Micro-finance Competition: Motivated Micro-lenders, Double-dipping and Default

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September, 2011

Abstract

We develop a tractable model of competition among motivated MFIs. We find that equilibria may or may not involve double-dipping (and consequently default), with double-dipping necessarily happening if the MFIs are very profit-oriented. Moreover, in an equilibrium with double-dipping, borrowers who double-dip are actually worse off compared to those who do not. Further, for intermediate levels of motivation, there can be multiple equilibria, with a double-dipping equilibrium co-existing with a no default equilibrium. Interestingly, an increase in MFI competition can lower efficiency, as well as increase the extent of double-dipping and default. Further, the interest rates may go either way, with the interest rate likely to increase if the MFIs are very motivated. The effect on borrower welfare is also ambiguous. Finally, an increase in MFI competition can cause a regime switch from a no default equilibrium to an equilibrium with double-dipping.

Key-words: Micro-finance competition; motivated MFIs; double-dipping; default.

JEL Classification No.: C72, D40, D82, G21

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1 Introduction

One of the salient features of the micro-finance movement is its rapid expansion. In India, for example, the average year-on-year increase in the portfolio of the Indian micro-finance sector over the period 2004-2009 was 107% (as compared with a mere 4% increase in commercial bank lending in 2008-09. See, Parameshwar et al., 2009). Other countries also witnessed similar expansions.

Between 1997 and 2005, the estimated number of microfinance institutions (MFIs) worldwide grew from 618 to 3133 (Hermes and Lensink 2007).

With increased micro-finance penetration, there has been a concomitant increase in competition among micro-finance institutions, with many areas being served by multiple MFIs. In India, for example, in the South Indian state of Karnataka, there were 7.31 million micro-finance accounts by the end of 2009 (Srinivasan, 2009). Even assuming that all the poor were covered, this comes to 2.63 accounts per household. In the context of Bangladesh, the Wall Street Journal (27.11.2001) reports that “Surveys have estimated that 23% to 43% of families borrowing from micro-lenders in Tangail borrow from more than one.”

One of the central concerns in this context, and the one we focus on in this paper, is that of ‘double-dipping’, i.e. borrowers taking loans from several MFIs. Closely connected to double dipping is the phenomenon of borrower default. Several empirical studies confirm the importance of double-dipping, as well as find evidence that suggest that double-dipping may be be linked to the phenomenon of borrower default. We next briefly report some of these studies.

McIntosh et al. (2005) find evidence for double dipping using Ugandan data, and find that repayment worsens with an increase in the number of competing MFIs. Vogelgesang (2003) finds multiple loan-taking to be an important trigger in borrower defaults in Bolivia and to result in low borrower repayment. While 13% of the borrowers of the Bolivian microfinancier Caja Los Andes took loans from other sources in 1996, this proportion increased to 24% in 2000.

In the Indian context, Srinivasan (2009) argues that borrowers often use loans from one MFI to repay other loans.

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1 McIntosh and Wydick (2005) provide evidence of increased MFI competition from Uganda and Kenya in East Africa, and Guatemala, El Salvador and Nicaragua in Central America.

2 Data from the MIX (Microfinance Information Exchange) indicates that the number of Indian MFIs reporting data shot up from 11 in 2000, to 144 in 2011. The corresponding numbers are 7 in 2000, and 73 in 2011 for Bangladesh, while those for Indonesia are 2 in 2000, and 60 in 2011. While these figures admittedly do not cover non-reporting MFIs, a more than tenfold increase in coverage over 2000-2011 is nonetheless strongly suggestive of a rapid increase in the number of MFIs.

3 The number would be higher if one takes into account that the loans generally go to women and the very poor are typically not covered (Srinivasan, 2009). In fact such increased MFI competition is happening all across India, including the Northern and Eastern states, and not just in the Southern states (which traditionally lead in micro-finance development) (Srinivasan, 2009).

4 The same Wall Street Journal report claims that villagers in Tangail district could quote rates from as many as 7 competing microlenders from memory.

5 Vogelgesang (2003) finds that some clients obtained loans far below their desired loan size and these clients were more likely to default. Similarly, some clients had to pay high interest rates, which also encouraged default.
McIntosh and Wydick (2005) find that in spite of the fact that competitive pressures among microlenders in Bangladesh reduced interest rates for some borrowers, 32% of the Grameen Bank’s loan portfolio in Tangail was overdue by 2 years or more. Marconi and Mosley (2005) find that the rapid increase in microfinance competition in Bolivia played a role in worsening the microfinance loan portfolio, with over-indebtedness affecting 15-17% of the portfolio of leading MFIs. In case of other MFIs, this proportion was as high as 32%, resulting in some institutions failing. Moreover, there is some evidence to suggest that not all borrowers’ welfare went up in spite of the increase in microfinance providers.

Theoretically, the effect of an increase in MFI competition is not very clear-cut. In fact, it is not even clear if an increase in such competition necessarily increases double-dipping. Even if it does, the implications of such double-dipping, as regards efficiency and borrower welfare, are yet to be explored. With the borrowers using loans from one MFI to repay another, it is of course obvious that such double-dipping can weaken borrower discipline and increase default (see Hoff and Stiglitz 1997). Others have argued however that competition, by reducing interest rates, will improve borrower welfare. It is therefore intriguing that empirical evidence on the effect of increased MFI competition on interest rates has yielded mixed results. For example, Porteous (2006) and Fernando (2006) provide evidence of a decrease in MFI interest rates in response to increased MFI competition over these periods (in Bolivia over the 1990s and in Cambodia over 2003-2006, respectively). On the other hand, there is scant evidence that interest rates fell in response to MFI competition for Bangladesh, or Uganda (Porteous 2006). Though interest rate caps were imposed in Bangladesh, this was an exogenous policy outcome, not an outcome of the increased competition itself. However, default does respond to interest rate changes (Vogelgesang 2003).

In an effort to understand this phenomenon in all its complexity, we seek to develop a tractable model capable of capturing the interplay between the various factors at work. In consonance with reality, we develop a framework that allows for two facts, first, that the MFIs are motivated, i.e. not only interested in their own profits, but also in the utility of the borrowers, and second, that money is fungible, thus allowing for double-dipping. That it is difficult for MFIs to assess their borrowers’ total liabilities, i.e. whether they are borrowing from other sources or not, is acknowledged by policy makers (Srinivasan 2009).

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6A survey by the Grameen Koota staff covering 200 borrowers (including 105 defaulters), suggests that 25 per cent of these borrowers had taken loans from 6 or more MFIs. In another extreme example, one woman was found to have borrowed Rs. 4 million from different MFIs (Srinivasan, 2009).

7As an example of the former stance, consider the following quote: “with the development of active competition between MFIs there has been a deluge of loan funds available to borrowers which has fueled excessive borrowing....Finally it is believed that in consequence of overborrowing, default rates have been climbing in some locations but these have not been disclosed because of ever-greening and multiple lending” (page 16, Malegam Committee Report 2011). As an example of the second stance, consider the quote “to reduce interest rates charged by MFIs or improve the service provided to borrowers....Ultimately, this can only be done through greater competition both within the MFIs and without from other agencies operating in the microfinance sector.” (page 32, Malegam Committee Report).
We use a variation of the Salop circular city model populated by borrowers, as well as motivated MFIs. In this framework the distance between an MFI and a borrower captures whether the loan product offered by the MFI is tailored to the needs of the borrower or not. That many NGOs (including MFIs) are motivated is well known in the literature. The United Nations Interagency Committee on Integrated Rural Development for Asia and the Pacific (1992) (henceforth UNICIRDAP) for example, defines NGOs as organizations with six key features: they are voluntary, non-profit, service and development oriented, autonomous, highly motivated and committed, and operate under some form of formal registration. Thus our approach is complementary to McIntosh and Wydick (2005) and Navajas et al. (2003) where the MFIs are taken to be largely client-maximizing. As we shall later see, several of the results in this paper depend on the fact that the MFIs may be motivated.

The essential tension in the model arises out of the interaction of two facts, namely that money is fungible, and that there is a moral hazard problem in that the MFIs cannot observe whether the borrowers spend the amount borrowed by them in production, which is efficient, or in consumption, which is not. Given that money is fungible, this allows the borrowers to borrow from two MFIs, and use the money to both invest, as well as consume.

We begin by summarizing our main results, before going on to discuss these in greater details. We find that equilibria may or may not involve double-dipping, with (a) both kinds of equilibria co-existing whenever the productive project is relatively productive, and the MFIs are neither too motivated, nor too profit-oriented, and (b) double-dipping necessarily happening if the MFIs are very profit-oriented. Moreover, in an equilibrium with double-dipping, borrowers who double-dip are actually worse off compared to those who do not. Further, increased MFI competition has some non-obvious implications. Interestingly, an increase in MFI competition can lower efficiency, as well as increase the amount of double-dipping and default. Further, the interest rates may go either way, with the interest rate likely to increase if the MFIs are very motivated. The effect on borrower welfare is also ambiguous. Finally, an increase in MFI competition can cause a regime switch from an equilibrium without double-dipping, to an equilibrium with double-dipping.

We next discuss these results in somewhat greater length. We find that there is a symmetric equilibrium as long as competition is not so severe such that feasibility is threatened. An equilibrium with double-dipping (henceforth DDE) exists whenever either the project is not too profitable, or it is profitable and the MFIs are not too motivated, whereas an equilibrium without double-dipping (henceforth SDE) exists whenever the project is very efficient, and the MFIs are not too profit-oriented. The intuition being that when projects are not very efficient, double-dipping is relatively attractive as then the gains from double-dipping, i.e. the consumption utility, is likely to outweigh the losses arising out of the fact that the borrower does not gain anything out of the project. Interestingly, whether a double-dipping equilibrium, i.e. DDE, exists or not is tied up with the motivation level of the MFIs, with a DDE existing whenever the MFIs are very profit-oriented.
This follows because with profit-oriented MFIs the interest rate is likely to be high, which makes double-dipping relatively attractive.

We next examine some properties of equilibria with double-dipping. Interestingly, borrowers who double-dip are actually worse off compared to those who do not. This is because borrowers who are close to the MFIs, i.e. have loan products tailored to their needs, prefer to take a single loan and invest it productively. For borrowers who are further from the MFIs, while their utility from single-dipping is lower, their utility from double-dipping remains unaffected as their aggregate distance from the two neighboring MFIs remains the same. Thus the borrowers who double-dip do so because the loan products are not really tailored to their needs, so that even double-dipping is not too productive for such borrowers.

Furthermore, the equilibrium interest rates may be decreasing in the profitability of the project if the MFIs are highly motivated. Intuitively, a rise in project profitability reduces borrowers’ incentives to double dip as they become more keen to reap the returns of investing in a single productive project. The resulting reduction in the mass of defaulting borrowers makes it possible for highly motivated MFIs - who just break even in equilibrium - to do so at a lower interest rate.

In fact, a similar result - an interest rate which declines with project profitability - obtains under a SDE as well, though this time for relatively profit-oriented MFIs. The intuition here is that the marginal utility of a rise in interest rates for these MFIs is decreasing in project profitability. Hence, a fall in project profitability induces these profit-oriented MFIs to increase the interest rates they charge. Further, we find that an increase in motivation reduces the interest rates and makes borrowers better off in the aggregate.

