Credit constraints, productivity shocks and consumption volatility in emerging economies

Rudrani Bhattacharya Ila Patnaik∗

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Abstract

How does access to credit impact consumption volatility? Theory and evidence from advanced economies suggests that greater household access to finance smooths consumption. Evidence from emerging markets, where consumption is usually more volatile than income, indicates that financial reform further increases the volatility of consumption relative to output. We address this puzzle in the framework of an emerging economy model in which households face shocks to trend growth rate, and a fraction of them are credit constrained. Unconstrained households can respond to shocks to trend growth, and financial reform increases the share of such households. The interaction creates a novel impact. Easing of financial constraints allows more households to smooth consumption in response to shocks to trend growth in productivity. In this setting financial reform leads to greater relative consumption volatility. Calibration of the model for pre and post financial reform in India provides support for the model’s key predictions.

JEL Classification: E10, E32
Keywords: Macroeconomics, Real Business Cycles, Emerging Market business cycle stylized facts, financial development.

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1 Introduction

Emerging economies have been seen to witness an increase in consumption volatility relative to output volatility after financial development. This behaviour appears puzzling since traditional models and evidence from advanced economies suggest that consumption should become smoother after credit constraints are reduced. This puzzle can be explained in a model featuring credit constraints and shocks to trend growth of productivity. The model predicts that relative consumption volatility rises when more consumers can smooth consumption.

The presence of credit constraints in an economy explains the excess volatility of consumption and its sensitivity to anticipated income fluctuations. A model featuring credit constrained consumers predicts that consumption cannot be smoothed fully. But in such a model, the volatility of consumption can be at least as high as income volatility, or at most one. Further, if constraints are eased the model predicts a reduction in relative consumption volatility.

Another feature of emerging economy models is the presence of shocks to trend growth of productivity. When households anticipate a higher growth rate of income which eventually leads to a rise in future income, they respond to this permanent income shock by increasing current consumption more than the rise in current income via borrowing against the future income or reducing current savings. As a result, consumption fluctuates more than income in emerging economies. This feature results in the relative volatility of consumption in emerging economies becoming greater than one.

A common feature of reform in emerging economies is financial sector reform. The increase in the access of households to finance resulting from reform allows households to smooth consumption over their lifetimes. But at the same time, emerging economies witness shocks to trend growth. The combination of the response of households to shocks to trend growth and the easing of credit constraints can yield an increase in the relative volatility of consumption.

The goal of this paper is to understand the joint impact of easing of liquidity constraints and shocks to trend growth on consumption volatility. We present a model in which some households do not have access to finance. They can neither save nor borrow. These credit constrained households cannot smooth consumption over their lifetimes. The rest of the households in the economy are unconstrained and respond to a perceived income shock by smoothing consumption. Shocks to income that are perceived to be permanent lead to
an increase in current period consumption higher than the increase in current period income. Only unconstrained households can smooth consumption. Constrained households can only increase consumption by the amount income has increased. Financial sector reform gives more households access to finance. Now more households become unconstrained and can respond to the income shock that they perceive to be permanent. The key prediction of this model is that financial development in an emerging economy leads to an increase in relative consumption volatility.

This prediction can be tested. We calibrate the model to Indian data for pre and post reform years, where we keep all other parameters constant and only change the share of credit constrained consumers. We find support for our model’s key prediction.

Our paper makes a contribution towards understanding the joint impact of financial development and shocks to trend growth on consumption volatility. It contributes to a growing literature that studies the effects of financial frictions on volatility. Earlier work mainly analyses the effect of domestic financial system development on output and consumption volatility through its effect on firms (Aghion et al., 2003, 2010). Some papers focus on the impact of financial globalisation on volatility (Aghion et al., 2003; Buch et al., 2005; Leblebicioglu, 2009). The effect of domestic financial system development on output and consumption volatility is explored in a limited strand of literature. Iyigun and Owen (2004) propose a theory of income inequality in rich and poor countries as the cause of consumption volatility whose mechanics partly resemble those of our theory, once appropriately re-interpreted.

Our model takes into account the broadly acknowledged fact that in emerging economies all consumers do not have access to finance (Honohan, 2006). The framework includes shocks to trend growth as in Aguiar and Gopinath (2007). Liquidity constrained households are modelled as in Hayashi (1982); Campbell and Mankiw (1991).

The rest of the paper is organised as follows. Section 2 presents evidence on relative consumption volatility and financial development in emerging economies. Section 3 presents the model and its predictions. Section 4 contains the calibration exercise. Section 5 concludes.
<table>
<thead>
<tr>
<th>Region &amp; reform date</th>
<th>Pre-reform</th>
<th>Post-reform</th>
<th>Change</th>
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<td><strong>Std. dev.</strong></td>
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<td>0.30</td>
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Source: Datastream, Author’s calculations
2 Consumption volatility and financial development

Recent empirical evidence on emerging economy business cycles shows an increase in the volatility of consumption relative to that of output after financial sector reform in Asia, Turkey and in India (Kim et al., 2003; Alp et al., 2012; Ghate et al., 2011).

We measure relative volatility of consumption in the pre and post financial sector reform period for a number of developing countries. The choice of the date on which reform took place is based on Kim et al. (2003); Singh et al. (2005); Rodrik (2008); Alp et al. (2012); Aslund (2012). The analysis is based on annual data for a set of emerging economies. The span of the analysis varies across countries given the availability of the data. Table 10 in Appendix I lists period of analysis for each country.

