 2) is of H type assigned by the updated posterior belief after the disclosure stage. (ii.a) Suppose the revealed type of firm 1 is H. Suppose further that

$$
\frac{V_H - V_L}{c_L - c_H} \geq \frac{x}{1-x}.
$$

Then, firm 1 charges $[c_L + (V_H - V_L)]$, sells only in the state where rival is of type L and earns gross expected profit $(1 - x)(c_L + V_H - V_L - c_H)$. Firm 2 of type L sells zero with probability one and follows a mixed strategy; firm 2 of type H charges $c_L$ and sells to all consumers and earns gross profit equal to $(c_L - c_H)$. Next, suppose that (5) does not hold. Then, firm 1 follows a mixed strategy that has a mass point at $(c_L + V_H - V_L)$ and a continuous distribution on an interval $[p, c_L]$ where $p < c_L$ while firm 2 of type H follows a mixed strategy that has a mass point at $c_L$ and whose support is the interval $[p, c_L]$; the equilibrium (gross) profits of both firms are equal to $(1 - x)(c_L + V_H - V_L - c_H)$. Firm 2 of type L follows a mixed strategy and sells zero, earning zero gross profit. (ii.b) Suppose the revealed type of firm 1 is L. Then, firm 1 as well as both types of firm 2 charge a common price $c_L$ and all consumers buy from firm 2 with probability one; firm 1 as well as firm 2 of L type earns zero gross profit while firm 2 of H type earns gross profit equal to $(c_L - c_H)$.

(iii) Suppose that neither firm’s type is revealed fully at the end of the disclosure stage. In particular, consider the symmetric situation where the updated posterior belief assigns identical probability $x \in (0, 1)$ to the event that either firm is of H type. The unique symmetric equilibrium is one where both firms of type L charge price $c_L$ earning zero (gross) profit while each firm of type H follows a mixed strategy with continuous distribution on support $[(1 - x)c_L + xc_H, c_L]$ earning (gross) expected profit equal to $(1 - x)(c_L - c_H)$.

The proof of this lemma is contained in the appendix. The result in part (i) of the lemma is self-explanatory. The result in part (ii.a) relates to the situation where firm 1 is
fully revealed to be of type $H$ but the type of firm 2 is not revealed fully, requires some explanation. In this game, in the event that firm 2 is of type $L$, firm 1 can always attract consumers away from its rival by charging $c_L + (V_H - V_L)$, and therefore guarantee itself an expected profit of at least $(1 - x)(c_L + V_H - V_L - c_H)$. On the other hand, firm 1 of type $H$ may also have an incentive to undercut its rival in the event that the latter is of type $H$. It is easy to see that firm 2 of type $L$ must earn zero profit. This, however means that if firm 2 of type $H$ charges any price above $c_L$ with positive probability, it will be imitated by firm 2 of type $L$. Indeed, as the equilibrium profit of firm 2 of type $L$ as zero, as long as the equilibrium profit of firm 2 of type $H$ is strictly positive, the $D1$ criterion requires that consumers’ beliefs assign probability one to the event that firm 2 is of type $L$ when it charges an out of equilibrium price higher than $c_L$. Therefore, in equilibrium, the price charged by firm 2 of type $H$ does not exceed $c_L$. The decision problem for firm 1 (which is of type $H$) is then whether to forsake the market in the state where firm 2 is of type $H$ and sell only in the state where the latter is of type $L$ charging a deterministic price $c_L + V_H - V_L$, or to compete for the market even when its rival is of type $H$. In the latter case, both firms randomize over prices below $c_L$, while firm 1’s mixed pricing strategy has a mass point at $c_L + V_H - V_L$; as firm 2 of type $H$ cannot charge a price above $c_L$ to take advantage of this potential high price of its rival, it too places a positive probability mass on $c_L$. Which of the two options is exercised by firm 1 in equilibrium depends on whether or not (5) holds. Firm 2 of type $L$ sells zero with probability one but follows a mixed strategy with prices above $c_L$ (and zero mass at $c_L$) so that firm 2 of type $H$ reveals its type fully even when it charges $c_L$. The result in part (ii.b) is easy to understand. As firm 1 is revealed to be of $L$ type, it cannot make positive profit in the state where rival is of $H$ type. Price competition therefore ensures that firm 2 of $L$ type earns zero profit. The latter in turn creates a strong incentive for firm 2 of $L$ type to imitate any price above $c_L$ charged by firm 2 of $H$ type. In equilibrium, both types of firm 2 pool at price equal to $c_L$, firm 1 charges $c_L$ too, and all consumers strictly prefer to buy from firm 2 as it has better expected quality. Finally, the result in part (iii) reflects the fact that each $H$ type firm enjoys market power in the state where the rival is of $L$ type but also has an incentive to undercut rival in the state where the latter is of $H$ type leading to mixed strategy pricing by the $H$ type. Once again, $L$ type firm can sell only in the state where rival is of $L$ type and so price competition drives the profit of $L$ type firms to zero. As a result, $H$ type firms cannot charge price above $c_L$ without being imitated and therefore, $c_L$ is the upper bound of the support of $H$ type’s price distribution.