Interestingly, we find that multiple equilibria can obtain over some parameter ranges, with both SDE and DDE co-existing. Specifically, this can happen when MFIs are neither too motivated nor too profit-oriented, and when project productivity is relatively high. Intuitively, since the MFIs are not too motivated, they charge a relatively high interest rate in the DDE, sustaining some borrowers’ incentives to double dip. At the same time, high project productivity and the fact that MFIs are not too profit-oriented combine to result in a relatively low interest rate in the SDE, sustaining borrowers’ incentives to borrow from a single source and repay interest on their loans.

We next turn to the central question of this paper, namely the effect of an increase in MFI competition. We find that the effects are quite nuanced. First, an increase in MFI competition necessarily lowers efficiency, and leads to an increase in default, both at the aggregate level, as well as in default per MFI. With an increase in competition, there is an increase in product variety. While this increases borrower utility from both single-dipping, as well as double-dipping, relative to single-dipping double-dipping becomes more attractive. This increases default, as well as lowers efficiency.

Second, we find that the effect on the interest rate can go either way. Somewhat paradoxically, while an increase in MFI competition leads to an increase in interest rates when the MFIs are
highly motivated, lower motivation levels generate the opposite implication (an increase in the number of profit oriented MFIs pushes interest rates down). With highly motivated MFIs, the interest rates are likely to be low, so that the MFIs will be just breaking even. With an increase in competition, double-dipping becomes even more attractive, so that the MFIs are forced to increase the interest rates so as to break-even. Interestingly this is in line with empirical evidence on the effect of MFI competition on interest rates being ambiguous. Nor does the existing theory predict an unambiguous fall in interest rates in response to competition. For example, Hoff and Stiglitz (1997) show that an increase in the number of lenders can push up the price of loans.

Third, the effect of increased competition on borrower welfare is also quite nuanced. While the borrowers gain, in the aggregate, from a decline in the interest rates, the effect of an increase in interest rates can affect different borrowers differently. While the utility of double-dipping borrowers increases (because of increased product variety), that of single-dipping borrowers may either increase, or decrease.

Fourth, an increase in competition increases the likelihood of multiple equilibria obtaining and widens the parameter space over which it obtains. Intuitively, the increase in competition makes borrowing from a single source attractive even for customers of relatively profit-oriented MFIs by lowering the distance such borrowers have to travel as well as through the indirect effect of lowered interest rates. At the same time, it also increases incentives to double dip, even for customers of relatively profit-oriented MFIs, by reducing the distance that defaulters have to travel to borrow from multiple sources. Hence, it increases the likelihood that a SDE and a DDE coexist and widens the range of motivation parameters over which this will happen.

We finally examine if an increase in competition can cause a regime switch from an SDE to a DDE, finding that such a regime switch may happen if the project is neither too productive, nor too unproductive. Again the intuition has to do with the fact that an increase in MFI competition makes double-dipping relatively more attractive vis-a-vis taking a single loan. Moreover, in that case there can be an increase in interest rates, with a consequent decline in aggregate borrower welfare.

Given the emphasis on modeling MFIs as motivated agents, it may be of importance to point out that several of the results in this paper hinge on the fact that MFIs are motivated. For one, if the MFIs are not motivated then an SDE can never exist, and consequently multiple equilibria cannot emerge. For another, the result that under a DDE an increase in competition can lead to an increase in interest rates, also emerges only when the MFIs are motivated.

Finally turning to some policy issues, one broad policy lesson seems to be that apart from any direct effects of any intended policy change, there are likely to be other indirect effects working via its effect on the level of competition, the implications of which are quite nuanced. We find this to be true for some suggested policy reforms in India, namely an interest rate cap, as well as providing the MFIs access to subsidized credit.
1.1 Related Literature

Having already alluded to related empirical literature on MFI competition, double dipping, and default, we now briefly relate our paper to the small, though growing theoretical literature on MFI competition. McIntosh and Wydick (2005), as well as Navajas et al. (2003) examine competition between a client-maximizing incumbent MFI, and a profit-oriented entrant. McIntosh and Wydick (2005) show that non-profit MFIs cross-subsidize within their pool of borrowers. Thus when competition eliminates rents on profitable borrowers, it is likely to yield a new equilibrium in which poor borrowers are worse off. Navajas et al show that the richest and most productive borrowers of the incumbent would switch to the entrant. Our paper however differs from both these papers in several respects. First, in contrast to both these papers, our paper explicitly allows for motivated MFIs. As we argue earlier, allowing for motivated MFIs yield interesting new insights. Second, while McIntosh and Wydick (2005) focus on the issue of cross-subsidization, our concern is with the issue of double-dipping. Finally, in terms of results, while all three papers identify some possible negative aspects of MFI competition, these are driven by different reasons. Recall that in McIntosh and Wydick (2005) the negative result arises from a reduction in cross-subsidization. Similarly in Navajas et al. it arises from the fact that the entrant does not cross-subsidize, and therefore is able to attract the more productive borrowers of the incumbent. In the present paper however the negative effect of competition arises from an interaction of several factors, namely (a) an increase in double-dipping, (b) a regime switch to a double-dipping equilibrium, and (c) a possible increase in the interest rates.

Finally, as in Hoff and Stiglitz (1997), the present paper shows that an increase in lender competition may push up interest rates. However, unlike Hoff and Stiglitz (1997), in this paper this effect is a function of MFI motivation; if the MFIs are profit-oriented, this result will not obtain and increased competition pushes interest rates down. Moreover, while in Hoff and Stiglitz (1987) the result arises out of the fact that in a monopolistically competitive market entry by new lenders leads to a loss of economies of scale, in the present paper the result is driven by (a) competition increasing borrowers’ incentives to default, (b) the break-even constraint confronting MFIs, and (c) the fact that highly motivated MFIs just break even.

The rest of the paper is organized as follows. The next section develops the basic framework. Sections 3 and 4, analyze SDE and DDE equilibria respectively. Section 5 analyzes the implications of an increase in MFI competition, while Section 6 concludes. Finally, some of the proofs have been collected together in the Appendix.

\footnote{Aldashev and Verdier (2010) examine a model of NGO competition, where the NGOs allocate their time between working on the project and fundraising. They find that if the market size is fixed and there is free entry of NGOs, then the equilibrium number of NGOs can be larger or smaller than the socially optimal one.}
2 Framework

We frame the problem using a variation of the Salop circular city framework. The model is populated by borrowers of mass one, as well as several micro-finance institutions (MFIs). The borrowers are uniformly distributed over a circle of unit length, whereas the MFIs, \( n \) in number, are located symmetrically along the circumference of the circle. Let us denote these MFIs as \( M_1, M_2, \ldots, M_n \), and let these MFIs be located in the same order on the circle.

Every borrower has access to one productive project that requires a setup cost of 1, and yields a return of \( F \). She can also spend 1 unit of money on consumption, when she obtains a utility of \( u \). These two activities are however fundamentally different in that while production is efficient, consumption is not, so that \( F > c' > u \), where \( c' \) is the opportunity cost of capital. The MFIs however can access capital at a subsidized interest factor of \( c < c' \).

The borrowers however have no money, or assets, so that in case they want to invest, or consume, they must borrow the required amount from some MFI. In case a borrower borrows one unit of capital from an MFI located at a distance of \( x_i \) from her, she also incurs a transportation cost of \( t.x_i \), where \( t.x_i \) can be interpreted as non-monetary transactions cost of borrowing.

We then impose a series of conditions so as to focus on the case of interest. We first assume that productive project is efficient enough even after allowing for transportation costs, i.e. \( F > c + t/4 \). Note that this ensures that even when there are exactly two MFIs (i.e. \( n=2 \)), it is efficient for all borrowers to take a loan for productive purposes (this is because doing so is efficient even for those borrowers who are located the furthest from an MFI, and consequently have the highest transaction costs of \( t/4 \)). Further, the project is not too efficient relative to the cost of capital, in the sense that \( F/2 < c \). This ensures that double-dipping will lead to default (and also captures the ground reality that, for various reasons, productive projects may not be too plentiful in less developed countries). Finally we assume that the subsidy received by the MFIs is not too large in the sense that \( u > c' \).

\[ A1. \quad F - t/4, u > c > F/2. \]

Our second assumption is made purely to reduce the number of cases under consideration. It states that the utility borrowers obtain from consumption should not be too small.

\[ A2. \quad u > 2c - F/2. \]

The MFIs can observe whether the productive project is being undertaken or not. Whether the borrowers consume or not is, however, unobservable. This moral hazard problem has important

\[ ^9 \text{We shall later briefly discuss the implications if subsidy is small, e.g. if } c' = c. \]

\[ ^{10} \text{However, this is not a serious restriction as if this assumption is relaxed, the only change would be that we would have an additional sub-case to consider; none of our current results would be ruled out.} \]
implications as to how the amount borrowed by a borrower is used. Thus if a borrower takes a single loan from an MFI, then the concerned MFI can ensure that the borrower invests productively, and thus recover its money. In case a borrower takes two loans, of 1 unit each, from two different MFIs, she can however invest one unit in the productive project, but consume the other unit of capital. In that case the MFIs may not be able to recover their capital. It is this moral hazard problem that lies at the heart of the present paper.

Each MFI lends exactly one unit of capital to a borrower who approaches it. The transactional costs of borrowing are neither observable nor verifiable, so that the MFIs can only charge an uniform interest from all borrowers who approach them. Let \( r_i \) denote the uniform interest factor being charged by the \( i \)-th MFI. Without loss of generality, let \( F \geq r_i \geq c \), for all \( i \).

All MFIs maximize their utility subject to a break-even constraint. As discussed in the introduction, we are interested in analyzing MFIs that are motivated, so that the utility of the \( i \)-th MFI is a weighted sum of its profits, call it \( \pi_i(r_1, \cdots, r_n) \), and the aggregate utility of its own borrowers, call it \( W_i(r_1, \cdots, r_n) \). Thus the utility of the \( i \)-th MFI can be written as

\[
U_i(r_1, \cdots, r_n) = \mu W_i(r_1, \cdots, r_n) + (1 - \mu) \pi_i(r_1, \cdots, r_n),
\]

where \( 0 \leq \mu \leq 1 \) denotes the motivation level of the MFIs. \( M_i \) therefore maximizes (1) subject to the constraint that \( \pi_i(r_1, \cdots, r_n) \geq 0 \).

We consider a scenario where the MFIs simultaneously decide on their interest factors. Given the configuration of interest rates, the borrowers then make their borrowing and investment/consumption decisions, with the MFIs giving out a loan of 1 unit of capital to all borrowers that approach them.

We look for subgame perfect Nash equilibria of this game in pure strategies. Let \( U_i(r_1, \cdots, r_n) \) and \( \pi_i(r_1, \cdots, r_n) \) denote the utility and profit respectively of the \( i \)-th MFI, i.e. \( M_i \), when the interest vector is \( (r_1, \cdots, r_n) \) and the borrowers are responding optimally to the announced interest rates, i.e. \( (r_1, \cdots, r_n) \).

