Talent Utilization and Search for the Appropriate Technology

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Abstract

This paper analyzes a model of economic development which is propagated by matching between technologies and the talents they require. It shows that differences in productivity across countries are amplified by three dimensions of talent utilizations. First, the range of different talents utilized. Second, the density of a specific talent utilized. Third, the average match quality in the economy. Assuming set up costs, the equilibrium number of technologies increases with productivity. A larger number of technologies enables a better matching between individuals' talents and requirements of technologies. Workers' search for the appropriate technology reinforces the benefits of development by increasing the extent to which talents are utilized.

Keywords: income level, total factor productivity, technological density, appropriate technology, talent utilization, search.

JEL Classifications: J21, L16, O11, O33, O47.

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1 Introduction

Total factor productivity (TFP) is an important determinant of development. However, measured by the Solow residual, it is no more than “The Measure of Our Ignorance” (Abramovitz (1956)). This research provides a theoretical explanation of differences in TFP and, thus, income differences across countries. It harnesses Adam Smith’s idea of the division of labor to explain how small differences in productivity across countries are amplified through search and matching. The premise of the paper is that each technology requires a different set of talents, which are distributed across individuals. When more individuals are properly matched to the appropriate technology, talent utilization increases, and with it output. The paper describes an economy in which exogenous productivity affects overall talent utilization through product variety and individuals’ incentives to search for the appropriate technology.

The paper has the following results. Higher productivity yields a larger variety of technologies. Such an environment fosters better matches between technologies requirements and individuals’ talents. In addition, the range of different talents being utilized is larger. This better environment also induces individuals to increase their search effort, contributing to the extent to which talents are matched. A larger variety and higher search effort result in a smaller average mismatch between technologies and talents on the one hand, and in a higher number of individuals finding the appropriate technology on the other hand. Our main result shows that small differences in economies’ productivity are amplified through higher talent utilizations and higher average match quality.

The idea of the paper is presented by a model of economic development where the final output is produced by many intermediate goods. Each country produces a different variety of intermediate goods. Each intermediate good corresponds to a specific technology and is produced by a continuum of entrepreneurs with heterogeneous talents. To implement a particular technology a specific entrepreneurship talent is required. The extent to which entrepreneur’s talent matches the technology requirements determines the efficiency units of labor that an entrepreneur supplies. Entrepreneurs’ efficiency units of labor is combined with raw labor to produce Intermediate goods.
With decreasing returns to accumulated factors, a fixed cost of importing each technology determines the number of intermediate varieties. Higher productivity increases entrepreneurial rent. As a result, a smaller continuum of entrepreneurs is needed to cover the fixed cost, yielding a higher average match. At the same time, lower labor resources are needed to produced each variety, leading to a larger number of varieties when labor market clears. The number of varieties affects individuals’ incentives to search for their appropriate technology, given that search is required to overcome information frictions. We show that investment in search increases with development, acting as another source of amplification by increasing the extent to which talents are being utilized.

This paper belongs to a strand of literature which tries to explain why some countries are so much richer than others. The answer that this literature provides lies between factor accumulation and the efficiency with which these factors are used. On the one hand, Mankiw, Romer and Weil (1992), Parente, Rogerson and Wright (2000), Weil (2005) and Manuelli and Seshadri (2005) find that most of the cross-country differences in per capita output are induced by factors accumulation. On the other hand, Chari, Kehoe and McGrattan (1996), Prescott (1998), Hall and Jones (1999), Parente and Prescott (2000), Klenow and Rodriguez-Clare (1997), Bils and Klenow (2000), Hendricks (2002) and Jeong and Townsend (2007) find that most of the cross-country differences in per capita output are induced by TFP.

Given the importance of TFP in explaining large cross-country differences in income leaves us with the need to understand the underlying technological differences across countries. Zeira (1998), Basu and Weil (1998) and Acemoglu and Zilibotti (2001) are theoretical contributions that emphasize the role of appropriate technologies for explaining TFP differences. Zeira (1998) focuses on the range of technologies adopted

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1For an updated survey of such development accounting literature see Caselli (2005).
2Mankiw et al. (1992) addresses the role of human capital, Parente et al. (2000) emphasize the role of home production, Weil (2005) examines the role of health and Manuelli and Seshadri (2005) stresses the importance of controlling for the quality of education when examining this question.
due to differences in capital labor ratios. Basu and Weil (1998) addresses the role of learning-by doing that influences technological progress at the capital labor ratio. Acemoglu and Zilibotti (2001) emphasizes skill supply for utilizing advanced technologies. In these papers, differences in factor distribution across countries drive the adoption or invention of the appropriate technology. In our model, appropriateness is at the micro level. Each individual can be appropriately matched to a technology, or not. Thus, countries may have the same input distribution, yet differ in the appropriateness of technology.