We are now ready to state the one of the main results of this section.

**Proposition 7** Assume (3). There exists an equilibrium where the product qualities of both firms are revealed with probability one after the voluntary disclosure stage of the game (and before prices are set) if, and only if,

$$(1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H) \geq d.$$
In particular, under (6), there is a symmetric equilibrium where each firm voluntarily discloses quality if, and only if, it is of $H$-type.

**Proof.** First, we show that under (6), there is a symmetric equilibrium where each firm voluntarily discloses quality when it is of $H$ type, but not if it is of $L$ type. On this equilibrium path, the price competition game is one of complete information and the outcome is as indicated in part (i) of Lemma 6 and equilibrium payoff of each firm of $L$-type is 0 and that of each firm of $H$-type is $(1 - \alpha)(c_L + V_H - V_L - c_H) - d$. The out of equilibrium beliefs (after the price setting stage) for a firm does not disclose type is as follows: if it charges price $> c_L$, is is of type $L$ with probability one, and if it charges price $< c_L$, is is of type $H$ with probability one. Note that these satisfy the D1 criterion; the equilibrium profit of $L$ type firm is zero and therefore it has a higher incentive (than $H$ type) to deviate and charge a price above $c_L$ while only a type $H$ firm would find it profitable to charge price below $c_L$. Suppose that firm 2 of $H$-type deviates and does not disclose its type. Then, firm 1 believes that it is in a complete information pricing game (in part (i) of Lemma 6) where the type of firm 2 is $L$ for sure. If firm 1 has revealed its type to be $H$, it will charge price $c_L + (V_H - V_L)$ and expect to sell to all consumers. If firm 1 has revealed its type to be $L$, it will charge price $c_L$. If firm 2 of type $H$ charges price above $c_L$, out of equilibrium beliefs of consumers assign probability one to the event that firm 2 is of type $L$ and therefore, strictly prefer to buy from firm 1. So this deviation cannot be gainful. If firm 2 of type $H$ charges a price below $c_L$, its maximum expected profit is $c_L - c_H$, and this does not exceed its equilibrium payoff $(1 - \alpha)(c_L + V_H - V_L - c_H) - d$ if (6) holds. Next, suppose that (6) does not hold i.e.,

$$(1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H) < d.$$  \hspace{1cm} (7)

and suppose that, contrary to the proposition, there exists $d > 0$ and an equilibrium where the type of both firms are fully revealed with probability one at the voluntary disclosure stage. In any such equilibrium, the market outcome is identical to the full information outcome (except for the disclosure cost being incurred) and therefore the equilibrium profit (gross of any disclosure cost) of each firm of $L$-type is 0 and that of each firm of $H$-type is at most $(1 - \alpha)(c_L + V_H - V_L - c_H)$ as such a firm can make money only if its rival is of $L$-type. The equilibrium strategy of firm 1 at the voluntary disclosure stage can be one of three kinds: (1) Disclose if, and only if, realized type is $H$; (2) Disclose if, and only if, realized type is $L$, (3) Disclose independent of realized type. Consider case (1). Suppose firm 2 of type $H$ deviates and does not disclose its type. Given the equilibrium strategy of firm 2, firm 1 must then infer that firm 2 is of type $L$ with probability one. It would then be rational for firm 1 to believe that firm 2 would never charge a price lower than $c_L$ which means that independent of firm 2’s type, it would never charge a price strictly less than $c_L$. The deviation strategy of firm 2 of type $H$ would then be to charge a price $c_L - \epsilon$ for $\epsilon > 0$ arbitrarily small. Upon observing this out of equilibrium price set by firm 2, consumers must infer that firm 2 is of type $H$ with probability 1 (under D1 criterion as only $H$ type
firms could gain by charging price below \( c_L \). All consumers would therefore buy from firm 2 yielding firm 2 of type \( H \) a deviation profit \( c_L - c_H - \epsilon \) and this exceeds its expected equilibrium payoff for some \( \epsilon > 0 \) as long as \((1 - \alpha)(c_L + V_H - V_L - c_H) - d < c_L - c_H \) which follows from (7); thus, the deviation is gainful. Next, consider cases (2) and (3). Here, firm 1 of type \( L \) earns negative payoff after disclosure (it makes zero profit under full information) while it can certainly ensure zero payoff by not disclosing (and charging \( c_L \) in the continuation game). This completes the proof.