**Definition.** We say that \( (r_1, \cdots, r_n) \) constitutes a subgame perfect Nash equilibrium if

(i) \( \pi_j(r_1, \cdots, r_n) \geq 0 \ \forall j \), and

(ii) there is no \( r'_i \) such that \( U_i(r_1, \cdots, r'_i, \cdots, r_n) > U_i(r_1, \cdots, r_i, \cdots, r_n) \) and \( \pi_i(r_1, \cdots, r_n) \geq 0 \).

A subgame perfect equilibrium is said to be a *double-dipping equilibrium* (*DDE* for short) if at least some of the borrowers take loans from multiple sources. We can define a *single-dipping equilibrium* (*SDE* for short) along similar lines.

**Proposition 1.** A symmetric pure strategy equilibrium exists whenever the MFI competition is not too severe, i.e. \( n \) is not too large. Further, there can be multiple equilibria when the project is reasonably profitable, and the MFIs are neither too motivated, nor too profit-oriented.
The proof of this proposition follows from Propositions 2 and 4 later on. However, when MFI competition is very intense we find that no pure strategy equilibrium exists. As we shall later argue, in this case all borrowers will have an incentive to double-dip irrespective of the interest rates, so that MFIs will necessarily make losses. Thus Proposition 1 implies that an equilibrium (in pure strategies) exists whenever the MFI competition is sufficiently strong, but not so strong that feasibility is destroyed.

3 Equilibria with Single-dipping

We begin by examining single-dipping equilibria, this case providing a benchmark for the subsequent analysis. Further, it also allows one to examine, later on, the possibility of multiple equilibria, as well as regime switch from single-dipping to double-dipping equilibria. We find that such single-dipping equilibria exist whenever the projects are relatively productive, and the MFIs are not too profit-oriented. Further we derive some comparative statics properties of such equilibria.

The argument proceeds in two stages. We begin by solving for the equilibrium rate of interest in a symmetric SDE under the assumption that the MFIs make a non-negative profit, and that there is no double-dipping in equilibrium. This follows, as usual, from the utility maximizing conditions for the MFIs. The complete solution is then obtained by incorporating these two criteria into the analysis.

Consider a candidate symmetric single-dipping equilibrium where every borrower takes a single loan, and all firms charge the same interest rate, say $r$. Under single-dipping, recall that the MFIs can ensure that the productive project is undertaken, so that there is no default. Thus the utility of a borrower who takes a single loan from an MFI located at a distance $x$ from her is

$$U = F - r - tx.$$  

In an effort to pin down $r$, we proceed by examining if one of the MFIs, say $M_1$, has an incentive to deviate to a different interest factor, say $r'$. We first solve for the demand for loans facing the deviant MFI. Such a deviation will affect the borrowing decisions of the borrowers located on both sides of $M_1$, i.e. those in between $M_1$ and $M_n$, as well as $M_1$ and $M_2$. As usual, solving for the demand facing $M_1$ involves identifying the borrower who is indifferent between borrowing from $M_1$ and $M_2$. Let us consider the consumer who is located at a distance of $d$ from $M_1$ and is indifferent (see Figure 1). Using (2) we find that

$$d = \frac{r - r'}{2t} + \frac{1}{2n}. \tag{3}$$

Thus $M_1$’s utility

$$U_1(r', r, \ldots, r) = 2\mu\left[\frac{r - r'}{2t} + \frac{1}{2n}\right](F - r') - t \int_0^d xdx$$

$$+ 2(1 - \mu)[\frac{r - r'}{2t} + \frac{1}{2n}](r' - c). \tag{4}$$
As usual the unconstrained solution, ignoring feasibility and the possibility of double-dipping, follows from setting \( r' = r \) in the first order condition. Denoting the solution by \( r^* \), we have that it has a unique solution with

\[
r^* = \frac{(1 - \mu)c - \mu F - \frac{(3\mu - 2)t}{2n}}{1 - 2\mu}.
\]

For \( r^* \) to qualify as the equilibrium interest factor, it must however be the case that (a) none of the borrowers have an incentive to double-dip, and (b) the MFIs break even.

First consider the incentive to double-dip. Given that money is fungible, a borrower who takes a loan from two MFIs, can spend on both the productive project, as well as consumption. She can then show the productive project to both the MFIs, claiming that she has taken only a single loan from that particular MFI. Given that \( r \geq c \geq F/2 \), the MFIs cannot of course both be repaid out of the project income. We assume that in that case the two MFIs will share the project return symmetrically, obtaining \( F/2 \) each. Clearly, the borrower’s utility in this case is:

\[
u - \frac{t}{n}.
\]

This is because in this case the whole of the return \( F \) from the productive project is taken away by the MFIs, so that the borrower is only left with her consumption utility net of transaction costs. Further, the total transaction costs of borrowing from two lenders is constant at \( t/n \) and independent of the borrower’s location.

To rule out double dipping, the utility from doing so must be less than the utility of a borrower who takes a single loan from an MFI located at a distance \( x \) from her, i.e. \( F - r^* - tx \).

From (2), it is enough to consider a borrower located at a distance of \( \frac{1}{2n} \) from both \( M_1 \) and \( M_2 \) since such a borrower has the greatest incentive for double-dipping. (Other borrowers would be closer to one MFI than to another and hence be more tempted to just borrow from the closest one). Given that the utility of such a borrower in case she does not double-dip is \( \frac{(1 - \mu)(F - c) + (5\mu - 3)t/2n}{1 - 2\mu} \), there is no double-dipping provided

\[
\frac{(1 - \mu)(F - c) + (5\mu - 3)t/2n}{1 - 2\mu} \geq u - \frac{t}{n},
\]

which simplifies to

\[
\mu \geq \hat{\mu} = \frac{u - (F - c - t/2n)}{2u - (F - c - t/2n)} < \frac{1}{2}.
\]

Intuitively, if the MFIs are very motivated, so that \( \mu \) is large, then \( r^* \) would be small. This is because a higher interest rate reduces borrower welfare, which is unappealing to a motivated MFI. A low interest however makes double-dipping unattractive for the borrowers, hence the requirement that \( \mu \geq \hat{\mu} \).

We next turn to the break-even constraint. We find that there is some \( \mu' \), \( 0 < \mu' < 1/2 \), such that whenever \( \mu < \mu' \), the MFIs obtain a positive profit in case they all charge \( r^* \). The intuition
again follows from the fact that, for \( \mu \) small, \( r^* \) is large, so that MFIs make a positive profit charging \( r^* \). Otherwise, for \( \mu \geq \mu' \), they all charge exactly \( c \), and break-even.

Moreover, we can show that a SDE exists if and only if the project is relatively productive, i.e. \( F > u + c - t/2n \). This is because for \( F \) small, not only is the return from the project low, but moreover, \( r^* \) is large, so that taking a single loan is not very attractive to the borrowers. Formally, a SDE exists if and only if \( \mu' > \hat{\mu} \) (so that the range where MFIs earn positive profits is non-empty), which simplifies to \( F > u + c - t/2n \).

Summarizing the preceding discussion we obtain Proposition 2 (the detailed proof can be found in the Appendix. The parameter zone for which a SDE exists is graphically shown in Figure 4(a)).

**Proposition 2.** A symmetric equilibrium with single-dipping exists if and only if the project is productive enough, i.e. \( F > c + u - t/2n \), and the MFIs are not too profit-oriented, i.e. \( \mu \geq \hat{\mu} \).

Under these conditions we find that:

(i) This equilibrium is unique in the class of symmetric SDE.

(ii) Whenever the MFIs are neither too motivated, nor too profit-oriented, i.e. \( \hat{\mu} \leq \mu < \mu' \), then the equilibrium involves all the MFIs charging an interest factor of \( r^* = \frac{(1-\mu)c-\mu F - \frac{3\mu-2}{2n}t}{1-2\mu} \), and earning positive profits. If the MFIs are extremely motivated, i.e. \( \mu \geq \mu' \), then the equilibrium involves all MFIs charging \( c \) and just breaking even.

Proposition 2 demonstrates that for a SDE to exist it is necessary that \( F \) be large, i.e. \( F > u + c - t/2n \). As \( F \) decreases, the borrowers benefit less from investing in the productive project. Moreover, from (5) we see that for \( \mu < \mu' \), a lower \( F \) leads to an increase in \( r^* \), so that the net utility of the borrowers decreases on both counts. Consequently, the attractiveness of borrowing from only one source diminishes, and borrowers are tempted to double-dip.

The intuition for the result that for small \( \mu \), the equilibrium interest \( r^* \) is decreasing in \( F \), can be traced back to the utility function of the MFIs, i.e. (4). From (4) we have that the marginal utility of \( r' \), is decreasing in \( F \), so that an increase in \( F \) implies that the MFIs have an incentive to decrease their own rate of interest.\footnote{This follows as \( \frac{\partial^2 U_1}{\partial r' \partial F} < 0 \).}

Further note that this result is only true for motivated MFIs, and not if \( \mu = 0 \).

### 3.1 Comparative Statics

We then examine the effects of changes in MFI competition level, as well as MFI motivation, on the interest rate and borrower welfare under a SDE.
Proposition 3. Let $F > u + c - t/2n$ and $\mu \geq \hat{\mu}$, so that a symmetric single-dipping equilibrium exists. Whenever $\mu < \mu'$, so that the equilibrium interest is $r^*$, an increase in MFI competition leads to a decrease in the interest factor, and an increase in borrower welfare. An increase in MFI motivation has similar effects.

The intuition for this result is quite straightforward. With an increase in number of MFIs, an increase in competition forces a lowering of interest. This, along with the fact that there is a reduction in transactions costs of the borrowers, implies that borrower welfare increases. Similarly, as the MFIs become more motivated, charging a higher interest becomes less appealing to the MFIs, as this will lead to a lowering of borrower welfare. Thus the equilibrium interest is lower, leading to greater utility for the borrowers.

4 Equilibria with Double-dipping

We next turn to analyzing equilibria with double-dipping. We show that an equilibrium with double-dipping exists whenever either the project is not too profitable, or it is profitable and the MFIs are not too motivated. Moreover, such an equilibrium exhibits some interesting properties.

As in the case of single-dipping equilibria, we proceed by first solving for the equilibrium interest factor under the assumption that the MFIs make a non-negative profit, and that double-dipping does happen in equilibrium. We next examine what are the implications once these considerations are allowed for.

Consider a candidate symmetric double-dipping equilibria where every MFI charges the same interest factor, say $r$, and at least some borrowers take two loans, one each from the two MFIs located closest to them. As earlier, we then examine if one of the MFIs, say $M_1$, has an incentive to deviate to a different interest factor, say $r'$.

Consider borrowers located in between $M_1$ and $M_2$. The utility of a double-dipping borrower is given by (6), i.e. $u - t/n$. Next recall that in case a borrower takes a single loan from a MFI located at a distance $x_i$ from her, her utility is $F - r - tx_i$. Consequently in an equilibrium with double-dipping, it is intuitive that borrowers who are close to $M_1$ (respectively $M_2$) will take a single loan from $M_1$ (respectively $M_2$), whereas borrowers who are at an intermediate distance from both the MFIs will indulge in double-dipping.

