Co-movement of Skill Premium and Stock Prices

Kausik Gangopadhyay†, Atsushi Nishimura‡ and Rupayan Pal♯
† Indian Institute of Management Kozhikode (IIM-K), India
‡ Ernst & Young Shinnihon Tax, Japan
♯ Indira Gandhi Institute of Development Research (IGIDR), India

Abstract

We examine the empirical phenomenon of co-movement of skill premium and share prices by appealing to the techniques of cointegration. The US data (1984–2010) reveals that stock prices and skill premium are cointegrated indicating a long run relationship. We carry out a further investigation into panel data of six OECD countries (1984–2005). The stock price and the skill premium are found to be cointegrated in this panel data, too. We posit that the co-movement of skill premium and stock prices is driven by unexpected technological progress. Building on a three sector model of endogenous growth through Schumpeterian creative destruction a la Aghion and Howitt (1992, Econometrica: 60(2)), we reconcile these two apparently different ideas in a single model. Our model demonstrates that technological revolution can lead to rise in both skill premium and stock price at the same time. We calibrate our model based on the US data and find that a large part of the change, quantitatively speaking, is explained by our model.

Key words: Technological revolution, drastic innovation, skill premium, stock prices
JEL Classifications: O33, O41, E24, J31

Corresponding author and address: Kausik Gangopadhyay, Indian Institute of Management Kozhikode (IIM-K), IIMK Campus P.O., Kozhikode 673570, India. Telephone: +91 495 2809118, Fax: +91 495 2803010.
E-mails: †kausik.gangopadhyay@gmail.com; ‡atsushi.nishimura@jp.ey.com; ♢rupayan@igidr.ac.in
1 Introduction

The rise in skill premium, defined as wage of skilled labour relative to unskilled labour, in the 1980s for the United States economy have been documented well in the literature (Bound and Johnson, 1992). Primarily it is manifested through a wider dispersion in the wage distribution. During the same time, the U.S. stock market has also registered a very high growth in terms of market capitalization (Greenwood and Jovanovic, 1999). Such co-movement (Figure 1) of skill premium and stock prices in the 1980s was not confined in the U.S. only. Similar change in the distribution of wages as well as increase in stock prices was also observed in the U.K. during the same time (Gosling et al., 2000; Shiller, 1989). These two phenomenons - rise in skill premium and growth of stock prices - have been addressed in the existing literature to a large extent, but separately.\(^1\) We conjecture these two phenomenons as different consequences of the same phenomenon of the technological progress, beyond expectation.

In this paper, we rigorously examine the long run relationship between the variables of stock price and skill premium. In case of two stochastic variables, existence of a cointegrating vector indicates a long run relationship. We use average gross earnings of full-time dependent employees at the ninth decile and the corresponding figure at the first decile, as a measure of skill premium. The stock prices are measured through share price indices, averaged annually. For the U.S. data (1984–2010), we perform tests laid down by Engle-Granger cointegrating procedure and Johansen’s cointegrating procedure to conclude that there is a cointegrating relationship between them. We extend our analysis with OECD data using panel data econometric techniques. Panel unit root tests and panel cointegration tests affirm our hypothesis for the OECD countries as well.

After empirical examination and confirmation of our hypothesis, we posit a theoretical

\(^{1}\)For example, Bound and Johnson (1992), Berman et al. (1998), Krusell et al. (2000) and Mitchell (2005), to name a few, examined the reasons for the rise in skill premium. On the other hand, Greenwood and Jovanovic (1999), Datta and Dixon (2002), Madsen and Davis (2006) and Pastor and Veronesi (2009), among others, studied the factors associated with changes in stock prices.
explanation for this co-movement. Building on a three sector model of endogenous growth through Schumpeterian creative destruction, as proposed by Aghion and Howitt (1992), we demonstrate that a technological revolution – technological progress beyond expectation – can lead to rise in both skill premium and stock prices. We, thus, reconcile the two apparently different ideas that (a) technological progress is a major driving force to rise in skill premium, and (b) innovation leads to increase in firm value, in a single model. The intuition behind our result is as follows. A technological revolution increases the productivity of the skilled labor, beyond expectations. The increased demand for skilled labor leads to disproportionate increase in their wage. In addition to this increase in skill premium, technological revolution corresponds to unanticipated larger size of innovation which increases the revenue of modern firms disproportionately more than the increase in costs. Thereby, technological revolution increases the stock prices as well.

We note here that there are two papers, which are closely related to the present study. Hall et al. (2000) and Manuelli (2000) also attempted to explain the co-movement of skill premium and stock prices. However, our paper differs from these studies. Hall et al. (2000) introduced the idea of e-capital in a four-sector model. A rise in the value of e-capital triggers the workings of their model. The mechanism is entirely different in our model. Moreover, our framework models arrival of new firms with the adoption of new technology which is abstracted in the existing frameworks. Manuelli (2000) presents the phenomenon of co-movement in the light of a modified version of Mortensen and Pissarides (1994) model, and predicts an initial diminish in the wage followed by a resumption of usual wage.

The rest of the paper proceeds as follows. In Section 2, we elaborate our empirical findings. In this section, we state the methodology to estimate a cointegrating relationship, both for the time series data and for a more general panel data. Our results are stated afterwards. We postulate our economic environment in Section 3. Proposition 1 shows that both skill premium and share prices will rise on the event of a technological revolution. In Section 4, we calibrate our model to quantitatively assess the explanatory power of our economic environment. Section 5 concludes.
2 Empirical Findings

2.1 Data

The Organization for Economic Co-operation and Development (OECD) maintains data on various economic variables. We gathered two variables: Share Price and Skill Premium. The share price indices are part of the Monthly Monetary and Financial Statistics (MEI) database. They are usually calculated by the stock exchange, although occasionally agencies such as central banks compile the series. Monthly data are averages of daily quotations, quarterly and annual data are averages of monthly figures. This variable (denoted by $SP$) is normalized in such a manner that the value for 2005 remains at 100. We calculated the skill premium through the ratio of average gross earnings of full-time employees at the ninth decile, and the corresponding figure at the first decile. This variable (henceforth, $DR$) is part of the Labour Force Statistics (LFS) database.