Proposition 7 provides a necessary and sufficient condition under which competing firms communicate information about their own product quality exclusively through voluntary disclosure rather than signaling. This condition (6) is more likely to hold if the disclosure cost is small, the \textit{a priori} probability of high quality is small, consumers’ quality consciousness (or premium) is high, and the relative cost advantage of high quality firm is small. Note that the cost advantage of the high quality firm may be related to the expected penalty that the low quality producer may face in the future and the regulatory framework. If this expected penalty is high relative to quality consciousness, so that the ratio \( (c_L - c_H) / (V_H - V_L) \) is large, the left hand side of (6) is negative; in that case, no matter how small the disclosure cost, there is no equilibrium where both firms reveal their private information fully through voluntary disclosure.

**Corollary 8** Assume (3). If

\[
\frac{(c_L - c_H)}{(V_H - V_L)} \geq \frac{1 - \alpha}{\alpha}
\]  

(8)

then for all \( d > 0 \) i.e., no matter how small the cost of voluntary disclosure of quality, there does not exist any equilibrium where both product qualities are revealed with probability one through voluntary disclosure.

When condition (6) does not hold, a symmetric equilibrium can be of only two possible types: either type of any firm discloses (so that we have a pure signaling outcome), or high quality firms randomize between disclosure and non-disclosure. Our last proposition provides necessary and sufficient conditions for these outcomes.

**Proposition 9** Assume (3).

(a) There exists a symmetric equilibrium where neither firm discloses its product quality voluntarily (with any positive probability) if

\[
d \geq (1 - \alpha)(V_H - V_L).
\]  

(9)

(b) Both firms of \( H \)-type randomize between disclosure and non-disclosure if

\[
\max\{0, (1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H)\} < d < (1 - \alpha)(V_H - V_L).
\]  

(10)
The proof of this proposition is contained in the appendix. Observe that conditions (6), (9) and (10) are mutually exclusive and exhaust the parameter space. Interestingly, an increase in the *ex ante* probability of being high quality type appears to make non-disclosure more likely.

8 Conclusion

This paper has considered the question whether firms have an incentive to disclose information concerning the quality of the product they produce (or the quality of the production process) in a world where there is severe (Bertrand like) competition in the market. Information disclosure is a long-term decision that cannot be revoked. If a firm decides not to disclose, its pricing decision may still signal information concerning private product quality information.

Non-disclosure is the unique symmetric equilibrium outcome if low quality generates more surplus than high quality. Non-disclosure remains an equilibrium outcome when there are no disclosure cost if the surplus high quality generates is not much larger than the surplus low quality generates. In this case, however, there exists a second equilibrium where firms disclose their information if, and only if they produce high quality. When multiple symmetric equilibria exist, there is a large set of parameter values for which the nondisclosure equilibrium generates higher profits for all firms than the disclosure equilibrium.

There are several general mechanisms that underly these results. First, once a firm discloses it has subjected itself to the standard uncercutting logic of Bertrand competition leading to low market prices as there is nothing left for consumers to infer from prices. By not disclosing, a firm may protect itself from this profit-eroding pricing game as consumers refuse to buy from firms that undercut prices. We have shown that this refusal to buy is a result of consumers rationally conjecturing that undercutting prices are more likely to be set by low quality firms as they have a stronger incentive to undercut. Thus, not disclosing can act as a pre-commitment not to undercut. Second, once quality is disclosed a firm is more easily the target of a rival firm trying to provide a more competitive price/quality offer. If quality is not known by the rival firm, it is impossible for that rival firm to know which price/quality offer it should undercut. Third, not revealing may give a firm the possibility to cheat (deviate), pretend to be of a different (low) quality and to signal quality through prices to consumers in the price competition phase. If quality is directly disclosed, this cheating is not possible anymore.