The interplay between TFP and division of labor is augmented by a search mechanism. With higher TFP and hence more varieties, workers have a higher probability of finding a better match for their talents, and thus search might be more intense. The information friction and search decision provides an endogenous margin which amplifies initial differences in TFP. The inefficiency of matching between heterogenous workers and heterogenous firms in the presence of information frictions has been put forth by Shimer (2005). Decreuse’s (2008) setup of the search and matching process is most similar to ours, as it allows workers to have heterogenous sector specific skills and make a choice about the number of market segments to search. While this literature is concerned with the matching friction underlying unemployment (see also Mortensen and Pissarides (1999) and Shimer (2007)), we focus, rather, on the informational friction underlying talent utilization. Workers do not have direct information on the best match for their talent, and hence have to search. Most importantly, search is embedded in a larger model which also specifies the production side of the economy, and solves for the number of varieties.

The rest of the paper is organized as follows. Section 2 formalizes the arguments, section 3 solves for the equilibrium, section 4 provides a cross country analysis, section 5 presents some concluding remarks and proofs appear in the Appendix.

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4Our research is motivated by evidence provided by Baumgardner ((1988a), (1988b)) and Garicano and Hubbard (2009) that find that individuals’ specialization differs across regions.

5See also Gautier and Teulings (2004)
2 The Model

Consider a small open economy in a world with one final good, which is used for consumption only. This final good is produced using a continuum of intermediate goods. For simplicity the model assumes no physical capital and, therefore, intermediate goods are produced using labor only. All markets are assumed to be perfectly competitive. The final good as well as each intermediate good is assumed to be perfectly tradable, but labor is not tradable, and its market is domestic. For simplicity there is no population growth and population size is normalized to one.

2.1 Production

2.1.1 Production of the final good

The final good is produced by the following continuous log-linear production function

$$\log Y = \int_0^1 \log x(j) \, dj$$  \hspace{1cm} (1)

where $Y$ is the total output produced in an economy, $x(j)$ is the input of intermediate good $j$.

2.1.2 Production of intermediate goods

Each country produces a discrete variety of intermediate goods out of a potential continuum, which is the interval $[0,1]$. Each point on this unit segment represents a different type of intermediate good which requires a specific talent to operate the technology by which it is produced. This specific talent will be henceforth called the “job requirements”.

Individuals are indexed on the unit segment with uniform density. The index of each individual represents her talent. As job requirements represent the location

$^6$A different location on the unit segment reflects a different type of talent. More specifically, the location of a specific individual: ($i = 1$) does not indicate a maximum level of talent; rather, it represents a different talent from any ($i \neq 1$).
of an entrepreneur whose talent accurately matches these requirements, individuals and intermediate goods are both indexed on the same unit segment without any ambiguity.

Each intermediate good is produced by a continuum of entrepreneurs, each endowed with a specific talent which matches to some extent the job requirements of the technology used. The extent to which an entrepreneur’s talent matches the job requirements determines the number of efficiency units of labor this entrepreneur supplies, according to the following function.

\[ h(j, i) = h_0 - bd(j, i) \]  

where \( h(j, i) \) is the ex-post efficiency units of labor that entrepreneur \( i \) supplies for producing intermediate good \( j \), \( h_0 \) is the maximum efficiency units of labor that an entrepreneur can have and \( d(j, i) \) is the distance between the location of intermediate good \( j \), which reflects its job requirements, and that of entrepreneur \( i \), which reflects her entrepreneurship talent. This distance expresses the level of mismatch between the two. The larger the distance is, the greater the mismatch.

Each individual is a potential entrepreneur, who can produce an intermediate good \( j \) according to the following production function

\[ x(j, i) = A \left[ l(j, i) \right]^\alpha \left[ h(j, i) \right]^{(1-\alpha)} \]  

where \( \alpha \in (0, 1) \), \( x(j, i) \) is the output of intermediate good \( j \) produced by entrepreneur \( i \), \( l(j, i) \) is the number of workers employed by her and \( A \) is a productivity parameter, which is country specific. This coefficient may reflect geography: land quality, climate and access to sea, resource endowments: land abundance and natural resources or even infrastructure, and should therefore differ across countries.

Each intermediate good is produced by a continuum of entrepreneurs taking prices as given. Namely, each entrepreneur \( i \) takes the equilibrium wage, \( w \), the cost \( r(j) \) of technology \( j \)’s blueprint, and the price \( P(j) \) of a unit of intermediate good \( j \) and
maximizes:

$$\pi(j, i) = P(j)A[l((j, i)]^a [h(j, i)]^{(1-a)} - w[l((j, i)] - r(j)$$  \hspace{1cm} (4)

2.1.3 The monopolistic market for technologies

The final good is produced by many intermediate goods, where each one requires knowledge of a specific technology. This knowledge is owned by a monopoly. Since intermediate goods are substitutes in the production of the final good, competition arises among monopolies.