Figures 1 and 2 plot these two series for the United States and Japan, respectively. Moreover, Figure 3 illustrates these two variables for other six OECD countries, namely Australia, Finland, France, Korea, Netherlands and United Kingdom.

2.2 Methodology to Estimate a Cointegration Relationship for the US Data

Use of Ordinary Least Squares (henceforth, OLS) regression or correlation coefficient in determining time series variables may give rise to spurious relationship, particularly when these variables are integrated of order one. Relationship between a set of stochastic time series variables could be calculated appealing to the technique of cointegration and error correction model. We used Engle-Granger methodology for this purpose. This method tests for unit roots of selected variables. When several time-series variables under inspection are found to be integrated of order one, an OLS regression with these variables is performed. If estimated residuals for this particular regression are found to be stationary or devoid of any unit root, these variables-in-question are cointegrated.
Our examination of long run relationship based on the Engle-Granger methodology involved the following three steps.

1. We tested for unit root in the variables of $DR_t$ and $SP_t$. We appealed to Augmented Dicky Fuller (ADF) test for performing unit root test. For example, we estimated the following equation for testing unit root of $DR_t$:

$$\Delta DR_t = \beta_0 + \alpha_0 \cdot DR_{t-1} + \sum_{i=1}^{p} \alpha_i \cdot \Delta DR_{t-i} + e_t$$  \hspace{1cm} (1)

In this equation, $e_t$ is the error term of the equation. We constructed the Dicky Fuller statistic from the OLS estimate of the above equation. The distribution of this statistics under null hypothesis is given by MacKinnon (1996). We started with a maximum lag length of 4 as $p$ and tested for the significance of $\hat{\alpha_p}$. In case, it was found to be significant, we performed ADF test with $p$ lags. In the case of its being insignificant, we reduced $p$ by 1 and continued with the same exercise.

2. We ran the following OLS regression:

$$SP_t = \alpha_{US} + \beta_{US} \cdot DR_t + u_t$$  \hspace{1cm} (2)

We estimated the parameters of $\alpha_{US}$ and $\beta_{US}$ as well as $\{\hat{u_t}\}$ from the above regression.

3. We tested for absence of a unit root in the series of $\{\hat{u_t}\}$. We perform the ADF test for these estimated residuals. When we fail to accept the unit root for these estimated residuals, it is indicative of stationary nature of the residuals.

For our data-set, we restricted our examination of the cointegrating relationship to the time period in which the co-movement had happened. The time-period was posited to be in the 1980s by the concerned literature. We used the year of 1984 as the beginning of this period of co-movement as this provided us the best possible result in favour of our proposition.
Moreover, there is considerable divergence of opinion among economists regarding the end period of this co-movement. We used the longest possible period, 1984–2010, to test our hypothesis.

Additionally, we employed Johansen’s methodology (Hamilton, 1994, Chapter 20) for detection of a unit root. Under this methodology, there are two tests, trace test and $\lambda_{\text{max}}$ test. We constructed the matrix of long run relationships, the rank of which is equivalent to number of potential cointegrating relationships. The trace test examines the null hypothesis of rank $\leq r$ against the alternative of rank $> r$ for a number $r = 0, 1$. The $\lambda_{\text{max}}$ test investigates into the validity of rank $= r$ against the alternative of rank $= r + 1$ for $r = 0, 1$. We considered the case of restricted constant in our study.

2.3 Methodology to Estimate a Cointegrating Relationship for the OECD Panel Data

When a cointegrating relationship between stock price and skill premium is uncovered for the US data, it leads to the conclusion that these two variables are associated in the U.S. A more compelling evidence will be an association between these variable for other OECD countries. We utilized the panel econometric techniques in examination of this idea. These techniques provided us with powerful statistical tests and make statistical inferences allowing for cross country heterogeneity and cross-country dependence in the data. The underlying model between relationship of these two variables in a panel set-up is described below:

$$SP_{it} = \mu_i^{OECD} + \beta_i^{OECD} \cdot DR_{it} + e_{it}$$  \hspace{1cm} (3)

where $i$ is the index for various countries in the panel and $t$ is the index for time.

The following three steps broadly describe our procedure to examine a cointegrating relationship in the OECD panel data.

1. Panel Unit Root Tests: We examined the stationarity of variables-in-question through panel unit root tests. Panel unit roots function based on the null hypothesis
that the variable-in-question is non-stationary across all series in the panel. The alternative hypothesis can either be stationarity of one series or all series in the panel.

There are two groups of panel unit root tests. The first group of tests, for example Im et al. (2003), Levin et al. (2002), Harris and Tzavalis (1999), Maddala and Wu (1999), assumes cross-sectional independence. The statistics of interest under these tests are denoted by $\bar{W}_t$, $t^*$, $\rho$, and $\chi^2$, respectively. The second group of tests which include Breitung (2000) and and Pesaran (2007) relapses this assumption of cross-sectional independence. The corresponding statistics are given by $\lambda$ and $Z_t$, respectively.

Our basic model is described by (1). We described the p-values for a one-sided test. For the test proposed by Levin et al. (2002), use of some semi-parametric corrections are required. We employed the Bartlett kernel along with Newey and West (1994) bandwidth selection algorithm for this purpose. All bandwidth and lag orders are set according to the rule, $4(T/100)^{2/9}$. The optimal number of lags in (1) are chosen according to Akaike information criterion.