The total demand for loans facing $M_1$ consists of two elements, that from those who single-dip, and that from double-dippers. We then identify two borrowers, A and B, located in between $M_1$ and $M_2$, and at distances $a$ and $b$ respectively from $M_1$ (see Figure 2). Let A be indifferent between borrowing from $M_1$ alone, and borrowing from both $M_1$ and $M_2$. Similarly, let B be indifferent between borrowing from $M_2$ alone, and both the MFIs. It is clear that the total demand for loans for $M_1$ is $2a$ from single dipping borrowers, and that from double-dipping borrowers is $2(b - a)$.

Equating the payoffs from single-dipping and double-dipping (using (2) and (8)), it is straight-
forward to see that

\[ a = \frac{F - u - r'}{t} + \frac{1}{n}, \]  
(9)

and,

\[ b = \frac{u - F + r}{t}. \]  
(10)

Thus, the ‘number’ of borrowers who are double-dipping is given by

\[ b - a = 2u - 2F + r + r' - 1/n. \]  
(11)

Next observe that the profit of \( M_1 \), the deviating MFI, is

\[ \pi_1(r', r) = 2[ar' + (b - a)F/2 - bc], \]  
(12)

and the aggregate utility of its clientele is given by

\[ W_1(r', r) = 2[a(F - r') - t \int_0^a xdx] + 2(b - a)(u - t/n), \]  
(13)

where the term in square brackets denote the aggregate utility of those borrowers who take a single loan from \( M_1 \), and the second term represents the aggregate utility of those borrowers who double-dip. Substituting these expressions into (1), we have that

\[ U_1(r', r) = 2\mu[a(F - r') - t \int_0^a xdx + (b - a)(u - t/n)] + 2(1 - \mu)[ar' + (b - a)F/2 - bc]. \]  
(14)

Consequently, \( M_1 \) maximizes its utility \( U_1(r', r) \) subject to the break even condition that

\[ \pi_1(r', r) = 2[ar' + (b - a)F/2 - bc] \geq 0. \]  
(15)

We observe that the first order condition for an equilibrium only depends on \( r' \). Denoting the solution by \( r^{**} \), we have that

\[ r^{**} = \frac{(2\mu - 1)(u - t/n) + (3 - 5\mu)F/2}{2 - 3\mu}. \]  
(16)

We then incorporate the non-negative profit-constraint into the analysis. Using (9)-(11), we find that all MFIs charging \( r^{**} \) yields a non-negative profit if \( \mu \leq \mu^* \) (where \( \mu^* < 1/2 \)), but not otherwise. The intuition is again that as the MFIs become more motivated, charging a higher interest becomes less and less appealing, as this decreases borrower welfare.

We then examine if the equilibrium indeed involves double-dipping. We first focus on relatively profit-oriented MFIs. We find that whenever the MFIs are not very motivated, i.e. \( \mu \) is small, there will be double-dipping with all MFIs charging \( r^{**} \). This is because if \( \mu \) is small, then \( r^{**} \) is going to be high, making double-dipping more attractive. We find that this result obtains either
when (a) the project is not very productive, i.e. $F < u + c - t/2n$, and $\mu \leq \bar{\mu}$, or (b) the project is relatively productive, i.e. $F > u + c - t/2n$, and $\mu \leq \bar{\mu}$, where $\bar{\mu} = \frac{u - F/2}{u - F/2 + t/2n} < \mu$. In case (a), project productivity, i.e. $F$, being small, has a negative effect on the utility of a single-dipping borrower. While $F$ being small also tends to make $r^{**}$ smaller, thus mitigating the direct effect of $F$ being small to some extent, it is the direct effect of a change in $F$ that dominates. In case (b), $F$ being large of course makes double-dipping less attractive, thus in this case the MFIs have to be more profit-oriented for a DDE to be sustainable (this is captured in the fact that $\bar{\mu} < \mu$).

On the other hand, if the MFIs are relatively motivated, i.e. $\mu > \bar{\mu}$, then we find that for $F$ small, i.e. $F < u + c - t/2n$, the equilibrium involves all MFIs charging $r$ such that they just break-even. With MFIs being very motivated they would prefer to charge low interest so that borrower welfare is high. Thus in equilibrium they charge the lowest possible interest that is consistent with feasibility.

We need one more notation before we can present the main result for this section. Let $\bar{n}$ be the highest possible $n$ such that MFIs remain viable at that level of competition (that is, they make non-negative profits). We can show that $\bar{n} < \frac{t(u+c-F)}{2(u+c-F)^2-(u-c)^2}$ (this is proved in the appendix. The parameter zone for which a DDE exists is graphically shown in Figure 4(b)).

**Proposition 4.** Suppose that the number of MFIs is not so large that MFIs necessarily make losses, i.e. $n \leq \bar{n}$. A symmetric equilibrium with double-dipping exists whenever either (a) the project is not too profitable, i.e. $F < u + c - t/2n$, or (b) the project is profitable, i.e. $F > u + c - t/2n$, and the MFIs are not too motivated, i.e. $\mu \leq \bar{\mu} = \frac{u - F/2}{u - F/2 + t/2n}$. In case either of these conditions hold then:

(i) This equilibrium is unique in the class of DDE.

(ii) Suppose the project is not too profitable, i.e. $F < u + c - t/2n$. In case the MFIs are not too motivated, i.e. $\mu < \bar{\mu}$, then all MFIs charge $r^{**}(\mu) = \frac{(2\mu-1)(u-t/n)+(3-5\mu)F/2}{2-3\mu}$, when they earn positive profits. Whereas if the MFIs are very motivated, i.e. $\mu \geq \bar{\mu}$, then they charge $r$, when they just break even.

(iii) Suppose the project is highly profitable, i.e. $F > u + c - t/2n$. If the MFIs are highly profit-oriented, with $\mu \leq \bar{\mu} = \frac{u - F/2}{u - F/2 + t/2n}$, then a DDE exists with all MFIs charging $r^{**}(\mu)$ and earning strictly positive profits.

**Proof.** Given Observations 5-8, for existence it is sufficient to show that there will be full market coverage. Note that the utility of the borrowers who take a single loan decreases the further she is

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12The formal argument leading to this paragraph is provided in Observations 6-8 in the Appendix.

13In case competition is very high, but not so high as to make MFIs unviable, very profit-oriented MFIs charge $r$ in equilibrium, earning zero profits. To reduce the number of cases we focus on the case where competition is not strong enough for this to happen.
from the concerned MFI. Moreover, the lowest utility enjoyed by such a borrower equals $u - t/n$, which also equals the utility of all those borrowers who double-dip. Given that this is positive, there is full market coverage.

We then observe that the first order condition does not depend on the interest factor being charged by the other MFIs. Consequently, all firms face the same optimization problem. Thus the equilibrium is unique and symmetric.

While the comparative statics properties of this equilibrium will be examined in greater details in Section 5 later on, let us mention some interesting properties of this equilibrium.

First, consider the utility level of the borrowers located in between $M_1$ and $M_2$. For borrowers who take a single loan, note that their utility decreases the further they are from their lender (See (2)). Whereas the utility of the borrowers who double-dip are all the same at $u - t/n$. Further, note that the borrowers who double-dip are actually worse off compared to borrowers who do not (see Figure 3).

Second, given Propositions 2 and 4, we can now prove Proposition 1. From Proposition 2 an SDE exists if and only if the project is productive enough, i.e. $F > c + u - t/2n$, and the MFIs are not too profit-oriented, i.e. $\mu \geq \hat{\mu}$. Next consider Proposition 4. Given that $n \leq \tilde{n}$, existence of a DDE is assured whenever either (a) the project is not too profitable, i.e. $F < u + c - t/2n$, or (b) the project is profitable, i.e. $F > u + c - t/2n$, and the MFIs are not too motivated, i.e. $\mu \leq \tilde{\mu} = \frac{u - F/2}{u - F/2 + t/2n}$. Combining the two, we obtain the first part of Proposition 1. The second part of the proposition also follows from Propositions 2 and 4, as well as proposition 6 (to follow).

Third, in case the MFIs are very motivated, i.e. $\mu > \mu$, the interest rate - which is set at the break-even interest rate $\bar{r}$ - may be decreasing in $F$ at relatively high levels of $\bar{r}$ (and increasing at low levels). Intuitively, an increase in project productivity increases the number of single-dipping borrowers (by making investment and borrowing from a single source more attractive). On one hand, this implies that the mass of single-dippers can be kept constant through a corresponding rise in the interest rate. On the other hand, a higher mass of single-dippers means that the MFIs can break even by charging a lower interest rate than before. The second factor dominates at high levels of $\bar{r}$, since at high rates of interest the marginal utility gain from a further increase in interest is small.

Finally, note that the equilibrium interest $r^{**}$ does not depend on the MFI's cost of capital, i.e. $c$. This is because $c$ enters an individual MFI's objective function only via its impact on aggregate costs, which in turn depends on the aggregate number of borrowers who take loans from this MFI, i.e. $2b$. The result now follows as $2b$ however only depends on the interest factor of the neighboring MFIs, but not on the MFI's own interest factor. As we later argue in the concluding section, this result has some policy implications, especially in the Indian context.
4.1 Increase in Motivation

We now consider the effects of an increase in the motivation parameter, i.e. $\mu$, on default. A priori the effect is unclear. On the one hand, it may be argued that more motivated MFIs will charge a lower interest rate, thus lowering default. On the other hand, however, it may be argued that more motivated MFIs will be more tolerant of default, and thus increased motivation may lead to increased default. This argument would be in line with Roy and Roy Chowdhury (2009).

Our analysis suggests that an increase in the motivation parameter necessarily reduces default. The reason however is somewhat deeper than what is usually argued. In a double-dipping equilibrium the double-dippers actually obtain a utility lower than those who do not double-dip. Thus the MFIs in fact have an incentive to reduce double-dipping, and the way to do that is to reduce the incentive to double-dip by reducing the rate of interest.

**Proposition 5.** Let there be a double-dipping equilibrium. If $\mu < \mu_{\text{c}}$, then for a small increase in the motivation of the borrowers there is a decrease in the rate of interest, as well as the extent of default. Further, there is an improvement in borrower welfare.

This is interesting given that in the Indian context some commentators have been concerned about mission-drift, i.e. the MFIs becoming more profit oriented, which can happen either because of the incumbent MFIs losing motivation, or due to the entry of new, more profit-oriented MFIs. The Malegam Committee Report (page 33) states “it has been suggested that the entry of private equity in the microfinance sector has resulted in a demand for higher profits by MFIs with consequent high interest rates and the emergence of some of the areas of concern which have been discussed earlier.” Thus Proposition 5 does seem to suggest that, in case of mission drift, such increase in interest rates, and consequently, default may take place.

4.2 Multiple Equilibria

Given Propositions 2 and 4, we can show that exists parameter zones over which multiple equilibria exist. This is not only of theoretical importance, but has implications for the effect of increased competition. We shall later argue that this allows for the possibility of regime switch from a SDE to a DDE as a result of an increase in MFI competition.