The market for technologies operates as follows. A monopolistic owner of a technology incurs a setup cost, $C$. This cost, which is measured in terms of the final good, can be interpreted as the cost of importing on the shelf technology for producing intermediate good $j$. Her revenues are $R(j)$, which consist of total payments collected from all entrepreneurs using technology $j$. Assuming an owner does not observe entrepreneurs’ talents, she cannot discriminate and thus charges a uniform price, $r(j)$. Therefore, profit generated by monopolistic owner $j$ is

$$\pi(j) = R(j) - C$$  \hspace{1cm} (5)

Where $R(j) = \int_{i \in E(j)} r(j) \, di$ and $E(j)$ is the set of entrepreneurs using technology $j$.

2.2 Individuals

Each individual derives utility from consuming the final good and, thus, individuals’ maximization problem collapses to income maximization problem. An Individual can either work as an entrepreneur, utilizing her talent and earning some profits or be employed as a simple worker, earning the equilibrium wage, $w$.

For a non trivial number of technologies to arise in equilibrium, an entrepreneur must earn at least as much as a simple worker. However, to be an entrepreneur, an individual must search and find an appropriate technology. The information
friction is such, that each individual does not know how well her talent matches
with the existing technologies. This could either be because she does not know her
own talent or she does not know the technological requirements of $j$.

**Assumption 1**  The probability that entrepreneur $i$ finds the closest technology $j$ is
independent of her distance from technology $j$.

This assumption captures the symmetry in individuals’ ignorance regarding tech-
nological requirements. Individuals are as likely to find the most appropriate tech-
nology for their talents whether they are very close to it or further away. This
assumption implies that investment in search is equal across individuals.

An individual invests $s$ in search, incurs a cost $g(s)$ and finds the closest technology
with probability $q(s)$. Accordingly, individuals choose search effort to maximize,

$$I = [1 - q(s)]w + q(s)I_{Informed} - g(s)$$

(6)

Where $I_{Informed}$ is the average income of the set of individuals who find the location
of the closest technology $j$, thus:

$$I_{Informed} = [E(\pi(j,i) | \pi(j,i) > w)]\rho + w(1 - \rho)$$

(7)

Such that $\rho$ is the probability that labor market clearing conditions enable an in-
formed individual to operate as an entrepreneur, that is, $\pi(j,i) > w$.

2.3 Labor market

Labor market consists of entrepreneurs producing using different technologies and
employing workers. Let $J$ denotes the equilibrium number of technologies and $\phi(j,i)$

As will be seen later, the average distance is shorter in more developed countries. Thus,
relaxing this assumption and allowing for higher success probability for shorter distances will add
another dimension of amplification.
is the density of entrepreneurs $i$ who buy technology $j$. Each entrepreneur $i$ producing with technology $j$ demands $l(j,i)$ workers. Let $\{j_1, \ldots, j_J\}$ be the set of technologies arising in equilibrium. Aggregate demand for labor is

$$\sum_{j \in \{j_1, \ldots, j_J\}} \int \phi(j,i) l(j,i) \, di$$

and aggregate supply of labor is

$$1 - \sum_{j \in \{j_1, \ldots, j_J\}} \int \phi(j,i) \, di$$

### 3 Equilibrium

An equilibrium is a vector $\{s, r(j), E(j), P(j), l(j,i), w, J\}$ which is a solution to (i) individuals' maximization of income; (ii) the monopolistic market for technologies: profit maximization and (iii) zero profits condition; (iv) the final good maximization problem; (v) the intermediate goods maximization problems; (vi) a threshold condition on individual’s choice of employment; and (vii) labor market clearing condition.

#### 3.1 Final good market

Let the final good serves as a numeraire. Profit maximization by firms, which produce the final good, leads to the following first-order condition

$$P(j) = \frac{\partial Y}{\partial x(j)} = \frac{Y}{x(j)} \tag{8}$$

Substituting equation (8) into equation (11) we get that the condition \( \int_0^1 \log P(j) \, dj = 0 \) must hold at the optimum. Due to symmetry and to the world competition in markets for intermediate goods all prices must be equal. Hence $P(j) = P = 1$. 