2. Panel Cointegration Tests: When two variables were found to possess a unit root, we carried out panel cointegration tests to examine whether these variables are cointegrated based on the evidence obtained in this panel. We considered tests which verifies the null hypothesis of absence of any cointegrating relationship between the variables-in-question. Therefore, we appealed to structural tests described by Westerlund (2007) instead of residual based tests. When assumptions of weak stationarity for the residuals are valid, Westerlund (2007) tests possess higher power compared to residual based tests. Moreover, cross-sectional independence is not a pre-requisite for validity of these tests.

The idea of these structural tests is construction of the following conditional error
correction model using the model (3) as a cointegration relationship.

\[
SP_t = \alpha_t^{OECD} + \rho_t^{OECD} \cdot (SP_{t-1} - \beta_t^{OECD} DR_{t-1}) \\
+ \sum_{s=1}^{p} \delta_{ts}^{OECD} \cdot \Delta SP_{t-s} + \sum_{s=1}^{p} \lambda_{ts}^{OECD} \cdot \Delta DR_{t-s} + \epsilon_{it}
\] (4)

If \( \rho_t^{OECD} \) s are all equal to zero, the null hypothesis of no error correction or no cointegrating relationship is valid. Westerlund (2007) constructed four different tests, for which test statistics is given by \( G_\tau \), \( G_\alpha \), \( P_\tau \), and \( P_\alpha \). Under null hypothesis of no cointegration, the distributions of these statistics converge to normal distributions with some specified means and specified variances. Under the alternative hypothesis, values of these statistics diverge toward negative infinity. These tests are, therefore, consistent. For robust standard errors, we used the technique of bootstrap replications.

In our study, we added one lag and one lead of estimated residuals in the model. In our small sample, we also computed p-values of test statistics based on 500 bootstrap replications. The relevant kernel bandwidth was set according to the rule, \( 4(T/100)^{2/9} \). All reported p-values are based on a one-sided test.

3. Examination of Assumptions of Panel Cointegration Tests: Westerlund (2007) tests are more powerful compared to residual based tests under the condition of weak exogeneity for regressors in (3). In particular, this assumption ensures that regressors will not be part of an error-correcting model when the dependent variable is regressed on them. Therefore, we constructed the following model and perform Westerlund (2007) tests for this model:

\[
DR_{it} = \mu_{i,t}^{WE} + \beta_{i,t}^{WE} \cdot SP_{it} + e_{it}^{WE} \\
DR_{it} = \alpha_{i,t}^{WE} + \rho_{i,t}^{WE} \cdot (DR_{i,t-1} - \beta_{i,t}^{WE} SP_{i,t-1}) \\
+ \sum_{s=1}^{p} \delta_{is}^{WE} \cdot \Delta DR_{i,t-s} + \sum_{s=1}^{p} \lambda_{is}^{WE} \cdot \Delta SP_{i,t-s} + e_{it}^{WE}
\] (5)

If the null hypothesis is accepted, it implies no evidence of cointegration which, in turn, supports our assumption of weak exogeneity.
Additionally, we examined cross-sectional independence by considering the estimated residuals in (4). For that purpose, we ran a fixed effect model on those residuals with country specific effect and a noise component. Subsequently, we appealed to two tests to examine the cross-sectional independence: Breusch-Pagan Lagrange Multiplier (LM) test and Pesaran (2007) CD test. Both of these tests are based on the null hypothesis of cross-sectional independence.

We are constrained by the availability of data for the skill premium. Nevertheless, we found six OECD countries (France, Japan, Korea, Netherlands, United Kingdom, and United States) with available data for the period of 1984–2005. We justify our choice of countries and years with the following reasoning: The Australian data can not be used for having a missing data point in the year of 1996 in the panel data analysis. The Finnish data is available from 1986 onwards and the Dutch data does not extend beyond 2005. Therefore, We can use the panel data from seven countries (France, Finland, Japan, Korea, Netherlands, United Kingdom, and United States) for 1986–2005. We find quantitatively quite similar results except that Westerlund (2007) tests indicate p-values for statistics of $P_\alpha$ and $P_\tau$ at 0.148 and 0.122, respectively. We included the year of 1984 to remain consistent with our findings using the U.S. data. Also, we avoided the years of 2007–2009 on account of the sub-prime crisis. All these justifies our use of the panel data of six specified countries for the time-span specified.

2.4 Results

ADF tests conclude that share price (SP) and skill premium (DR) possess unit roots (Table 1). Because of low power of ADF tests, the acceptance of unit root can be inconclusive. Kwiatkowski et al. (1992) proposed a test which uses no unit root as the null hypothesis. We found that Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test rejects the null hypothesis at 1% level (Table 2) for both variables, confirming our findings. Also, the estimated residuals in running the regression (2) do not possess any unit root. A cointegrating relationship, therefore, exists between these two variables. We used Johansen’s Cointegration test (Table
3) to the same effect.

Without time trend in the equation of unit root, the panel unit root tests (Table 4), unequivocally, accept the null hypothesis of existence of unit root for both variables. With a time trend, the panel unit root tests pass do not reveal any unanimity in their conclusions. Nevertheless, tests allowing for cross sectional dependence, strongly accept the null hypothesis of existence of unit root for both variables. We also performed the cross sectional independence tests (Table 7) on the assumption of existence of a cointegrating relationship. The rejection of null hypothesis indicates cross sectional dependence. This, in turn, provides consistency for our methodology.

Once both variables were found to be non-stationary, we employed Westerlund (2007) tests to examine the possibility of a cointegrating relationship between these two variables. Results tabulated in Table 5 suggest that all tests reject the null hypothesis of no cointegration at 5% level. This is a strong indication of cointegrating relationship between two variables-in-question. Table 6 shows acceptance of null hypothesis as the outcome of weak exogeneity tests. Since this assumption is found to be valid, we need not consider any other cointegration test.