From this perspective, mandatory disclosure rules make the market more competitive, certainly in the short run, and therefore benefitting consumers. Mandatory disclosure rules may however have two disadvantages. First, with mandatory disclosure rules there will be too much disclosure as some unnecessary disclosure costs are made. Second, if the resulting market competition is severe (like in this paper) mandatory disclosure will lead
either high or low quality firms to exit the market as they will not be able to recover their disclosure cost. This may give rise to future market power of the remaining firm(s). A proper comparison of current and future benefits and costs of mandatory versus voluntary disclosure rules is an interesting area for future research.

APPENDIX

Proof of Lemma 6

The proof of part (i) is obvious.

Consider (ii.a). In the event that firm 2 is of type \( L \), firm 1 can sell to the entire market at price \( c_L + (V_H - V_L) > c_H \), and therefore its equilibrium expected (gross) profit \( \geq (1 - x)(c_L + V_H - V_L - c_H) > 0 \). If firm 1 sells only in the state where rival is of \( H \) type, price competition would drive its profit to zero. Therefore, it must sell in the state where rival is of \( L \) type, and thus firm 2 of type \( L \) must earn zero gross profit in equilibrium. If firm 2 of type \( H \) sells at any price above \( c_L \) with positive probability, it will be imitated by firm 2 of type \( L \). Therefore, in equilibrium, the price charged by firm 2 of type \( H \) does not exceed \( c_L \). The decision problem for firm 1 (which is of type \( H \)) is then whether to forsake the market in the state where firm 2 is of type \( H \) and sell only in the state where the latter is of type \( L \) charging a deterministic price \( c_L + V_H - V_L \), or to compete for the market even when its rival is of type \( H \). First, suppose (5) holds. It is optimal for firm 1 to forsakes the market when rival is of type \( H \) and therefore, charge \( c_L + (V_H - V_L) \) with probability one. Firm 2 of type \( L \) sells zero with probability one and follows a mixed strategy whose distribution function \( \phi \) is continuous with support is \([c_L, \infty)\) where

\[
\phi(p) = 1 - \frac{c_L + V_H - V_L - c_H}{p + V_H - V_L - c_H}, \quad p > c_L.
\]

This distribution function makes firm 1 of type \( H \) indifferent between charging \( c_L + (V_H - V_L) \) and any price above that. Firm 2 of type \( H \) charges \( c_L \) with probability one. Next, suppose that (5) does not hold i.e., \( \frac{V_H - V_L}{c_L - c_H} < \frac{1}{1 - x} \). In this case, firm 1 follows a mixed strategy that has a mass point at \((c_L + V_H - V_L)\) and a continuous distribution on the interval \([p, c_L]\) where \( p \) is given by

\[
p - c_H = (c_L + V_H - V_L - c_H)(1 - x).
\]

Note that \( p < c_L \). In particular, the distribution function \( F^H(p) \) followed by firm 1 is given by:

\[
F^H(p) = \begin{cases} 
0, & p \leq p \\
1 - (1 - x) \frac{c_L + V_H - V_L - c_H}{p - c_H}, & p \in [p, c_L] \\
1 - (1 - x) \frac{c_L + V_H - V_L - c_H}{c_L - c_H}, & p \in [c_L, c_L + V_H - V_L) \\
1, & p \geq c_L + V_H - V_L.
\end{cases}
\]
Firm 2 of type $H$ follows the distribution function $\Gamma^H(p)$ where

$$
\Gamma^H(p) = \begin{cases} 
0, & p \leq \frac{c_L}{x} \\
1 - \frac{1-x}{x} \left[ \frac{c_L + \frac{V_H}{p} - c_H}{p - c_H} - 1 \right], & p \in [\frac{c_L}{x}, c_L) \\
1, & p \geq c_L.
\end{cases}
$$