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### 3.2 Intermediate goods market

Profit maximization by entrepreneur $i$ who produces intermediate good $j$ leads to the following demand for labor

$$ l(j, i) = \left(\frac{\alpha A}{w}\right)^{\frac{1}{1-\alpha}} [h_0 - bd(j, i)] $$

(9)

The process of establishing new firms by new entrepreneurs takes place until it becomes unprofitable to set up a new firm for producing the same intermediate good $j$. This is generated by a threshold condition that applies to the marginal entrepreneur in sector $j$, who is indifferent between being an entrepreneur in sector $j$ or working as an employee in any firm. Formally, the threshold condition is:

$$ \pi(j \pm \tilde{d}(j)) = (1 - \alpha)A \left[\tilde{l}(j)\right]^\alpha \left[\tilde{h}(j)\right]^{(1-\alpha)} - r(j) = w $$

(10)

where $\tilde{h}(j)$ is the number of efficiency units of labor that the marginal entrepreneur has and $\tilde{l}(j)$ is the number of workers employed by her. Recall from equation (2) that $\tilde{h}(j) = h_0 - bd(j)$, where $\tilde{d}(j)$ is the maximal distance between the requirements of sector $j$ and the talent of the marginal entrepreneur.

### 3.3 The monopolistic market for technologies

Entrepreneurs join sector $j$ from both sides, the size of sector $j$ is represented by the width of that sector which is the interval $[j - \tilde{d}(j), j + \tilde{d}(j)]$. Thus, the size of each sector, which is $2\tilde{d}(j)$, represents the continuum of firms that produces the same intermediate good $j$. The density of entrepreneur $i$ working in sector $j$ is known by the following two results:

**Lemma 1** At the macro level, density of potential entrepreneurs in an economy is $\phi(j, i) = q(s_{ij})$.

**Proof.** Follows directly from corollary (1). ■
Corollary 1 Density of entrepreneur $i$ is independent of her distance from her closest technology $j$, i.e. $\forall i, j$ s.t. $i \in E(j)$, $\phi(j, i) = q(s)$.

Proof. Follows directly from assumption (1) and lemma (1) ■

Thus, the density of entrepreneurs for a given skill-technology match is the probability that an entrepreneur matches with her closest technology. This probability depends on search effort $s$ which is the same for all individuals. Since density does not depend on $i$, it is now convenient to integrate using the distance $t$ defined by: $t = |j - i|$. Then (5) becomes:

$$
\pi(j) = r(j) \cdot 2 \left[ \int_0^{d(j)} q(s) \, ds \right] - C
$$

(11)

Where $q(s)$ is the density of entrepreneurs at any given location. This density is a function of search effort, $s$, and it is independent of distance, $t$. From equation (10) it follows that the price that owner $j$ charges for selling her technology to other entrepreneurs, $r(j)$, affects entrepreneurs’ surpluses and therefore affects the size of sector $j$. First order condition with respect to the monopolistic rent yields:

$$
\frac{\partial \bar{d}(j)}{\partial r(j)} r(j) + \bar{d}(j) = 0
$$

(12)

substituting equation (9) into (10) and applying the implicit function theorem implies that:

$$
\frac{\partial \bar{d}(j)}{\partial r(j)} = \frac{-w^{1-\alpha}}{\alpha \bar{d}(j)^{\alpha} (1 - \alpha) b A^{1-\alpha}}
$$

(13)

substituting equation (13) into equation (12), isolating $w$,

$$
w = \alpha (1 - \alpha) \bar{d}(j)^{\alpha} \left( \frac{\bar{d}(j)}{r(j)} \right)^{\frac{\alpha}{1-\alpha}}
$$

(14)
substituting equation (14) and (9) into (10) and isolating \( r(j) \) yields:

\[
 r_j = \gamma \frac{b \bar{d}(j)}{(h_0 - 2bd(j))^\alpha} A 
\]

where \( \gamma = \alpha^\alpha (1 - \alpha)^{(1-\alpha)} \).

Another potential entrepreneur, \( j \) far from \( j \) finds it profitable to initiate a new sector that produces a different intermediate good. She incurs the set up cost, \( C \), and through the above described market for technologies she sells the blueprint to other entrepreneurs close to her. Ultimately, many sectors are being established, where each sector produces a unique intermediate good by a continuum of firms. The larger the variety of intermediate goods, the smaller the surplus for each owner. This conclusion is driven by the assumption of substitution of the intermediates in producing the final good. As a result, at equilibrium, the variety of intermediate goods in an economy is determined by applying the zero profit condition for all owners, which yields:

\[
 \gamma \frac{b \bar{d}(j)}{(h_0 - 2bd(j))^\alpha} A \cdot 2 \left[ \int_0^{\bar{d}(j)} q(s) \, ds \right] = C 
\]

By corollary (1), equation (16) collapses to

\[
 2q(s)\gamma \frac{b [\bar{d}(j)]^2}{[h_0 - 2bd(j)]^\alpha} A = C 
\]

**Corollary 2** All sectors are symmetric, i.e., each sector has the same size and, therefore, charges the same price for selling technology.