To sum up, we found existence of a long run relationship between share price and skill premium post 1984 not only in the US data but also in the panel data of available OECD countries.

3 Economic Environment

We consider a three sector economy consisting of a consumption goods sector, an intermediate goods sector and a research sector. In this economy there is a continuum of infinitely lived individuals who inelastically supply their labor endowment in the market. There are two types of labor, skilled labor and unskilled labor. Let \( L \) and \( H \) denote the supply of skilled labor and unskilled labor, respectively, in the economy.

The consumption goods sector produces one good using unskilled labor and intermediate good. The production function of the consumption good is of Cobb-Douglas form with
constant returns to scale:
\[ y = Ax^\alpha L^{1-\alpha}, \]  \hspace{1cm} (6)

where \( y \) is the quantity of the consumption good produced, the parameter \( A \) indicates the productivity of the intermediate good, \( x \) denotes the quantity of the intermediate good used as input and \( L \) denotes the amount of unskilled labor employed. We assume that unskilled labor is fully employed in the consumption goods sector. We further assume that the consumption goods sector is competitive and the price of the consumption good is normalized to 1. Therefore, prices of the inputs used in the consumption goods sector are determined as follows.

\[ L \cdot w_L = (1 - \alpha) y \]
\[ x \cdot p_x = \alpha y, \]  \hspace{1cm} (7)

where \( w_L \) and \( p_x \) denote the wage rate of unskilled labor and price of the intermediate good, respectively.

The intermediate good is produced by a profit maximizing monopolist using skilled labor as the sole input. The production technology of the intermediate good is assumed to be linear:

\[ x = h, \]  \hspace{1cm} (8)

where \( x \) is the amount of the intermediate good produced and \( h \) is the amount of skilled labor used in the intermediate goods sector. For simplicity, it is assumed that there is no inventory of the intermediate good. The price of the intermediate good is determined by its marginal value in equilibrium, according to equation (7). This marginal value of the intermediate good depends on its quantity available in the market. By choosing the optimal quantity of the intermediate good to be supplied, the monopolist can charge the desired price of the good to maximize his profit. The profit expression of the monopolist is given by

\[ \pi (x) = (p_x - w_H) x, \]

where \( w_H \) is the wage rate of the skilled labor. From (6) and (7), we derive the price of intermediate good: \( p_x = \alpha A x^{\alpha - 1} L^{1-\alpha} \). Without any loss of generality, we consider that
\( L = 1 \). Therefore, the profit expression of the monopolist can be written as follows.

\[
\pi(x) = (\alpha Ax^{\alpha-1} - w_H)x
\]  

(9)

From the first order condition of profit maximization, we derive the following expression:

\[
w_H = \alpha^2 Ax^{\alpha-1}.
\]  

(10)

Substituting the value of \( w_H \) from (10) in to (9), we have the expression of monopolist profit: \( \pi(x) = \alpha(1 - \alpha)Ax^\alpha \).

### 3.1 The Research Sector

The research sector produces a stochastic set of innovations, using skilled labor as the only input. Let \( n \) be the total amount of skilled labor employed in the research sector. An innovation in the research sector follows a Poisson process and occurs with a probability of \( \lambda n \), where \( \lambda \) is a constant parameter.

Note that the rate of innovation is assumed to be dependent only on the amount of skilled labor employed in the research sector. More precisely, there is no contemporaneous spill-over in research. An innovation leads to a better technology that improves the quality of the intermediate good produced, without affecting its production function. As mentioned before, better quality of the intermediate good leads to higher productivity in the consumption goods sector by increasing the value of the parameter \( A \), by a factor \( \gamma \) which is larger than unity.

We also consider that innovations are drastic and important enough to affect the entire economy. Once an innovation occurs, earlier innovations become obsolete. In other words, innovation leads to Schumpeterian creative destruction in this model, as in Aghion and Howitt (1992).

Innovations, in our model, are discrete by definition. Nevertheless time is continuous. We denote the time interval starting from the \( i \)th innovation to the \((i + 1)\)th innovation by \( T_i \), where the subscript \( i = 0, 1, 2, \ldots \). So, if \( n_i \) amount of skilled labor is employed in the
research sector during ith interval, the \((i + 1)^{th}\) innovation occurs with probability of \(\lambda n_i\). That is, with probability \(\lambda n_i\), the value of the productivity parameter \(A\) changes from \(A_i\) to \(A_{i+1}\). Let us denote the size of innovation by \(\gamma\), where
\[
\gamma = \frac{A_{i+1}}{A_i}. \tag{11}
\]
Mathematically, the length of a time interval, \(T_i\), is a random variable. Since the underlying innovation follows a Poisson process with parameter \(\lambda n_i\), \(T_i\) is exponentially distributed with the parameter \(\lambda n_i\).

Let \(V_i\) denote the value of the \(i^{th}\) innovation, which occurs with probability \(\lambda n_{i-1}\), to a research firm. Therefore, the problem of the research firm in the \(i^{th}\) interval can be written as follows.
\[
\max_{n_i} (\lambda n_i V_{i+1} - w_{H_i} n_i),
\]
where \(w_{H_i}\) denotes the wage rate of high skilled labor in the \(i^{th}\) interval. Clearly, the above optimization problem has a corner solution. The only possible solution with a positive and realistic level of R&D satisfies the following:
\[
w_{H_i} = \lambda V_{i+1} \text{ when } n_i > 0. \tag{12}
\]

The value of the \((i + 1)^{th}\) innovation to a research firm, \(V_{i+1}\), is the expected present value of the flow of the intermediate good producing monopolist’s profits \(\pi_{i+1}\) over the interval of \(T_{i+1}\). Therefore, if the discount rate for future relative to present is \(\rho\), the flow of the \((i + 1)^{th}\) innovation to a research firm is \(\rho V_{i+1}\). In a competitive economy, this has to be same as the expected value of profit for the monopolist. The monopolist earns \(\pi_{i+1}\) each period but faces a risk of losing the monopoly value of amount \(V_{i+1}\) with probability \(\lambda n_i\).
\[
\rho V_{i+1} = \pi_{i+1} - \lambda n_{i+1} V_{i+1},
\]
\[
\text{implying } V_{i+1} = \frac{\pi_{i+1}}{\rho + \lambda n_{i+1}}. \tag{13}
\]

### 3.2 Definition of Stationary Equilibrium

We define a stationary equilibrium below:
1. There is full employment of unskilled and skilled labor.