**Proposition 6.** Let the productive project be relatively productive so that $F > u + c - t/2n$, and the MFIs be neither too motivated, nor too profit-oriented, i.e. $\hat{\mu} \leq \mu \leq \tilde{\mu}$. Then there are multiple equilibria, one SDE, and one DDE. Further, for any given set of parameter values, the interest under the DDE exceeds that under the SDE.

Consider the parameter values for which multiplicity may obtain. Given that $\mu$ is not too large, $r^{**}$ is going to be large, so that single-dipping is not that attractive for borrowers. Hence a DDE
exists. Whereas given that \( F \) is large and \( \mu \) is not too small, \( r^* \) is going to be small, so that single-dipping is quite attractive, and an SDE exists.

Intuitively, the fact that interest rate under a DDE exceeds that under an SDE can be traced to the fact that under a double-dipping equilibrium the MFIs suffer losses on some borrowers. Thus they must charge a relatively higher rate of interest so as to ensure that the profit component of their utility is not too low. In a DDE, at least the borrower at a distance of \( 1/2n \) from \( M_1 \) must have an incentive to double dip (otherwise, there will be no double dipping). Hence this borrower must get more out of double dipping than single dipping, i.e. we must have \( u - t/n > F - r^{**} - t/2n \) or \( r^{**} > F - u + t/2n \). Now consider the SDE. In an SDE, no one double dips, hence even the borrower at a distance \( 1/2n \) from \( M_1 \) (whose incentive to double dip is strongest) must obtain more from single-dipping than double-dipping. This implies that \( F - r^* - t/2n > u - t/n \), or \( r^* < F - u + t/2n < r^{**} \).

\textit{Example.} As an example, consider \( F=20 \), \( c=11 \), \( u=12 \), \( t=36 \). Then \( F > u + c - t/2n \) for \( n < 6 \). Now we can show that both a SDE and a DDE exist at \( n=5 \). In this case we have \( \hat{\mu}=0.3548 \) and \( \tilde{\mu}=0.3571 \), so the zone of multiple equilibria is [0.3548,0.3571]. Interest rate in the SDE falls from 11.6 at \( \mu = \hat{\mu} \), to 11.452 at \( \mu = \tilde{\mu} \). As expected, interest rates in the SDE fall with motivation, though note that positive profits are earned over the entire zone. Interest rates in the DDE over this zone fall from 11.614 for \( \mu = \hat{\mu} \), to 11.6 at \( \mu = \tilde{\mu} \) (again, corresponding to a positive profit). We find that though the interval for multiple equilibria is not necessarily non-empty, it is more likely to be non-empty when \( n \) is not too low (subject to still being in the zone such that \( F > u + c - t/2n \)).

5 MFI competition: Default, interest rates and borrower welfare

We then turn to analyzing the effect of an increase in MFI competition. As discussed in the introduction, this is an issue that has become extremely important in the recent years and, consequently much debated in the literature.

We first consider the effect of increased MFI competition on double-dipping equilibrium, abstracting, for the moment, from the issue of regime switch. We find some interesting results. First, we show that contrary to popular wisdom, default increases with competition. This is true both of aggregate default, as well as of default per MFI. Thus an increase in MFI competition necessarily leads to an increase in inefficiency. Second, we find that the effect of MFI competition on the interest rate can go either way. Somewhat paradoxically, with higher motivation levels we find that an increase in MFI competition leads to an increase in equilibrium interest rates, whereas lower motivation levels generate the opposite implication. Finally, the effect on borrower welfare is also quite nuanced. While all borrowers gain from a decline in the interest rates, the effect of an increase in interest rates can affect different borrowers differently. While the utility of double-dipping borrowers increases, that of single-dipping borrowers may either increase, or decrease.
Proposition 7. Let the hypotheses of Proposition 4 hold, so that an equilibrium with double dipping exists. Further, suppose that there is an increase in MFI competition. Consider equilibria with double-dipping:

(i) There is an increase in aggregate default, as well as the number of defaulters per MFI. Consequently, inefficiency increases.

(ii) Suppose that either (a) the project is not too profitable, i.e. $F < u + c - t/2n$ and the MFIs are not too motivated, i.e. $\mu \leq \bar{\mu}$, or (b) the project is profitable, i.e. $F > u + c - t/2n$, and the MFIs are not too motivated, i.e. $\mu \leq \bar{\mu}$. Then an increase in the number of MFIs reduces interest rates and increases aggregate borrower welfare.

(iii) In case the project is not too profitable, i.e. $F < u + c - t/2n$ and the MFIs are relatively motivated, i.e. $\mu > \bar{\mu}$, an increase in the number of MFIs leads to an increase in interest rates. While the aggregate utility of double-dipping borrowers increase, that of single-dipping borrowers may either increase, or decrease.

In the Indian context, it is interesting that the recent Malegam committee report argues that it is of the utmost importance “to reduce interest rates charged by MFIs or improve the service provided to borrowers....Ultimately, this can only be done through greater competition both within the MFIs and without from other agencies operating in the microfinance sector.” (page 32, Malegam Committee Report). As Proposition 7 suggests however, an increase in competition is not an unmixed blessing as it necessarily reduces efficiency, and may also adversely affect the borrowers’ utility.

Proposition 7(i) is intuitive. With an increase in the number of MFIs to $n'$, double-dipping becomes more attractive compared to taking a single loan. This follows since while the transactions cost under single dipping is $t/2n$, that under double-dipping is $t/n$. Ceteris paribus, this increases default. While in some cases there may a countervailing effect in the form of a decrease in interest, it is the direct effect which dominates.

Proposition 7(ii) deals with relatively profit-oriented MFIs. From Proposition 7(i), an increase in MFI competition increases default, so that the MFIs reduce their interest rates in a bid to decrease default. This, along with the fact that there is a decline in aggregate transaction costs imply that there is an increase in aggregate borrower welfare.

Proposition 7(iii) deals with relatively motivated MFIs. Interestingly, in this case an increase in MFI competition may lead to an increase in the rate of interest, with adverse welfare implications for at least some borrowers. The intuition is simple though. Recall that these MFIs just break even. When $n$ increases, double dipping becomes relatively more attractive, resulting in more losses and therefore requiring a higher interest rate to break even. This however does not affect the double-dipping borrowers whose utility increases because of the transaction cost effect, while
for the single-dippers, the increase in the interest rate and the reduction in transactions costs have opposite effects on utility.

5.1 MFI competition and regime switch

We then examine if an increase in MFI competition can cause a regime switch. We shall say that a change in parameter values cause a regime switch from a SDE to a DDE provided initially the equilibrium involved a SDE, whereas after the parametric shift there is a unique equilibrium which is a DDE. A regime switch from a DDE to a SDE can be defined symmetrically.

We show that a regime switch from a SDE to a DDE may happen if the project is neither too productive, nor too unproductive. Moreover, in that case there can be an increase in interest rates with a consequent decline in borrower welfare.

**Proposition 8.** Let there be an increase in MFI competition, so that the number of MFIs increase from \( n \) to \( n' \).

(i) Suppose the project is very productive in the sense that \( F > u + c - t/2n' \). Then an increase in MFI competition can never cause a regime switch from a SDE to a DDE, or from a DDE to a SDE.

(ii) Suppose that the project is neither too productive, nor too unproductive, in the sense that \( u + c - t/2n < F < u + c - t/2n' \). Then an increase in MFI competition can cause a regime switch from a SDE to a DDE. In this case there can be an increase in interest rates, with consequent decline in aggregate welfare.

(iii) Suppose the project is very productive in the sense that \( F > u + c - t/2n' \). Then an increase in MFI competition increases the size of the motivation levels over which it obtains, i.e \( \tilde{\mu} - \hat{\mu} \).

Proposition 8(iii) follows from the fact that with an increase in \( n \), \( \hat{\mu} \) falls, while \( \tilde{\mu} \) increases. Recall that an increase in \( n \) leads to a fall in \( r^* \), the interest rate charged in an SDE by MFIs that are not too motivated. This factor makes single-dipping more attractive. It also reduces the distance that a borrower who borrows from only one source has to travel. These factors then combine to reduce \( \hat{\mu} \), so that a SDE becomes feasible at a relatively smaller level of motivation. At the same time, the rise in \( n \) increases the tendency to default by reducing the distance a double-dipper has to travel. Hence a DDE becomes likely even for relatively motivated MFIs (an increase in \( \tilde{\mu} \)).

Proposition 8(i) then follows from 8(iii), since 8(iii) indicates that, subject to the fact that the project is very productive, an increase in competition increases the interval of motivation over which multiple equilibria obtains.

Proposition 8(ii), on the other hand, follows since an increase in MFI competition makes double-dipping more attractive vis-a-vis taking a single loan, generating the possibility of a regime switch.
from a SDE to a DDE. Moreover, this happens for intermediate values of $F$ because for $F$ large, the equilibrium interest is going to be low under a candidate SDE, so that a SDE necessarily exists. Interest rates may increase as a result of the regime switch. As an example, consider $F=50, c=26, u=30, t=96$. Now consider $n=7$. At this point we have $F > u + c - t/2n$. Now we have $\hat{\mu}=.3$; MFIs with this level of motivation charge a SDE interest rate of 26.857. Now let $n$ increase to 8 at which point there is a regime switch to a DDE. Keeping motivation constant at .3, the interest rate in this DDE - with greater competition - has increased to 27.54.

Interestingly, both Propositions 7 and 8(ii) show that an increase in MFI competition can lead to an increase in equilibrium interest rates. However, while in Proposition 7 the result is driven by the fact that the MFIs are breaking even in equilibrium, in Proposition 8(ii) it is driven by the possibility of a regime switch. Interestingly though, both these can be traced to the fact that an increase in MFI competition makes double-dipping relatively attractive.

6 Conclusion

We conclude with a brief discussion of some policy implications of our framework. One main insight is that the predicted effect of policy changes may be nuanced because of the indirect effect of these changes on the level of MFI competition.

6.1 Providing subsidised loans to MFIs

Policy-makers have argued in favor of a reduction in the MFIs’ costs of lending, recommending the provision of subsidized loans to MFIs to this end (see, Malegam, 2011). In our framework, this translates into a reduction in $c$. Interestingly, in case the DDE equilibrium involves a positive profit for the MFIs, a fall in $c$ (arising out of providing subsidized loans to MFIs), does not affect the interest factor, and hence default and borrower welfare. The only effect is an increase in MFI profits. It is interesting that, even with motivated MFIs, a fall in $c$ need not be passed on to the borrowers. Of course, in case the MFIs are very highly motivated, and the DDE equilibrium involves the MFIs just breaking even, then a fall in $c$ leads to a fall in the equilibrium interest factor, which improves borrower welfare. However, in case such a decrease in $c$ attracts entry by more MFIs, then the implications, as we have already argued, may be complex.\[14\]

6.2 A cap on interest charged by the MFIs

In response to increasing borrower defaults (notably in some Indian states like the Andhra Pradesh), some policy-makers are advocating a cap on interest rates that MFIs are allowed to charge (the

\[14\] In contrast, if all subsidies on MFI loans were removed, so that $c = c' > u$, our results would change in the following way; when projects are relatively unproductive, a DDE would exist only for relatively profit-oriented MFIs, which would earn positive profits (but not for very highly motivated MFIs). Other results would remain the same.
Malegam Committee Report, 2011\textsuperscript{[15]} The policy-makers’ rationale for advocating such a cap is that the poor may be less able to repay unless the interest rate is kept low. On the other hand, many economists disagree with this measure fearing that the cap may be too low to enable MFIs to break even\textsuperscript{[16]} What would a cap on interest rates mean in the context of our model?