- (i) \( \forall j, r(j) = r \)
- (ii) \( \forall j, \bar{d}(j) = \bar{d} \)

**Proof.** Follows directly from equation (17) and (15). ■

Using corollary (2) and substituting (15) into (14) yields

\[
 w = \gamma (h_0 - 2\bar{d})^{1-\alpha} A 
\]
3.4 Labor market clearing

Recall that $t$ is the distance between an entrepreneur and her technology. Given that $J$ is the equilibrium number of sectors, and by corollary (2) labor market clearing implies that

$$2J \left[ \int_0^d q(s)l(t) \, dt \right] = 1 - 2J \left[ \int_0^d q(s) \, dt \right]$$ (19)

The term $2 \int_0^d q(s) \, dt$ represents the size (measure) of entrepreneurs out of a normalized population. Therefore, the left hand side of (19) represents the demand for labor and the right hand side of (19) represents the supply for labor.

Using corollary (2) and substituting equation (18) into (9), equation (9) can be rewritten as a function of distance from technological requirements solely.

$$l(t) = \frac{\alpha}{1-\alpha} \frac{h_0 - bt}{h_0 - 2bd}$$ (20)

**Proposition 1** Firm’s size is positively affected by the match quality.

**Proof.** Follows directly from equation (20). □

Using assumption (1) and substituting equations (20) into (19), yields:

$$J = \frac{1}{q(s)\tilde{d} \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right)}$$ (21)

3.5 Individuals’ Optimization:

Corollary (2) yields that $\rho = 2\tilde{d}J$. Along with (7), (6) could be rewritten as

$$I = [1 - q(s)]w + q(s)\{2\tilde{d}JE(\pi) + (1 - 2\tilde{d}J)w\} - g(s)$$ (22)
where $E[\pi]$ is the expected rents from being an entrepreneur, which can be described by

$$E(\pi) = \int_0^d \{(1 - \alpha)A[l(t)]^\alpha [h(t)]^{1 - \alpha} - r\} f(t) \, dt \quad (23)$$

Where $f(t)$ is the density function of talent with distance $t$. Given that $t$ is uniformly distributed on $[0, \tilde{d}]$, $f(t) = (1/\tilde{d})$.

Substituting equation $(20)$ and $(15)$ into $(23)$ yields

$$E(\pi) = \gamma \frac{h_0 - \frac{3}{2}bd\tilde{d}}{(h_0 - 2bd)^\alpha} A \quad (24)$$

Maximizing equation $(22)$ yields the following first order condition

$$q'(s)2\tilde{d}J[E(\pi) - w] = g'(s) \quad (25)$$

The intuition behind equation $(25)$ is straightforward. The left hand side of $(25)$ is the gain from a marginal increase in $s$ and the right hand side is its cost.

Using lemma $(1)$ and substituting $(18)$, $(21)$ and $(24)$ into $(25)$ yields

$$\frac{q'(s)}{q(s)} \left( \frac{\gamma bd\tilde{d}}{\alpha \frac{2h_0 - bd}{h_0 - 2bd} + 2} \right) \frac{A}{(h_0 - 2bd)^\alpha} = g'(s) \quad (26)$$

Using corollary $(2)$ along with substituting equations $(17)$ into $(26)$ yields

$$\frac{q'(s)}{q(s)^2g'(s)} 2\tilde{d} \left( \frac{\alpha \frac{2h_0 - bd}{h_0 - 2bd}}{1 - \alpha \frac{2h_0 - bd}{h_0 - 2bd} + 2} \right) \frac{C}{2} = 1 \quad (27)$$

Ultimately, $(17)$ and $(27)$ solve for the equilibrium values of $s$ and $\tilde{d}$ and $(21)$, in turn, solves for $J$. 

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4 Talent Utilization Across Countries

This section examines the extent to which different economies utilize differently their human resources. More specifically, we will show next how small differences in productivity are amplified through different channels. (i) Higher productivity yields more diversification reflected by a larger variety of technologies. Such an environment potentially allows for better matching between individuals’ talents and technologies. (ii) Within this better environment, the range of different talents being utilized is larger. That is, more developed economies utilize some talents that are wasted in less developed economies. (iii) This better environment also increases the marginal cost of mismatch, yielding better matches by employing a smaller range of talents in each technology. Consequently, each technology accommodates a more homogenous range of talents, which increase the average match quality in the economy. (iv) Finally, this better environment, under reasonable assumptions, induces individuals to increase their search effort, resulting in a higher intensity of talent utilization, and therefore better match quality.

The two core mechanisms underlying the above channels are diversification and search. Although these two mechanisms are interrelated, it will prove useful to isolate the role of diversification by holding search effort constant. Thus, initially, we analyze an economy in which the density of entrepreneurs, \( \mu \), and hence the intensity of talent utilization is constant.