2. The wage rate of skilled labor in the research sector must be equal to that in the intermediate goods sector

3. The economic variables – wage rates of skilled and unskilled labor ($w$ and $w_H$, respectively), skilled labor employed in the research sector ($n$) and in the intermediate goods sector ($h$) do not change after the $i^{th}$ innovation and before the $i + 1^{th}$ innovation.

4. Agents expect the current value of all parameters to remain the same.

From now on, we will solely focus on stationary equilibrium(s). We have,

$$w_{H_i} = \lambda V_{i+1} = \alpha^2 A_i x_i^{\alpha-1} \text{ [From (10) and (12)]} \quad (14)$$

$$\Rightarrow \lambda \frac{\pi_{i+1}}{\rho + \lambda n_{i+1}} = \alpha^2 A_i x_i^{\alpha-1} \text{ [By (13)]} \quad (15)$$

$$\Rightarrow \frac{\lambda}{\rho + \lambda n_{i+1}} \alpha (1 - \alpha) A_{i+1} x_{i+1}^{\alpha} = \alpha^2 A_i x_i^{\alpha-1} \quad (16)$$

Also, without any loss of generality, let us normalize the total supply of skilled labor in the economy to be one ($H = 1$). Then, we must have the following for full employment of skilled labor.

$$n_i + h_i = 1, \ i = 0, 1, 2 ... \quad (17)$$

Appealing to (8), (16) and (17), we can write

$$\frac{\lambda}{\rho + \lambda n_{i+1}} \alpha (1 - \alpha) A_{i+1} (1 - n_{i+1})^\alpha = \alpha^2 A_i (1 - n_i)^{\alpha-1}. \quad (18)$$

Note that we must have $n_i = n_{i+1} = n_{i+2} = ... = \hat{n}$, say, for the equilibrium to be stationary. Therefore, in stationary equilibrium, the stock price (i.e., expected firm value) is as follows:

$$\hat{V}_i = \frac{\pi_i}{\rho + \lambda \hat{n}} = \frac{\alpha (1 - \alpha) A_i (1 - \hat{n})^\alpha}{\rho + \lambda \hat{n}}. \quad (19)$$

And, the stationary equilibrium allocation of skilled labor to the research sector is given by

$$\frac{\lambda}{\rho + \lambda \hat{n}} \alpha (1 - \alpha) A_{i+1} (1 - \hat{n})^\alpha = \alpha^2 A_i (1 - \hat{n})^{\alpha-1}. \quad (20)$$

$$\Rightarrow \frac{\gamma \lambda^{\frac{1-\alpha}{\alpha}} (1 - \hat{n})}{\rho + \lambda \hat{n}} = 1, \text{ since } A_{i+1} = \gamma A_i \text{ by (7)}$$
Clearly, the stationary equilibrium employment in research sector is given by \( \hat{n} = \frac{\lambda \gamma}{\frac{\alpha}{1-\alpha} + \rho} \).

We assume that \( \lambda \gamma > \frac{\alpha}{1-\alpha} \rho \), for \( \hat{n} \) to be positive. Therefore, in the stationary equilibrium, the wage rates of the skilled labor and unskilled labor are, respectively,

\[
\hat{w}_L = (1 - \alpha) A_i \hat{x}^\alpha = (1 - \alpha) A_i (1 - \hat{n})^\alpha \tag{21}
\]

and

\[
\hat{w}_H = \alpha^2 A_i \hat{x}^{\alpha - 1} = \alpha^2 A_i (1 - \hat{n})^{\alpha - 1} \tag{22}
\]

Thus, the equilibrium skill premium, i.e., the wage rate of skilled labor relative to the wage rate of unskilled labor, is

\[
\hat{\omega}_i = \frac{\hat{w}_H}{\hat{w}_L} = \frac{\alpha^2}{1 - \alpha} \frac{1}{1 - \hat{n}}. \tag{23}
\]

It is easy to observe, from (20), that the equilibrium employment of skilled labor in the research sector is increasing in the size of innovation: \( \frac{\partial \hat{n}}{\partial \gamma} > 0 \). Further, for any given level of productivity of the intermediate good \( (A_i) \), higher allocation of skilled labor to the research sector leads to (a) decrease in wage rate of unskilled labor and (b) increase in wage rate of skilled labor, which in turn leads to increase in skill premium.