Note that firm 2 puts probability mass $\left( \frac{1-x}{x} \right)^{\left( \frac{V_H}{c_L} - c_H \right)} \in (0, 1)$ on price equal to $c_L$. Finally, firm 2 of type $L$ follows a mixed strategy with the distribution function $\phi$ as outlined in (??) which makes firm 1 of type $H$ indifferent between charging $c_L + (V_H - V_L)$ and any price above that. It is easy to check that given the strategy of firm 2, firm 1 of type $H$ is indifferent between charging $c_L + V_H - V_L$ and any price in $[\frac{c_L}{x}, c_L]$ and is strictly worse off charging a price in $(c_L, c_L + V_H - V_L)$. On the other hand, given the equilibrium strategy of firm 1, firm 2 of type $L$ can never make strictly positive profit and therefore, has no incentive to deviate from its prescribed strategy; firm 2 of type $H$ is indifferent between all prices in the interval $[\frac{c_L}{x}, c_L]$ and strictly prefers to not set a price below $\frac{c_L}{x}$.

Next, consider $(ii.b)$. As firm 1 is known to be of type $L$, it can never sell in the state where the rival firm is of type $H$ which leads to severe competition between $L$ type firms and an outcome where both $L$ types charge their marginal cost while firm 2 of type $H$ sells to all consumers though the latter cannot charge a price above of $c_H$ without being imitated by it’s own $L$ type; therefore both types of firm 2 charge price equal to $c_L$. All consumers (strictly prefer to) buy from firm 2. The out-of-equilibrium beliefs assign probability one to the event that firm 2 is of type $L$ if it charges a price above $c_L$.

Finally, consider $(iii)$. If a firm is of $H$ type, it can always charge a price just below $c_L$ and guarantee itself profit arbitrarily close to $(c_L - c_H)$ in the state where the rival firm is of $L$. Therefore, the equilibrium profit of the $H$ type firm must be strictly positive which also implies that in any symmetric equilibrium, a firm of $H$ type must sell in the state where the rival is of $L$ type (if $H$ type firms sell only in the state where both firms are of $H$ type, Bertrand price competition will lead to zero profit). This, in turn, implies that $L$ type firms must sell zero in the state where rival is $H$ type and therefore Bertrand competition between L type firms leads to marginal cost pricing for thos firms. Even though consumers would prefer to buy high quality at price slightly above $c_H$ rather than buy low quality at price $c_L$, a type $H$ firm cannot charge a price above of $c_H$ without being imitated by it’s own $L$ type (that earns zero profit in equilibrium). The unique symmetric $(D1)$ equilibrium of this game is one where both firms of type $L$ charge price $c_L$ while each firm of type $H$ follows a mixed strategy with distribution function $\Psi$ and support $[(1-x)c_L + xc_H, c_L]$ where

$$
\Psi(p) = 1 - \left( \frac{1-x}{x} \right) \left[ \frac{c_L - c_H}{p - c_H} - 1 \right], p \in [(1-x)c_L + xc_H, c_L].
$$

The out-of-equilibrium beliefs assign probability one to the event that firm 2 is of type $L$ if it charges a price above $c_L$. 

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Proof of Proposition 9

(a) In an equilibrium where both firms disclose with probability zero, the equilibrium path at the pricing stage is one where firms signal quality through prices, as described in part (iii) of Lemma 6. In particular, a firm charges $c_L$ for sure if it is of $L$ type and earns zero profit. If a firm is of $H$-type it follows a mixed strategy with no mass point whose support is an interval $[p_{\hat{H}}, c_L]$ where $c_H < p_{\hat{H}}$ and earns equilibrium payoff $(c_L - c_H)(1 - \alpha)$. The out of equilibrium consumer beliefs are that a firm charging price $> c_L$ is of low quality with probability one. Clearly, a low quality firm has no incentive to deviate and disclose quality. Suppose a firm, say firm 2, of type $H$ deviates and discloses its true quality. The continuation pricing game is as described in part (ii.a) of Lemma 6, firm 2 charges $c_L + (V_H - V_L)$ with positive probability and at that price, sells to all consumers only when it’s rival is of type $L$ and the deviation profit of firm 2 is

\[
(1 - \alpha)(c_L + V_H - V_L - c_H) - d \leq (c_L - c_H)(1 - \alpha), \text{ using (9)}.
\]