4.1 The Quality of Matches

The match quality in the economy could be measured by the average mismatch of entrepreneurs, \( \bar{\mu} \).

**Proposition 2** In more developed economies the average match quality is higher, which is reflected by a smaller continuum of talents employed in each sector. Formally, \( \frac{\partial \bar{\mu}}{\partial A} < 0 \).
Proof. Follows directly from applying the implicit function theorem on (17), which shows that
\[
\frac{\partial \bar{d}}{\partial u_1} = -\frac{\bar{d}}{2A\left(1 + \frac{\alpha bd}{h_0 - 2bd}\right)} < 0
\] (28)

Corollary 3 Monopolistic price for selling technologies increases with development. Formally, \(\frac{\partial r}{\partial A} > 0\).

Proof. Differentiating (15) with respect to \(A\) and substituting (28) yield,
\[
\frac{\partial r}{\partial A} = \frac{\gamma bd}{(h_0 - 2bd)^\alpha} \left(1 - \frac{1 + \frac{2\alpha bd}{h_0 - 2bd}}{2 + \frac{2\alpha bd}{h_0 - 2bd}}\right) > 0
\] (29)

The intuition of the result described in proposition (2) is as follows. Entrepreneurs in more developed economies are more productive and thus are not only willing to pay higher prices, but their willingness to pay declines more steeply with their distance \(d_i\). The monopoly which faces a steeper demand, sets a higher price. In addition, as we shall see later, wages in this economy are higher. Thus, the marginal entrepreneur at distance \(\bar{d}\) faces both higher wages and higher prices, and thus must be more productive, i.e. better matched, in a more developed country.

Next we show how development increases both the variety of technologies and the variety of talents utilized.

4.2 The Variety of Technologies

Proposition 3 Higher productivity induces more diversification: a larger number of intermediate goods. Formally, \(\frac{\partial d}{\partial A} > 0\).

Proof. Follows directly from (28) and (21).
Intuitively, in more developed countries, both factors, entrepreneurs and workers, are more productive. Consequently, less factors are needed to cover the same fixed costs. As each technology captures a smaller share of the factors of production, more technologies arise in equilibrium as a result of labor market clearing.

4.3 The Range of Talents

**Proposition 4** Higher level of development is associated with a larger range of talent utilized. Formally, \( \frac{\partial t}{\partial A} > 0 \), where \( t = \frac{2}{\bar{h} - \bar{h}_0} \).

**Proof.** Rewriting (21) as

\[
B = 2J\bar{d} = \frac{2}{q(s) \left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right)}
\]

Shows that \( B \) decreases with \( \bar{d} \). ■

Proposition (4) states that although the size of each sector is smaller in more developed economies the increase in the variety of sectors dominates. Thus, higher productivity increases the share of entrepreneurs in the population. Since in this model talents play a role only through entrepreneurship activities, it turns out that in more developed countries a larger variety of talents are utilized, albeit the same ex-ante distribution of talents in all countries.

Proposition (2) and Proposition (4) together, imply that the match quality is higher for more individuals in more developed countries. Another way to relate these two results is that individuals are more likely to receive returns to their skill, in other words, there is less randomness in income.

4.4 Amplification through Search

In this section we would like to learn how individuals’ choice of search effort for the appropriate technology varies across economies. As described above, different
economies foster different environments which shape the incentives individuals face when searching for the appropriate technology.

To prove our result regarding the role of search in the economy, we move to a specific probability and cost functions. We will later prove that our results generalizes to any concave probability and any convex cost functions.

4.4.1 An Example

To simplify we assume that both functions are linear and equal, \( q(s) = g(s) = s \). Consequently, (26) collapses to

\[
s = \frac{\gamma b \tilde{d}}{\left(\frac{\alpha}{1-\alpha} \frac{2h_0 - b \tilde{d}}{h_0 - 2b \tilde{d}} + 2 \right)} (h_0 - 2b \tilde{d})^\alpha A
\]

(30)

In this specific case we would like to examine the impact of development, \( A \), on search and diversification.

Proposition 5 Higher level of development is associated with:

(i) A smaller range of talents employed at the sectoral level. Formally, \( \frac{\partial \tilde{d}}{\partial A} < 0 \).

(ii) A higher investment in search effort.

(iii) A larger continuum of talents employed at the macro level. Formally, \( \frac{\partial B}{\partial A} > 0 \), where \( B = 2J \tilde{d} \).

(iv) A higher number of intermediate goods. Formally, \( \frac{\partial I}{\partial A} > 0 \).

(v) A larger continuum of entrepreneurs employed at the macro level. Formally, \( \frac{\partial E}{\partial A} > 0 \), where \( E = 2q(s)J \tilde{d} \).