### 3.3 Technological Revolution

A one time technological revolution takes place in the \( t^{th} \) interval, which, going beyond the expectation of agents, increases the size of the innovation from \( \gamma_0 \) to \( \gamma_1 (> \gamma_0) \). In other words, technological revolution makes productivity of the intermediate good increase by a higher factor, \( \gamma_1 \), at the end of each of the time intervals starting from the end of the \( t^{th} \) interval. However, technological revolution can not be anticipated ex-ante. Therefore, the stock price (i.e., the expected firm value) in the \( t^{th} \) interval is as follows.

\[
\hat{V}_t = \frac{\alpha (1 - \alpha) A_i (1 - \hat{n}_0)^\alpha}{\rho + \lambda \hat{n}_0} = \frac{\alpha (1 - \alpha) A_i (1 - \hat{n}_0)^\alpha}{\gamma_0 \lambda \frac{\alpha}{1-\alpha} (1 - \hat{n}_0)} \tag{24}, \text{ from (20)}
\]

where \( \hat{n}_0 \) is the equilibrium employment of skilled labor in the research sector before technological revolution. However, the stock price (expected firm value) in the \( (t + 1^{th}) \) interval
\[ \hat{V}_{t+1} = \frac{\alpha(1-\alpha)A_{t+1}(1-n_1)^\alpha}{\rho + \lambda n_1} = \frac{\alpha(1-\alpha)A_t \gamma_1 (1-n_1)^\alpha}{\gamma_1 \lambda^{1-\alpha} (1-n_1)} = \frac{\alpha^2 A_t \lambda^{1-\alpha}}{\lambda (1-n_1)^{1-\alpha}}, \tag{25} \]

where \( n_1 \) denotes the equilibrium employment of skilled labor in the post revolution period.

We have (a) \( \hat{n}_1 > \hat{n}_0 \), since \( \frac{\partial n}{\partial \gamma} > 0 \); and (b) \( \gamma_0 > 1 \). Therefore, comparison of (24) and (25) yields \( \hat{V}_{t+1} > \hat{V}_t \). This signifies that technological revolution leads to increase in stock prices. Also, from (23) it is easy to observe that skill premium also increases due to technological revolution: \( \hat{\omega}_{t+1} = \frac{\alpha^2}{1-\alpha} \frac{1}{1-n_1} > \frac{\alpha^2}{1-\alpha} \frac{1}{1-n_0} = \hat{\omega}_t \). We formulate our findings in the proposition below.

**Proposition 1.** Technological revolution leads to rise in both stock prices and skill premium.

The underlying intuition behind the above result is as follows. Technological revolution increases the demand of skilled labor, which leads to increase in skill premium. Also, since technological revolution increases the size of the innovation, the value of firms using new technology and skilled labor to produce better quality products increases in spite of increase in relative wage of skilled labor.

## 4 Quantitative Analysis

We also performed a quantitative investigation into the explanatory power of our economic environment. Calibration of the parameters of our economic model was done for that purpose. There are five parameters in our model: \( \alpha, \rho, \gamma_0, \gamma_1, \) and \( \lambda \). The initial value of \( A \) is normalized to unity without any loss of generality. \( \rho \) is the discount factor for a firm. Given the risk neutrality of firms, \( \rho \) is equated to the real interest rate. Mehra and Prescott (1985) calculated the average real return on the Standard and Poor’s 500 Composite Stock Index over 1889–1978 to arrive at a figure of 6.98% annually. The parameters of \( \gamma_0 \) and \( \gamma_1 \) depend on historical measures of growth in total income. We considered \( \gamma_0 \) and \( \gamma_1 \) as...
the pre- and post- technological revolution growth rates for the U.S. economy. The timing of technological revolution was maintained at 1984, at par with our empirical findings. We used Maddison (2003) to compute the growth rate of per capita U.S. Gross Domestic Product from 1871 to 1983 without considering the World War II years. This figure is 1.52% annually. The corresponding figure stood at 2.14% during 1984–2007. In our calibration of $\gamma_0$ and $\gamma_1$, we did not use the Solow Residual for two reasons. First, historical data does not extend beyond 1950s for calculating the Solow residual. Second, there is no difference between growth in consumption good and growth in total factor productivity in our model. Therefore, use of growth rate for GDP goes in tandem with the definition of $\gamma_0$ and $\gamma_1$.

The other two parameters of our interest are: $\lambda$ and $\alpha$. $\lambda$ determines the rate of innovation in our economic environment and is related to the frequency of innovation in the economy. $\alpha$ is a crucial parameter in determination of skill premium. We undertook an indirect approach to calibrate these two parameters. A particular value of $\lambda$ in model generates a value of $\hat{n}$, proportion of skilled labor employed in the R&D sector. Figure 4 portrays this relationship. We used the ratio of R&D expenditure to GDP as a proxy for the proportion of skilled employees in the R & D sector. In 2009, this figure had been at 2.90% which we used as our post technological revolution value of $\hat{n}$. In our economic environment, the value of skill premium depends on $\alpha$. Figure 5) illustrates this relationship. As mentioned before, we used the ratio of average earning at the ninth decile to that in the first decile for calculating the value of skill premium. This figure stood at 4.565 in the post technological revolution era, 1984–2009. Given the values of $\rho$, $\gamma_0$ and $\gamma_1$, we matched the prediction of the model regarding proportion of skilled employees in the R & D sector and the magnitude of skill premium to the calculated figures. This enabled us to find a unique set of values for the parameters of $\lambda$ and $\alpha$, estimated at 0.439 and 0.841 respectively. The calibrated values of all parameters are enlisted in Table 8.

A quantitative assessment of our model involves matching predictions of our model from external sources. The probability of innovation in a year is given by $\lambda \cdot \hat{n}$. Obviously, life expectancy of an innovation is the reciprocal of this quantity which was estimated at
76.736 years.\textsuperscript{2} de Geus (1997) stated that the average life expectancy of a multinational corporation –Fortune 500 or its equivalent– is between 40 and 50 years. However, this figure increases with firm-size. Data reveals that the median age of firms with more than ten thousand employees in 2008 was about 75 years (Luttmer, 2011).