We focus on the DDE equilibrium. Suppose that the interest cap, say \( \hat{r} \), is lower than the existing interest factor. In case the MFIs break-even if they all charge \( \hat{r} \), then it is easy to check that all MFIs charging \( \hat{r} \) constitutes a DDE. In this case an interest cap unambiguously reduces interest rate and defaults, and consequently increases borrower welfare.

Things however are different in case the MFIs make losses at the interest cap. If we expand the model by allowing for exit, then it is natural to conjecture that there will be exit, and the equilibrium will involve all the firms charging \( \hat{r} \). The implications for borrower welfare are now unclear, because while a lower interest rate would tend to increase welfare, welfare would tend to fall as there is a reduction in product variety.

In conclusion, we have constructed a tractable model of competition between motivated MFIs where some borrowers may double dip and default in equilibrium. We have also derived conditions that support a no-default (or single dipping) equilibrium, and analyzed the effect of competition on interest rates, default and borrower welfare. We obtain several interesting results. We find, for example, that while increasing MFI competition leads to lower interest rates if MFIs are relatively profit-oriented, competition can actually raise interest rates for highly motivated MFIs. We also see that while an increase in competition increases product variety, it worsens the incentive to double dip by a greater extent and hence increases default. Further, for projects of intermediate productivity, competition may induce a regime switch from a no-default (single dipping) equilibrium to one with double dipping and default, with an accompanying rise in interest rates and fall in borrower welfare. Our model also shows that a no-default equilibrium is easier to achieve when borrowers have access to high productivity projects, all else equal; besides directly boosting the attractiveness of productive investment relative to consumption, this also lowers interest rates reducing incentives to default.

**Acknowledgements:** Prabal Roy Chowdhury is grateful for the PPRU grant 870-G, as well as a visitorship to the Singapore Management University that made this collaborative effort possible.

\textsuperscript{15}The Committee advocates an interest cap of 24\% on individual loans.
\textsuperscript{16}See, for example, http://www.indianexpress.com/news/help-microfinance-dont-kill-it/716105/0
7 Appendix

Proof of Proposition 2. The proof of Proposition 2 follows from a series of observations, 1-4. We first state these observations, before providing the formal proofs later on. We begin by examining the incentive for double-dipping.

Observation 1. There exists \( \hat{\mu} \), where \( 0 < \hat{\mu} < 1/2 \), such that whenever \( \mu \geq \hat{\mu} \) and all MFIs charge \( r^* \), then the borrowers have no incentive to double-dip. Further, in this case there is full market coverage.

We then show that for an SDE to exist, it is necessary that the project be productive, i.e. \( F > u + c - t/2n \).

Observation 2. A necessary condition for a SDE to exist is that \( u < F - c + t/2n \).

We then check if \( r^* \) exceeds \( c \), which is necessary for the MFIs to break even. Straightforward calculations yield

Observation 3. There exists \( \mu' > 0 \), such that \( r^* \geq c \) if and only if \( \mu \leq \mu' \). Further, given \( u < F - c + t/2n \), \( \hat{\mu} < \mu' \).

Thus for any \( \mu \) in \([\hat{\mu}, \mu']\), all MFIs charging \( r^* \) constitute a SDE.

Observation 4. For \( u < F - c + t/2n \) and \( \mu > \mu' \), there exists an SDE where all MFIs charge \( c \) and just break even.

We next turn to proving these observations.

Proof of Observation 1. Note that from assumption A1, \( u > F - c - t/2n \), \(^{17}\) so that the no deviation condition does not hold for \( \mu = 0 \). Thus a SDE cannot exist in case the MFIs are pure profit maximizers. Then, there exists \( \hat{\mu} > 0 \), such that the no deviation condition holds if and only if \( \mu \geq \hat{\mu} \). We can check that the expression for \( \hat{\mu} \) is

\[
\hat{\mu} = \frac{u - (F - c - t/2n)}{2u - (F - c - t/2n)} < \frac{1}{2}.
\]

(17)

Finally note that if the no-deviation condition holds, then this implies that the borrowers have an utility of at least \( u - t/n > 0 \). Given that \( u - t/n > 0 \) (from A1), there is full market coverage. \( \blacksquare \)

Proof of Observation 2. Suppose we have \( u > F - c + t/2n \). Rearranging, \( u - t/n > F - c - t/2n \). Now if the interest rate charged were \( c \), this inequality implies that at least the borrower at a

\(^{17}\)This follows since from A1, we have \( u > F/2 \). So we can write \( u > F - F/2 > F - c > F - c - t/2n \), where the second inequality follows as \( c > F/2 \).

22
distance of $1/2n$ from both neighboring MFIs would prefer to double dip. However, note that to avoid losses, MFIs in a SDE must charge an interest rate of at least $c$. If the inequality holds for $r = c$ it also holds when $c$ is replaced by higher $r$. Hence, for all profitable interest rates, a nonzero mass of borrowers will double dip. Thus, for a SDE to exist, it is necessary that $u < F - c + t/2n$.

**Proof of Observation 3.** We find that there exists a threshold $\mu'$, such that $r^* \geq c$ only for $\mu \leq \mu'$. We observe that

$$\mu' = \frac{t/n}{F - c + 3t/2n}. \quad (18)$$

One can show that given $u < F - c + t/2n$, $\mu' > \hat{\mu}$.

**Proof of Observation 4. Step 1.** First consider highly motivated MFIs, with $\mu > 1/2$. Now note that for an individual MFI, setting $r' = r$ in the first order condition, we obtain

$$t \frac{\partial U_i}{\partial r'} = -\mu F + (1 - \mu)c - (3\mu - 2) \frac{t}{2n} + (2\mu - 1)r'.$$

(19)

Now note that given $\mu > 1/2$, we have $2\mu - 1 > 0$, so that the second derivative of utility with respect to $r'$ is positive. Therefore, the first order condition would minimize rather than maximize utility. Moreover, recall that for a single-dipping equilibrium to exist, a borrower at a distance of $1/2n$ from $M_1$ must get more utility out of single dipping, i.e. $F - r' - t/2n$, than out of double dipping, i.e. $u - t/n$. This imposes a ceiling on $r'$. We must have

$$r' \leq F - u + t/2n. \quad (20)$$

From this ceiling, we may check that $t \frac{\partial U_i}{\partial r'} < 0$ for all feasible $r'$. Hence, MFIs can maximize their utility by setting the lowest interest rate consistent with non-negative profits, i.e, $r = r' = c$. Moreover, no equilibrium where these MFIs set $r = r' > c$ exists; given $t \frac{\partial U_i}{\partial r'} < 0$ for all feasible $r'$, an individual MFI would deviate by charging a lower interest rate. Moreover, as long as $r' > c$, such a deviation would not violate the profit constraint.

**Step 2.** As shown in the text, for $\mu' < \mu < 1/2$, $r^* < c$. We now show that whenever $\mu \in (\mu', 1]$, when all other MFIs are charging $r = c$, an individual MFI will not deviate by charging a different $r'$. Evaluating $t \frac{\partial U_i}{\partial r'}$ at $r' = r = c$, we get

$$t \frac{\partial U_i}{\partial r'} = -\mu F + \mu c - (3\mu - 2) \frac{t}{2n}. \quad (21)$$

Now note that the RHS of this expression is decreasing in $\mu$, as its derivative with respect to $\mu$ is $-(F - c) - 3t/2n < 0$ given $c < F$. Also, note from (21) that the RHS is equal to 0 at $\mu = \mu'$. Therefore, for $\mu \in (\mu', 1]$, we have $t \frac{\partial U_i}{\partial r'} < 0$ when this derivative is evaluated at $r' = r = c$. If all other MFIs are charging $r = c$, an individual MFI never wants to charge a higher interest rate. It may want to charge a lower interest rate; however, that would violate the non-negative profit constraint and hence such a deviation would be infeasible.
Step 3. No symmetric equilibrium is possible for \( \mu \in (\mu', 1/2] \) where all MFIs charge \( r' = r > c \). Suppose all MFIs were charging \( r' = r > c \). Then for an individual MFI,

\[
\frac{t \partial U_i}{\partial r'} = -\mu F + (1 - \mu)c - (3\mu - 2)t/2n - (1 - 2\mu)r' < -\mu F + (1 - \mu)c - (3\mu - 2)t/2n - (1 - 2\mu)c,
\]

or

\[
\frac{t \partial U_i}{\partial r'} < -\mu F + (1 - \mu)c - (3\mu - 2)t/2n < 0 \quad (22)
\]
as shown in Step 2. Therefore, an individual MFI has an incentive to deviate by lowering its interest rate. As \( r' > c \), such a deviation does not violate the profit constraint. We have already shown in Step 1 that charging \( r' = r > c \) is also not an equilibrium for \( \mu > 1/2 \).

Step 4. The remaining condition needed for a SDE to exist for \( \mu \in (\mu', 1] \) at which the MFIs symmetrically charge \( c \) and earn zero profits, is that even the farthest borrowers should not double dip. This is equivalent to

\[
F - c - t/2n > u - t/n \quad \text{or} \quad F - c + t/2n > u.
\]

Thus subject to this condition, the part of the proof which deals with relatively motivated MFIs is complete.

Step 5. Consider relatively profit-oriented MFIs, with \( \mu \leq \mu' \). As argued in the text, these MFIs maximize their utility by symmetrically charging \( r^* \). Moreover, they earn positive profits by doing so. However, all borrowers will refrain from double dipping if and only if \( \mu \geq \hat{\mu} \). Note that given \( F/2 < u \), we have \( F - c - t/2n < u \) which implies that \( \hat{\mu} > 0 \). Moreover, it is evident from Observations 2 and 3 that \( \hat{\mu} < \mu' \) whenever a SDE exists. Hence the interval \([ \hat{\mu}, \mu' \]) is non-empty and involves all MFIs symmetrically charging \( r^* \) and earning positive profits. Moreover, subject to (7), the SDE automatically involves full market coverage.

Proof of Proposition 3. Consider \( F - c + t/2n > u \) and \( \mu \in [\hat{\mu}, \mu'] \) so that the MFIs are in a SDE and charging \( r^* \), where \( r^* \) is given by (5). Differentiating \( r^* \) with respect to \( n \),

\[
\frac{dr^*}{dn} = \frac{t(3\mu - 2)}{2n^2(1 - 2\mu)} < 0,
\]
as \( \mu < \mu' < \frac{1}{2} < \frac{2}{3} \). Differentiating \( r^* \) with respect to \( \mu \),

\[
\frac{dr^*}{d\mu} = \frac{-(F - c - t/2n)}{(1 - 2\mu^2)} < 0,
\]
as \( F > c + t/2n \) from A1. Thus a small increase in competition, or motivation, reduces interest rates. As the utility of a borrower located at a distance \( x \leq \frac{1}{2n} \) from \( M_1 \) in this SDE is given by \( F - r^* - tx \), and as \( F, t \) and \( x \) are unchanged, the fall in \( r^* \) following a small rise in motivation or competition therefore improves borrower welfare.