(vi) A higher monopolistic price for selling the right for using her technology. Formally, \( \frac{\partial r}{\partial A} > 0 \)

Proof. See the Appendix
5 Concluding Remarks

This paper argues that small differences in productivity are amplified by talent utilization. Talent utilization is a result of matching between technologies’ requirements and individuals’ talent. The amplification process works through three different channels. First, the variety of different talents utilized. Second, the density of a specific talent utilized. Third, the average match quality in the economy.

The analysis provides a tool to understand differences in economic structure across countries. It describes the forces that determine three different dimensions related to the structure of the economy: first, the number of sectors, each identified with a different technology; second, the size of each sector, which is reflected by the continuum of entrepreneurs utilizing their talents; third, the distribution of firms’ size mirrored by the distribution of workers employed by entrepreneurs.

The paper could also shed some light on the determinant of income inequality within economies. This inequality could be affected by three factors. First, constant income earned by simple workers. Second, differentiated income earned by entrepreneurs. Third, the relative sizes of these two groups as determined by the structure of the economy. Moreover, the model could be extended to deal with unemployment, an interesting dimension that we leave for future research.
References


APPENDIX

Proofs

Proof of part (i) of proposition (5).

Substituting (30) in (17) yields

\[ F(A, \tilde{d}) = 2\gamma^2 b^2 \frac{\tilde{d}^3}{(1 - \alpha \frac{2h_0 - bd}{h_0 - 2bd} + 2) (h_0 - 2bd)^{2\alpha}} A^2 = C \]

\[ \frac{\partial F}{\partial \tilde{d}} = 2\gamma^2 b^2 A^2 \tilde{d} \left[ 2 \left( 1 - \frac{2h_0 - bd}{h_0 - 2bd} \right) \left( h_0 - 2bd \right)^{2\alpha} \left( 1 + \frac{2bd}{h_0 - 2bd} \right) \right] - \frac{\alpha}{1 - \alpha (h_0 - 2bd)^{2 - 2\alpha}} \]

Thus

\[ \frac{\partial F}{\partial \tilde{d}} > 0 \]

\[ \iff \]

\[ 2 \left( 1 - \frac{2h_0 - bd}{h_0 - 2bd} \right) \left( h_0 - 2bd \right)^{2\alpha} \left( 1 + \frac{2bd}{h_0 - 2bd} \right) > \frac{\alpha}{1 - \alpha (h_0 - 2bd)^{2 - 2\alpha}} \]

\[ \iff \]

\[ 2 \left( 1 - \frac{2h_0 - bd}{h_0 - 2bd} \right) \frac{h_0}{h_0 - 2bd} > \frac{\alpha}{1 - \alpha (h_0 - 2bd)^2} \]

\[ \iff \]

22
\[
2 \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2bd} + 2 \right) (h_0 - 2b\bar{d}) > \frac{\alpha}{1 - \alpha} 3b\bar{d} \\
\]

\[
\iff \\
2 \frac{\alpha}{1 - \alpha} (2h_0 - b\bar{d}) > \frac{\alpha}{1 - \alpha} 3b\bar{d} \\
\iff \\
4h_0 > 5b\bar{d}
\]

Which always holds since \( h_0 > 2b\bar{d} \). Investigating \( F(A, \bar{d}) \) reveals that \( \frac{\partial F}{\partial A} > 0 \), implying that \( \frac{\partial \bar{d}}{\partial A} < 0 \). \( \blacksquare \)
Proof of part (ii) of proposition (5).

Substituting \( q(s) = g(s) = s \) in equation (27) and isolating \( s \) leads to

\[
s^2 = \frac{C}{2\tilde{d} \left( \frac{\alpha}{1 - \alpha} \frac{2b_0 - bd}{h_0 - 2bd} + 2 \right)}
\]

As part (i) proves that \( \bar{d} \) decreases with \( A, \frac{\partial s}{\partial A} > 0 \).

Proof of part (iii) of proposition (5).

Isolating \( q(s) \) in (17) and substituting it in (21) yields

\[
B = 2J\tilde{d} = \left( \frac{4\gamma b}{C} \right) V \left( \frac{1}{\frac{\alpha}{1 - \alpha} \frac{2b_0 - bd}{h_0 - 2bd} + 2} \right) \left( \frac{\tilde{d}^2}{(h_0 - 2bd)^\alpha} A \right)
\]

While \( V \) is constant, the denominator of \( U \) is increasing in \( \tilde{d} \) and since \( \frac{\partial \bar{d}}{\partial A} < 0 \), it decreases in \( A \), thus, \( U(\bar{d}) \) is increasing in \( A \). We next show that \( \frac{\partial W}{\partial A} > 0 \) yielding \( \frac{\partial B}{\partial A} > 0 \).