What about implications regarding dynamics of R&D sector, skill premium and share price? In the U.S. R&D performance has increased steadily over the years. Over the last 5 years (2004–09), annual growth in U.S. R&D spending averaged 5.8\%, compared to annual average growth of 3.3\% for U.S. gross domestic product (NCSES, 2012). Our model predicts an increase in number of employee ($\hat{n}$) by 3.57\%, annually. For skill premium and share price, we consider a five-year window before and after technological revolution.\textsuperscript{3} In those time periods, 1979–1983 and 1984–1988, we calculated average levels of skill premium and share price. From this calculation, the annualized rate of growth came as 1.71\% and 3.33\%\textsuperscript{4} respectively. The model predictions are relatively moderate levels of 0.10\% and 1.54\%.

The power of our model hinges upon the strength of R&D sector in bringing about a simultaneous rise in skill premium and share prices. The scale of operation for this sector is rather low as found in our calibration. Nevertheless, It can explain an important part of the change.

\textsuperscript{2}This is for post technological revolution era. For pre technological revolution, the corresponding figure is 81.395 years.

\textsuperscript{3}If we had chosen only the change in skill premium and share price in 1984, it would have contained a lot of noise. A wider time span mitigates that noise. However, use of far wider time span is detrimental to the spirit of a simple model to isolate the effect of the size of innovation by technological revolution. We assigned an arbitrary window of five years as a trade-off to both factors.

\textsuperscript{4}For share price, we calculated the change in value in excess of interest rate and inflation rate (averaged 2.04\%).

\[\]
5 Conclusion

Both share price and skill premium are related to inequality in society. The relation between skill premium and inequality is quite straight-forward. The limited nature of stock market participation and the concentration of stock wealth even among stockholders is well documented. For example, until the 1990s more than two-thirds of U.S. households did not own any stocks at all, while the richest 1% held 48% of all stocks (Poterba and Samwick (1995)). Therefore, a rise in share prices implies an augment in notions of inequality. We have empirically demonstrated a long run relationship between these two variables not only in the U.S. economy but also in five other OECD countries since 1984, which signifies the co-movement of these two variables.

A natural question poses, what makes these two variables move together? Introducing technological revolution in Aghion and Howitt (1992)'s three sector model of endogenous growth through creative destruction, this paper demonstrates that technological revolution can lead to rise in both skill premium and stock prices at the same time. It, thus, offers new insights to understand the phenomenon of co-movement of stock prices and skill premium. In essence, the rise in inequality can be traced to technological revolution or growth in innovation. The quantitative analysis of our economic environment shows that an important part of empirical change can be explained by our simple model with few parameters.

References


Figure 1: For the United States, the ratio of decile 9 to decile 1 earnings (skill premium) and share prices are plotted for the years 1960–2011 based on available data.
Figure 2: For Japan, the ratio of decile 9 to decile 1 earnings (skill premium) and share prices are plotted for the years 1960–2011 based on available data.
Figure 3: For six OECD countries under consideration, the ratio of decile 9 to decile 1 earnings (Skill premium) and Share prices are plotted for the years 1960–2011 subject to availability of data.
Figure 4: As $\lambda$ changes, the proportion of skilled labour (post technological revolution) in the R&D sector varies. Values for other parameters are fixed as per Table 8.
Figure 5: Variation in skill premium depending on the value of $\alpha$ is illustrated. Values for other parameters are fixed as per Table 8.
Table 1: Engle Granger Test for the US Data

First Step: Augmented Dicky Fuller Test for Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \tau )-Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share Price</td>
<td>-0.211</td>
<td>0.935</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>0.136</td>
<td>0.969</td>
</tr>
</tbody>
</table>

Second Step: Estimation of an Ordinary Least Squares Regression

Dependent Variable: Share Price

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-466.013</td>
<td>61.381</td>
<td>-7.592</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>116.027</td>
<td>13.4763</td>
<td>8.610</td>
</tr>
</tbody>
</table>

Third Step: Augmented Dicky Fuller Test for Estimated Residuals

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \tau )-Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residuals</td>
<td>-3.482</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Note: The data spans the years of 1984–2010 for the United States. ADF Tests consider presence of a unit root as the null hypothesis. All unit root tests are performed with a constant in the test equation. We use a maximum number of 4 lag orders in performing ADF tests.
### Table 2: Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test for the US Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test Statistic</th>
<th>1% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share price</td>
<td>0.962</td>
<td>0.708</td>
</tr>
<tr>
<td>Skill premium</td>
<td>0.957</td>
<td>0.708</td>
</tr>
</tbody>
</table>

Note: The data spans the years of 1984–2010 for the United States. The null hypothesis of KPSS tests considers absence of unit root. Tests are performed with a constant in the test equation and with 2 lags. We reject null hypothesis at some critical level if the test statistic is larger than the critical value of that level.

### Table 3: Johansen Test for the US Data

<table>
<thead>
<tr>
<th>Rank</th>
<th>Eigenvalue</th>
<th>Trace test</th>
<th>λ − max test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Statistics</td>
<td>p-Value</td>
</tr>
<tr>
<td>0</td>
<td>0.504</td>
<td>23.959</td>
<td>0.013</td>
</tr>
<tr>
<td>1</td>
<td>0.169</td>
<td>5.006</td>
<td>0.293</td>
</tr>
</tbody>
</table>

Note: The data spans the years of 1984–2010 for the United States. We have accounted for the case of restricted constant. The trace test examines the null hypothesis of Rank ≤ r against the alternative of Rank > r for r = 0, 1. The λ − max test investigates into the validity of Rank = r against the alternative of Rank = r + 1 for r = 0, 1.
Table 4: Panel Unit Root Tests