Derivation of \( \hat{n} \) and Proof of Proposition 4. That MFIs become unviable at large \( n \) can be seen intuitively from the following. If \( n \) is larger than \( \frac{t}{w+c} \), we have \( F - c > u - t/n \). But this implies that the utility a double-dipper gets, \( u - t/n \), exceeds the maximum utility of even a single-dipping borrower located at a distance of zero from \( M_1 \) (as such a borrower would incur no transport cost, but would need to pay at least an interest rate of \( c \)). In this situation, every one would double dip,
so that MFIs would incur losses. Thus while $\tilde{n}$ is necessarily less than this limit, we now derive a tighter bound that $\tilde{n}$ must satisfy.

We first examine when the non-negative profit constraint is satisfied. Observe that the non-negative profit constraint when all MFIs symmetrically charge $r$, reduces to

$$r^2 - Yr + Z \leq 0,$$

where $Y = 2F - c - u + t/n > 0$ and $Z = -(F - c)u + F(F - c + t/2n)$.\(^\text{18}\)

Let $r$ and $r^*$ be the two roots of (25), with $r$ being the minimal $r$ that satisfies (25) with equality. And moreover, let $\mu$ satisfy

$$r = r^*(\mu).$$

where it is easy to check that $\mu < 1/2$ (this is proved in Observation 5 below). MFIs make non-negative profits as long as (25) is satisfied. We will prove below that if the interest rate that MFIs optimally set (according to the first-order conditions) lies in between $r$ and $r^*$, they already make positive profits. For MFIs for whom this is not the case, we will show that the interest rate they end up charging corresponds to one of the two roots of (25). Hence, the condition that MFIs be viable is tantamount to the requirement that these roots be well-defined. Now this is equivalent to the requirement that $Y^2 - 4Z \geq 0$. Substituting in for $Y$ and $Z$ and rearranging, we find that this can be written as

$$(u - c)^2 > \frac{t^2}{n^2} + \frac{2t(u + c - F)}{n}.$$ \hspace{1cm} (27)

Given that $n < \frac{t}{u + c - F}$ the RHS of (27) is increasing in $n$. Therefore (27) is likely to hold when $n$ is not too large. Define $\tilde{n}$ as the largest $n$ for which (27) holds as an equality. Using $n < \frac{t}{u + c - F}$ and manipulating (27), we find that $\tilde{n}$ must satisfy $\tilde{n} < \frac{t(u + c - F)}{2(u + c - F)^2 - (u - c)^2}$. We then argue, that the break-even constraint is going to be satisfied at $r^*$ whenever the MFIs are not too motivated.

While the formal proof follows later, Figure 5 summarizes the basic argument. Note that $r^2 - Yr + Z$ is convex in $r$, with it being negatively sloped at $r = 0$. Thus there is an interval, $[\underline{r}, \bar{r}]$ over which it is negative. Given that $r^*$ is decreasing in $\mu$ and that $r^*(\underline{\mu}) = \underline{r}$, $r^* \geq \underline{r}$ for all $\mu \leq \underline{\mu}$. Given A2, and as long as $n$ is not too large, so that $(u - c)^2 - (c - F/2)^2 > \frac{t^2}{n^2} + \frac{2t(u + c - F)}{n}$, we necessarily have $r^*(0) < \underline{r}$. Consequently the upper limit implied by the non-negative profit constraint never binds. If $n$ is larger than the limit implied by the inequality above (call it $\hat{n}$) but smaller than $\tilde{n}$, we have an additional sub-case where very profit-oriented MFIs charge $\bar{r}$ in equilibrium, earning zero profits. To reduce the number of cases under study, we assume that competition is sufficiently restricted that this case does not arise.

\(^{18}\)Note that showing that $Y > 0$ as $Y = F + F - c - (u - t/n)$ and it is easy to show that the term in curly brackets in itself is positive given the restrictions on $\tilde{n}$.

\(^{19}\)It is clear that $\underline{r} = \frac{Y - \sqrt{Y^2 - 4Z}}{2}$. We shall later argue that both $\underline{r}$ and $\mu$ are well defined.
Observation 5. Let $\mu \leq \bar{\mu}$ and let $n < \tilde{n}$. Then in the symmetric outcome where all MFIs charge $r^{**}$, all MFIs have non-negative profits.

We then examine if the equilibrium indeed involves double-dipping. From (11) we see that $b - a$ depends positively on the interest rate. Therefore, if $b - a$ is positive at $\underline{r}$, then it is also positive for $r^{**} \geq \underline{r}$. Substituting $\underline{r} = r = r'$ in (11), the condition for positive $b - a$ boils down to $u > c + \sqrt{Y^2 - 4Z}$, which simplifies to

$$u > F - c + t/2n. \quad (28)$$

We thus have

Observation 6. Let $\mu \leq \underline{\mu}$ and $u + c - t/2n > F$. In case all MFIs charge $r^{**}$, then some borrowers double-dip.

Combining the Observations 5 and 6, it is easy to see that provided competition is not too strong, a double-dipping equilibrium exists whenever the MFIs are not too motivated, i.e. $\mu \leq \underline{\mu}$ and the project is not too profitable, i.e. $u > F - c + t/2n$.

We then consider the case where the MFIs are very motivated, i.e. $\mu > \underline{\mu}$. Recall that for $\mu = \underline{\mu}$, the MFIs just break-even at $\underline{r}$. Consider any higher $\mu$, and suppose that all MFIs apart from $M_i$ are charging $\underline{r}$. In that case one can show that while $M_i$ would like to charge a lower interest factor, doing so leads to a negative profit. Thus charging $\underline{r}$ maximizes its utility subject to the break-even constraint. Summarizing this discussion we have

Observation 7. Let $\mu > \underline{\mu}$, $n < \tilde{n}$ and that all MFIs apart from $M_i$ are charging $\underline{r}$. Then, for all $r \geq \underline{r}$, the utility of $M_i$ is decreasing in $r$.

We finally show that if the project is relatively profitable, in the sense that $u < F - c + t/2n$, then a DDE exists whenever the MFIs are not too motivated.

Observation 8. Suppose $F > u + c - \frac{t}{2n}$ and $n < \tilde{n}$. Then an equilibrium where some borrowers double dip exists if MFIs are relatively profit-oriented, i.e. for $\mu \in [0, \tilde{\mu}]$, where $\tilde{\mu} = \frac{u - F/2}{u - F/2 + t/2n} < \underline{\mu}$.

We then provide the proofs of Observations 5-8.

Proof of Observation 5. Step 1. Substituting for $a$, $b$ and $b - a$ (equations 9-11) into the zero-profit constraint, we find the latter can be expressed as a condition on a quadratic:

$$Q(r) = r^2 - Yr + Z < 0,$$

where $Y = 2F - c - u + t/n > 0$ and $Z = -(F - c)u + F(F - c + t/2n)$. Observe that $Y > 0$, so that $Q(r)$ is decreasing at $r = 0$, and convex. Consequently, $Q(r) < 0$ holds for intermediate values of $r$, $\underline{r} \leq r \leq \overline{r}$.
Step 2. Now it can be checked that $dr^{**} = \frac{u-t/n-F/2}{(2-3\mu)^2}$. The denominator is positive, and note that the numerator is negative given $u - t/n < F - c < F/2$ where the last inequality follows from $c > F/2$. Therefore, we have $dr^{**} < 0$.

Step 3. It can be checked that there is a threshold value of motivation $\mu$ at which the lower limit $r$ imposed by the zero-profit constraint becomes binding. Moreover, $\mu < 1/2$. To see this, note that first, we must have $r > c$. Since some double dipping occurs, MFIs have to accept losses of $c - F/2$ on a subset of borrowers, to counterbalance which the interest which they earn on non-defaulting borrowers has to strictly exceed their cost. Now note that $r^{**}(1/2) = F/2 < c$ so that losses are made. Given $dr^{**} < 0$, we therefore infer that there exists a $\mu < 1/2$ at which $r^{**}(\mu)$ has just fallen to $r$. Moreover, $\mu$ is strictly positive as it can be checked that $r^{**}(0) > r$. Hence there is a non-empty range $\mu \in [0, \mu]$.

Step 4: Finally, $A2$ combined with $n < \hat{n}$ is sufficient to show that MFIs charging $r^{**}(0)$ make strictly positive profits; hence we must have $r^{**}(0) < r$, so that the upper limit of the zero-profit constraint never binds; pure profit-maximizers always earn a positive profit.

Proof of Observation 6. Unless a positive mass of borrowers are double-dipping, we cannot have a double-dipping equilibrium. From Observation 1, for $\mu \leq \mu$, we have $r^{**} \geq r$. From (11), $b - a$ is increasing in the interest rate charged. Hence if it is positive at $r = r' = r$ it is also positive for $r^{**} > r$. Substituting in the expression for $r$ in (11), we find that the mass of defaulters evaluated at $r$ is $b - a = \frac{u-c-\sqrt{Y^2-4Z}}{t}$. Noting that the numerator of this fraction is always positive, the condition for positive $b - a$ boils down to

$$u > c + \sqrt{Y^2 - 4Z}$$

Substituting in the values of $Y$ and $Z$, and simplifying, this is equivalent to $F - c + t/2n < u$. Note that the condition does not depend on $\mu$.

Proof of Observation 7. Consider relatively motivated MFIs with $\mu \geq \mu$. If all other competing MFIs set their interest rate at $r$, will an individual MFI deviate by setting a higher or lower interest rate? We split our analysis into two sub-cases.

Case 1: $2/3 > \mu$.

First, note that for any MFI,

$$t \frac{\partial U_i}{\partial r} = [2\mu - 1](u - t/n) + (3 - 5\mu)F/2 - (2 - 3\mu)r'.$$

Thus $\frac{\partial U_i}{\partial r'} > 0$ for $r' < r^{**}$ and $\frac{\partial U_i}{\partial r'} < 0$ for $r' > r^{**}$. Now recall that as $r = r^{**}(\mu)$ and $dr^{**} < 0$, we have $r^{**} < r$ for $\mu$ in this range. Thus the utility of the MFIs are decreasing in $r'$ at and beyond $r$.

Now, check what happens to an individual MFI’s incentives if $r' = r = r$. In this range, $t \frac{\partial U_i}{\partial r'} < 0$ so an individual MFI will never increase its interest rate beyond $r$. While it would like to decrease its rate of interest, it cannot do so without violating the profit constraint. Hence no
deviation is possible if all MFIs charge \( r' = r = r_2 \). Consider an alternative scenario where \( r' = r > r_2 \) so that MFIs are earning positive profits. At any interest rate greater than \( r_2 \), utility is necessarily decreasing in \( r' \). An individual MFI can then increase its utility by deviating to a lower interest rate than \( r \). As \( r > r_2 \) this would not violate the profit constraint. Hence, there is no symmetric DDE in this range where MFIs earn positive profits, but there is one where they just break even.