(b) We begin by defining the equilibrium strategies. At the disclosure stage, each firm of type $L$ chooses not to disclose while each firm of type $H$ discloses with probability $p \in (0, 1)$. We will show later how this probability is determined. At the pricing stage, the strategies of firms on the equilibrium path are as follows. If both firms actually disclose their types, then they are both of type $H$, so that both firms charge price equal to $c_H$ and earn net profit (net of disclosure cost 0). If firm 2 discloses, but not firm 1, then the firms’ pricing strategies are as indicated in part (ii) of Lemma 6 where the posterior belief assigns probability $x = \frac{\alpha(1-p)}{1 - \alpha p}$, $1 - x = \frac{1 - \alpha}{1 - \alpha p}$ to $H$ and $L$ types for firm 2. The outcome is symmetric when firm 2 discloses but not firm 1. If neither firm discloses, the firms’ pricing strategies are as indicated in part (iii) of Lemma 6 where each firm is of type $H, L$ with probabilities

\[
x = \frac{\alpha(1-p)}{1 - \alpha p}, 1 - x = \frac{1 - \alpha}{1 - \alpha p}.
\]  

(11)

In this equilibrium, the expected net profit of a firm of $L$ type is 0 and no such firm can unilaterally deviate and gain strictly positive net profit. All we need to show now is that there is a value of $p \in (0, 1)$ so that each firm of type $H$ is indifferent between disclosure and non-disclosure if its rival plays according to the prescribed strategies. Given the strategy followed by firm 1, the reduced form expected profit of firm 2 of type $H$ when it discloses is 0 if its rival also discloses (and this occurs with probability $\alpha p$) and $(1 - x)(c_L + V_H - V_L - c_H) - d$ if its rival does not disclose (which occurs with probability $1 - \alpha p$), so that the expected net profit from disclosure is

\[
\pi^H_2(D) = (1 - x)(c_L + V_H - V_L - c_H)(1 - \alpha p) - d
\]

\[
= (1 - \alpha)(c_L + V_H - V_L - c_H) - d.
\]

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On the other hand, the reduced form expected profit of firm 2 of type \( H \) when it does not disclose, but its rival does disclose (this occurs with probability \( \alpha p \)) is \((c_L - c_H)\), if (5) holds, and \((1 - x)(c_L + V_H - V_L - c_H)\), otherwise; if the rival firm does not disclose (which occurs with probability \( 1 - \alpha \)), firm 2’s expected profit is \((1 - x)(c_L - c_H)\). The expected net profit of firm 2 of type \( H \) from nondisclosure can be written as

\[
\pi_H^2(ND) = \begin{cases} 
[1 - (1 - p)\alpha](c_L - c_H) & \text{if } p \geq 1 - \frac{1 - \alpha}{\alpha} \frac{V_H - V_L}{c_L - c_H} \\
\frac{1 - \alpha}{1 - \alpha p}[\alpha(V_H - V_L) + (c_L - c_H)] & \text{if } p < 1 - \frac{1 - \alpha}{\alpha} \frac{V_H - V_L}{c_L - c_H}
\end{cases}
\]

where the second line is valid only if \( \frac{V_H - V_L}{c_L - c_H} < \frac{\alpha}{1 - \alpha} \). It is easy to check that

\[
\lim_{p \to 1} \frac{1 - \alpha}{1 - \alpha p}[\alpha(V_H - V_L) + (c_L - c_H)] - [1 - (1 - p)\alpha](c_L - c_H) \to 0
\]

as \( p \to 1 - \frac{1 - \frac{V_H - V_L}{c_L - c_H}}{\alpha} \). Thus, \( \pi_H^2(ND) \) is continuous in \( p \) on \([0, 1]\). As \( p \to 0 \), \( \pi_H^2(ND) \to [1 - \alpha](c_L - c_H) \) so that, using (10),

\[
\pi_H^1(D) - \pi_H^1(ND) \to (1 - \alpha)(V_H - V_L) - d > 0
\]

On the other hand,

\[
\lim_{p \to 1} \pi_H^1(ND) \geq (c_L - c_H)
\]

so that, using (10),

\[
\lim_{p \to 1} [\pi_H^1(D) - \pi_H^1(ND)] \leq (1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H) - d < 0.
\]

From the intermediate value theorem, there exists \( \tilde{p} \in (0, 1) \) such that \( \pi_H^1(D) = \pi_H^1(ND) \) for \( p = \tilde{p} \). This completes the construction of the equilibrium.

**References**