<table>
<thead>
<tr>
<th>Variables</th>
<th>Without time trend</th>
<th>With time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Share Price</td>
<td>Skill premium</td>
</tr>
<tr>
<td>Tests</td>
<td>Im et al. (2003)</td>
<td>0.217 0.586 0.897 0.815</td>
</tr>
<tr>
<td></td>
<td>Levin et al. (2002)</td>
<td>-0.161 0.436 -0.616 0.269</td>
</tr>
<tr>
<td></td>
<td>Harris and Tzavalis (1999)</td>
<td>0.869 0.496 0.918 0.809</td>
</tr>
<tr>
<td></td>
<td>Maddala and Wu (1999)</td>
<td>10.241 0.595 7.072 0.853</td>
</tr>
<tr>
<td></td>
<td>Breitung (2000)</td>
<td>0.171 0.568 0.538 0.705</td>
</tr>
<tr>
<td></td>
<td>Pesaran (2007)</td>
<td>-0.670 0.252 1.466 0.929</td>
</tr>
</tbody>
</table>

Note: The data spans the years of 1984–2005 for the countries of France, Japan, Korea, Netherlands, United Kingdom, and United States. Tests consider presence of a unit root as the null hypothesis. P-values are for a one-sided test. All unit root tests are performed with a constant in the test equation. For semi-parametric corrections, the Bartlett Kernel is employed with Newey and West (1994) bandwidth selection algorithm. All bandwidth and lag orders are set according to the rule $4(T/100)^{2/9}$. The lags are chosen according to Akaike criterion.
Table 5: Panel Cointegration Tests

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>Value</th>
<th>z-Value</th>
<th>Asymptotic p-value</th>
<th>Robust p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\tau}$</td>
<td>-3.255</td>
<td>-4.029</td>
<td>0.000</td>
<td>0.040</td>
</tr>
<tr>
<td>$G_{\alpha}$</td>
<td>-15.364</td>
<td>-3.690</td>
<td>0.000</td>
<td>0.024</td>
</tr>
<tr>
<td>$P_{\tau}$</td>
<td>-7.236</td>
<td>-3.577</td>
<td>0.000</td>
<td>0.038</td>
</tr>
<tr>
<td>$P_{\alpha}$</td>
<td>-13.588</td>
<td>-4.929</td>
<td>0.000</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Note: We perform Westerlund (2007) tests for cointegration with the data for the countries of France, Japan, Korea, Netherlands, United Kingdom, and United States during 1984–2005. Tests consider absence of a cointegrating relation as the null hypothesis. The test regression considers share price as the dependent variable and skill premium as the independent variable. Additionally, the test regression is fitted with a constant, one lag and one lead. The kernel bandwidth is fixed according to the rule of $4(T/100)^{2/9}$. The asymptotic p-values and the robust p-values are calculated for a one-sided test. The asymptotic p-values are based on the normal distribution, whereas the robust p-values are based on 500 bootstrap replications.
Table 6: Weak Exogeneity Tests

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>Value</th>
<th>z-Value</th>
<th>Asymptotic p-value</th>
<th>Robust p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_\tau$</td>
<td>-1.430</td>
<td>0.999</td>
<td>0.841</td>
<td>0.804</td>
</tr>
<tr>
<td>$G_\alpha$</td>
<td>-6.361</td>
<td>0.380</td>
<td>0.648</td>
<td>0.702</td>
</tr>
<tr>
<td>$P_\tau$</td>
<td>-2.059</td>
<td>1.533</td>
<td>0.937</td>
<td>0.808</td>
</tr>
<tr>
<td>$P_\alpha$</td>
<td>-3.338</td>
<td>0.543</td>
<td>0.707</td>
<td>0.740</td>
</tr>
</tbody>
</table>

Note: Weak exogeneity tests are accompanied with Westerlund (2007) tests for cointegration, presented in Table 5. Tests consider absence of a cointegrating relation as the null hypothesis. Conclusions derived from Westerlund (2007) tests for cointegration are valid only if weak exogeneity is established by acceptance of null hypothesis in these tests. The test regression considers skill premium as the dependent variable and share price as the independent variable. Additionally, the test regression is fitted with a constant, one lag and one lead. The kernel bandwidth is fixed according to the rule of $4(T/100)^{2/9}$. The asymptotic p-values and the robust p-values are calculated for a one-sided test. The asymptotic p-values are based on the normal distribution, whereas the robust p-values are based on 500 bootstrap replications.

Table 7: Cross Sectional Independence Tests

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pesaran (2004) Cross-section Dependence (CD) Test</td>
<td>6.516</td>
<td>0.000</td>
</tr>
<tr>
<td>Breusch-Pagan Lagrange Multiplier (LM) Test</td>
<td>77.094</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: Cross sectional independence tests are accompanied with Westerlund (2007) tests for cointegration, presented in Table 5. Both tests consider cross-sectional independence of residuals in the cointegrating relationship as the null hypothesis.
Table 8: Parameters Calibrated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Method of Calibration</th>
<th>Value used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Annual Real Interest Rate</td>
<td>6.98%</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Pre-1984 growth rate of per capita GDP</td>
<td>1.52%</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Growth rate of per capita GDP (1974–2008)</td>
<td>2.14%</td>
</tr>
</tbody>
</table>

Matching model moments to statistics

| $\alpha$  | Value of skill premium is set to 4.565      | 0.841      |
| $\lambda$ | Ratio of R&D Expenditure to GDP is set to 2.9%| 0.439      |

Table 9: Quantitative Assessment of the Model

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model Prediction</th>
<th>Empirical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm’s life expectancy</td>
<td>77 years</td>
<td>40–50 Years/ 75 Years (Large firms)</td>
</tr>
<tr>
<td>Annualized Growth of R&amp;D sector</td>
<td>3.57%</td>
<td>5.80%</td>
</tr>
<tr>
<td>Annualized Growth of Skill Premium</td>
<td>0.10%</td>
<td>1.71%</td>
</tr>
<tr>
<td>Annualized Growth of Share Prices</td>
<td>1.54%</td>
<td>3.33%</td>
</tr>
</tbody>
</table>