**Case 2.** \( \mu > 2/3 \).

From examining the expression for \( \mu \rightd r \), we find that for \( \mu > 2/3 \), \( \partial U / \partial r < 0 \) for all feasible \( r' \), recalling that for non-negative \( a \), we must have \( r' \leq F - u + t/n \). The rest of the argument mimics the earlier case.

**Proof of Observation 8.** Note that \( F > u + c - t/2n \) implies, through rearrangement of terms, that \( u < c + \sqrt{Y^2 - 4Z} \), where recall that \( Y = 2F - c - u + t/n > 0 \) and \( Z = -(F - c)u + F(F - c + t/2n) \). Consequently, it is impossible to have \( b - a > 0 \) when the MFIs are relatively motivated (with \( \mu \geq \mu_0 \)). Thus it is clear that when projects are very productive, no DDE exists for \( \mu \geq \mu_0 \). Now consider the minimum interest rate above which some borrowers will always default. This is the interest rate at which the inequality \( F - r - t/2n \leq u - t/n \) holds as an equality (so that the borrower who is equidistant from \( M_1 \) and \( M_2 \) is just indifferent between single-dipping and double-dipping). This interest rate is therefore \( F - u + t/2n \): for higher interest rates, some borrowers will always double dip and default. Moreover, as this threshold interest rate is clearly greater than \( r_2 \) (given that no one defaults at \( r_2 \)) we may find a critical level of \( \mu \) (call it \( \mu_0 \)) such that \( r^{**}(\mu_0) = F - u + t/2n \). Using the formula for \( r^{**}(\mu) \), this works out to be \( \mu_0 = \frac{u - F/2}{u - F/2 + t/2n} \). Given that \( r^{**}(\mu) \) is decreasing in \( \mu \), we therefore find that \( r^{**} \geq F - u + t/2n \) for all \( \mu \in [0, \mu_0] \) so that for MFIs in this range, a positive mass of borrowers is always double dipping. Moreover note that given \( u > F/2 \), \( \mu_0 > 0 \). Moreover, in this parameter range, \( \mu_0 < \mu \) as at \( \mu \), the interest rate has already dropped to \( r < F - u + t/2n \). As MFIs in the range \( [0, \mu_0] \) are already setting their interest rates through optimization, and as \( r^{**} \) in this range satisfies the profit constraints, we therefore have an equilibrium with some double dipping for MFIs in this range.

**Proof of Proposition 5.** (i) From (16) recall that MFIs set their interest rate according to \( r^{**} \) for \( \mu < \mu_0 \). Now differentiation yields

\[
\frac{dr^{**}}{d\mu} = \frac{u - t/n - F/2}{(2 - 3\mu)^2} < 0.
\]

The negative sign follows since \( F/2 = F - F/2 > F - c > u - t/n \), given \( F/2 < c \). Thus for small \( \mu \) a rise in motivation causes a drop in the interest rate. Note that this proof - and the proof of the subsequent parts of this proposition- applies equally to MFIs in a DDE in the zone where \( F > u + c - t/2n \). In this case MFIs have \( \mu < \mu_0 < \mu_1 < 1/2 \) and always charge \( r^{**} \) earning positive profits.

(ii) From (11), the range of default, \( b - a \), is decreasing in interest rates.
(iii) Let $\mu < \bar{\mu}$, so that the MFIs set $r = r^{**}$. Consider borrowers located in between $M_1$ and $M_2$. As $\mu$ increases, there is a fall in $r^{**}$. Thus the number of borrowers who take a loan from $M_1$ alone increases, as $a$ increases. Those who did so previously still continue to do so, and there is a rise in their utility. This follows as the utility of such a borrower is denoted by $F - r^{**} - tx$ which is decreasing in $r^{**}$. The borrowers who switch from double-dipping to single-dipping also experience an increase in their utility, as their utility rises strictly above $u - t/n$. The utility of those who double-dip in both cases remains unaffected.

Proof of Proposition 6. From Observation 8, when $F - c + \frac{t}{2n} > u$, a DDE exists if MFIs are relatively profit-oriented, ie, for $\mu \in [0, \tilde{\mu}]$. From Proposition 2, when $F - c + \frac{t}{2n} > u$, a SDE exists when MFIs are relatively motivated, with $\mu \geq \hat{\mu}$. Combining these two results, we see that multiple equilibria can exist when projects are relatively productive, i.e. when $F - c + \frac{t}{2n} > u$. The second part of Proposition 6 (which deals with interest rates) is proved in the text.

Proof of Proposition 7(i).

Effect on default. Consider $F < u + c - t/2n$ and MFIs with $\mu < \mu_0$. These MFIs set interest rates according to $r^{**}$. From (12), we can substitute the solution for $r^{**}$ into (6) to derive an expression for $b - a$ as a function of $n$:

$$b - a = \frac{2(1 - \mu)(u - F/2) - \mu t/2n}{t(2 - 3\mu)}.$$  \hfill (29)

Differentiating with respect to $n$,

$$\frac{d(b - a)}{dn} = \frac{\mu}{(2 - 3\mu)n^2} > 0,$$  \hfill (30)

as $\mu < \frac{1}{2} < \frac{2}{3}$.

Therefore, the range of double dippers in between any two MFIs increases. Aggregate default includes $n$ such segments of double dippers. As each individual segment increases with a rise in $n$, and as the rise in $n$ also increases the number of such segments, aggregate default rises as well. Note that this proof applies equally to relatively profit-oriented MFIs (with $\mu \leq \tilde{\mu}$) in a DDE in the zone where $F > u + c - t/2n$ (of course, no DDE exists for more motivated MFIs in this zone). These MFIs, too, charge $r^{**}$ earning positive profits. To see directly that $\tilde{\mu} < 1/2$ in this zone, note that $F > u + c - t/2n$ is equivalent to $F - c > u - t/2n$. Now using $c > F/2$, we have $F/2 > F - c > u - t/2n$ or $t/2n > u - F/2$. Rearranging, we get $\tilde{\mu} = \frac{u - F/2}{u - F/2 + t/2n} < 1/2$. Hence, the comparative static effects of a rise in $n$ are identical. Now consider relatively motivated MFIs in a DDE in the zone where projects are not too productive, ie $F < u + c - t/2n$ and $\mu < \mu_0$. When $\mu \geq \mu_0$ and hence when the interest rate is $r$, the expression for $b - a$ is

$$b - a = \frac{u - c - \sqrt{c^2 - 2uc + 2(F - c)t/n + (u - t/n)^2}}{t},$$ \hfill (31)

29
Differentiation and simplification yields
\[ \frac{d(b - a)}{dn} = \frac{F - c - (u - t/n)}{n^2 \sqrt{Y^2 - 4Z}} > 0. \]
The positive sign follows from the sign of the numerator, which we can show is always positive given \( n < \tilde{n} \). Therefore, we find that \( b - a \) for highly motivated MFIs increases when \( n \) increases. Therefore, an increase in competition for all MFIs increases the range of defaulting borrowers in between any two MFIs, and also therefore increases aggregate default.

7(ii).

Effect on the interest factor and borrower welfare for profit-oriented MFIs. First consider the zone where \( F < u + c - t/2n \). Both \( r^{**} \) and \( \bar{r} \) are functions of \( n \). Hence the critical threshold \( \bar{\mu} \) also changes with a change in \( n \). However, it can be shown that it will still be less than 1/2. First consider what happens if the MFIs’ motivation parameter is less than the (new) critical threshold, so that they set interest rates according to \( r^{**} \). Differentiation yields
\[ \frac{d(r^{**})}{dn} = \frac{t(2\bar{\mu} - 1)}{n^2(2 - 3\bar{\mu})} < 0, \]
as \( \mu < \bar{\mu} < \frac{1}{2} < \frac{2}{3} \). Therefore, for MFIs with a relatively low degree of motivation, competition reduces interest rates. This proof applies equally to MFIs in a DDE in the zone where \( F > u + c - t/2n \). In this case MFIs have \( \mu < \bar{\mu} < \mu < 1/2 \) and always charge \( r^{**} \) earning positive profits. Now consider the effect on borrower welfare for relatively profit-oriented MFIs, who set their interest rates according to \( r^{**} \). Now, the welfare of double-dipping borrowers increases to \( u - t/n' \) as the total cost of double dipping, \( t/n' \), is now smaller. For the single-dipping borrowers while their utility tends to increase both because interest falls and the transactions costs are lower, some borrowers may be worse off as the MFIs may move away to a further location.

7(iii).

Effect on interest factor and borrower welfare for motivated MFIs. We are now dealing with relatively motivated MFIs, with \( \mu \geq \mu \). A DDE exists for these MFIs only in the zone where \( F < u + c - t/2n \), which is the zone we consider. It remains to check how \( \bar{r} \) changes with \( n \). Calculations show that \( \frac{d\bar{r}}{dn} \) has the same sign as \( 2\bar{r} - F \). However, we know that \( F/2 < \bar{r} \). Hence, \( \frac{d\bar{r}}{dn} > 0 \). For more motivated MFIs, an increase in competition raises interest rates. When MFIs are relatively motivated, an increase in competition to \( n' > n \) still increases the welfare of those borrowers who were previously double dipping. However, borrowers who continue to single dip may be worse off than before, as their utility falls due to a rise in the interest rate. They may however gain from the MFIs coming closer to them. Borrowers who switch from single-dipping to double-dipping may or may not be better off than before.

Proof of Proposition 8. (i) Suppose \( F > u + c - t/2n' \), i.e. projects are very productive, so that we do not transit to the zone where only DDE is possible. However, as shown in the proof of
Proposition 8(iii), the increase in \( n \) widens the zone of motivation levels for which we have multiple equilibria.

(ii) When projects are at an intermediate level of productivity, so that \( u + c - t/2n' > F > u + c - t/2n \), it is easy to see that a small increase in competition from \( n \) to \( n' \) will cause a regime switch from a zone where - depending on the motivation level - either SDE, DDE or both may be feasible, to a zone where only DDE is feasible.

(iii) The size of the multiple equilibria zone is given by \( \max[\hat{\mu} - \tilde{\mu}, 0] \). Now we have
\[
\frac{d\hat{\mu}}{dn} = -\frac{t}{2n^2} (F - c - t/2n) u^2 < 0
\]
\[
\frac{d\tilde{\mu}}{dn} = \frac{t}{2n^2} (u - F/2 + t/2n) > 0
\]
Hence \( \hat{\mu} - \tilde{\mu} \) is unambiguously increasing in \( n \), the number of MFIs.

8 References


D: The consumer located at D is indifferent between not taking a loan and taking a loan from M_1.

Figure 1: Single-dipping Equilibrium
Figure 2: Double-dipping Equilibrium
Figure 3: Borrower payoffs under DDE
Figure 4: Parameter zones for SDE, DDE and Multiple Equilibria.
\[ y = 2F - c + \frac{t}{n} \]
\[ z = - (F - c) u + F \left( F - c + \frac{t}{2n} \right) \]

Figure 5: Root of Proposition 4