\[
\frac{\partial W}{\partial A} = \left( \frac{\partial \bar{d}}{\partial A} A \right) \frac{2\bar{d}(h_0 - 2b\bar{d})^\alpha + 2\alpha b\bar{d}^2(h_0 - 2b\bar{d})^{\alpha - 1}}{(h_0 - 2bd)^2\alpha} + \frac{\tilde{d}^2}{(h_0 - 2bd)^\alpha} > 0
\]

\[\iff\]

\[
\left( - \frac{\partial \bar{d}}{\partial A} A \right) 2\bar{d}(h_0 - 2b\bar{d})^\alpha \left( 1 + \frac{\alpha b\bar{d}}{h_0 - 2bd} \right) < \bar{d}^2(h_0 - 2b\bar{d})^\alpha
\]

\[\iff\]

\[
\left( - \frac{\partial \bar{d}}{\partial A} A \right) 2 \frac{h_0 - 2b\bar{d} + \alpha b\bar{d}}{h_0 - 2bd} < \bar{d}
\]

Applying the implicit function theorem on \( F(A, \bar{d}) \) above yields
Thus

\[- \frac{\partial \tilde{d}}{\partial A} A = \frac{\partial E}{\partial \tilde{d}} A = \frac{\partial A}{\partial A} \frac{\partial E}{\partial \tilde{d}} = \frac{\partial E}{\partial \tilde{d}} A\]

\[4 \gamma b^2 A^2 \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right) \left( h_0 - 2bd \right)^{2\alpha} A^2 = 2 \gamma b^2 A^2 \left[ \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right) \left( h_0 - 2bd \right)^{2\alpha - 1} h_0 \right] + \frac{\alpha}{1 - \alpha} \frac{3h_0bd}{(h_0 - 2bd)^2} \]

\[2 \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right) \left( h_0 - 2bd \right)^{2\alpha} A^2 = \frac{\partial A}{\partial \tilde{d}} \frac{\partial E}{\partial \tilde{d}} \frac{\partial E}{\partial \tilde{d}} = \frac{\partial E}{\partial \tilde{d}} A \]

Thus

\[\frac{\partial B}{\partial A} > 0\]

\[\iff\]

\[2 \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right) \left( h_0 - 2bd + \alpha bd \right) < 2 \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right) h_0 - \frac{\alpha}{1 - \alpha} \frac{3h_0bd}{(h_0 - 2bd)^2}\]

\[\iff\]

\[2 \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right) \left( h_0 - 2bd + \alpha bd \right) < 2 \left( \frac{\alpha}{1 - \alpha} \frac{2h_0 - bd}{h_0 - 2bd} + 2 \right) h_0 - \frac{\alpha}{1 - \alpha} \frac{3h_0bd}{(h_0 - 2bd)}\]
\[ 2 \left( \frac{\alpha - 2h_0 - b\tilde{d}}{1 - \alpha h_0 - 2bd} + 2 \right) (-2b\tilde{d}(2 - \alpha)) < -\frac{\alpha}{1 - \alpha} \frac{3h_0b\tilde{d}}{h_0 - 2bd} \]

\[ \iff \]

\[ 2(2 - \alpha) \left( \frac{\alpha - 2h_0 - b\tilde{d}}{1 - \alpha h_0 - 2bd} + 2 \right) > \frac{\alpha}{1 - \alpha} \frac{3h_0}{h_0 - 2bd} \]

\[ \iff \]

\[ 2(2 - \alpha) \frac{\alpha - 2h_0 - b\tilde{d}}{1 - \alpha h_0 - 2bd} > \frac{\alpha}{1 - \alpha} \frac{3h_0}{h_0 - 2bd} \]

\[ \iff \]

\[ 2(2 - \alpha)(2h_0 - b\tilde{d}) > 3h_0 \]

\[ \iff \]

\[ 2(2h_0 - b\tilde{d}) > 3h_0 \]

\[ \iff \]

\[ h_0 > 2b\tilde{d} \]

Which always holds
Proof of part (iv) of proposition (5).
Follows directly from part (ii) and (iii) of proposition (5). □

Proof of part (v) of proposition (5).
Follows directly from part (i) of proposition (5) and equation (21). □

Proof of part (vi) of proposition (5).
Using Corollary (2), (15) can be rewritten as:

\[
\gamma = \frac{\bar{A}(\bar{\theta} - h_0 - 2\bar{b}d_j)^n}{Z(A, \bar{\theta})}
\]

Notice that while \( X \) is constant, \( Z = \frac{W(A, \bar{\theta})}{d} \). Since \( \frac{\partial W}{\partial A} > 0 \) and \( \frac{\partial \bar{A}}{\partial A} < 0 \) \( \implies \frac{\partial r}{\partial A} > 0 \). □