Table 1 shows the reform date and the volatility of consumption relative to that of output in the pre and post reform period. It shows that many emerging economies exhibit similar behaviour in that relative consumption volatility increases after reform.

Financial development has been a major component of reform. Table 2 shows the density of commercial bank branches and depositors with commercial banks in the beginning and in the end of the last decade in emerging economies. It indicates an increase in access of households to finance.

Figure 1 shows the trend in a commonly used indicator of financial development, namely, total bank deposits to GDP for a set of emerging economies. It shows a rise in the average bank deposits to GDP ratio for the set of emerging economies. The rising trend in the ratio is also visible for individual countries (Figure 1).

The above evidence suggests that the relative volatility of consumption rises after financial sector reform. This appears puzzling and cannot be explained by the existing literature. It supports the evidence in Kim et al. (2003); Alp et al. (2012); Ghate et al. (2011) who allude to the increase in relative consumption volatility after financial sector reform.

1The set of emerging economies consists of Chile, Columbia, Mexico, Peru, Indonesia, Malaysia, Philippines, Korea, Taiwan, Thailand, Turkey, Poland, Hungary, India, South Africa.
Table 2 Access to finance

<table>
<thead>
<tr>
<th>Country</th>
<th>Commercial bank branches per 100,000 adults</th>
<th>Depositors with commercial banks per 1,000 adults</th>
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<tbody>
<tr>
<td>Chile</td>
<td>13</td>
<td>18</td>
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<tr>
<td>Colombia</td>
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<td>Mexico</td>
<td>11</td>
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<td>4</td>
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<td>10</td>
<td>11</td>
</tr>
<tr>
<td>South Africa</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Source: Financial Inclusion, World development indicators

Figure 1 Financial development

This figure shows the average deposits to GDP ratio of a set of emerging economies and a few individual countries in the set.

Source: International Financial Statistics, IMF.
3 Credit constraints and consumption volatility: Theoretical framework

The theoretical literature on finance and macroeconomic volatility explores how financial integration and financial development affect output and consumption volatility through the channel of firms and households (Bernanke and Gertler, 1989; Greenwald and Stiglitz, 1993; Aghion et al., 2003; Iyigun and Owen, 2004; Buch et al., 2005; Leblebicioglu, 2009; Aghion et al., 2010).

The effect of financial integration on macroeconomic volatility dominates the literature. A limited strand of literature explores the role of domestic financial development in determining the pattern of macroeconomic fluctuations and the bulk of it focus on the channel of firms (Bernanke and Gertler, 1989; Greenwald and Stiglitz, 1993; Aghion et al., 2010).

The early literature predicts that financial development reduces macroeconomic fluctuations (Bernanke and Gertler, 1989; Greenwald and Stiglitz, 1993). More recent literature suggests that the nature of relationship between financial development and macroeconomic volatility can be non-linear (Aghion et al., 2003) and may depend on several factors, such as the composition of short-term and long-term investments in the economy (Aghion et al., 2010).

3.1 The model

Consider a closed economy populated by a continuum of infinitely lived households and firms, both of measure unity. There exist a fraction \( \lambda \) of households with no access to banking or other instruments to save. These consumers, who may be referred to as non-Ricardian households, are liquidity-constrained and unable to save or borrow to smooth consumption. They have no assets and spend all their current disposable labour income on consumption in each period.

Labour supply is inelastic as no labour-leisure choice is made by the representative household. In an emerging economy it is reasonable to assume that households allocate their available labour-time to production as much as possible. Hence, the representative household supplies one unit of labour inelastically.

Both Ricardian and liquidity-constrained households have identical prefer-
ences defined over a single commodity,
\[ U(C_t^i) = \ln(C_t^i) \quad i = R, L \]  
(1)
where \( C_t^i \) denotes total consumption of the household of type \( i \). Ricardian households are indexed as \( R \) and liquidity constrained households as \( L \).

A Ricardian household maximises discounted stream of utility
\[ V_t = E_t \sum_{t=0}^{\infty} \beta^t \log(C_t^R), \]  
(2)
subject to the following budget constraint,
\[ C_t^R + I_t^R = R_t K_t^R + W_t, \]  
(3)
where \( \beta \in (0,1) \) denotes the subjective discount factor. Here \( C_t^R \) is total consumption of the Ricardian household in period \( t \). The variables \( I_t^R \) and \( K_t^R \) denote investment and capital stock of the household respectively. The economy-wide return to capital and wage rate are given by \( R_t \) and \( W_t \). In each period the Ricardian household divides her disposable income comprised of wage and rental income into consumption and savings.

The stock of capital of the representative Ricardian household evolves via the following law of motion,
\[ K_{t+1}^R = (1 - \delta)K_t^R + I_t^R - \frac{\phi}{2} \left( \frac{K_{t+1}^R}{K_t^R} - \mu_3 \right)^2 K_t^R \]  
(4)
The investment is subject to quadratic capital adjustment cost as in [Aguiar and Gopinath (2007)](https://www.nber.org/papers/w13338).

Households who do not have access to financial services cannot save or borrow. Their behaviour is thus different from that of Ricardian consumers. Liquidity constrained households maximise instantaneous utility \( \log C_t^L \) subject to the following budget constraint in each period,
\[ C_t^L = W_t, \]  
(5)
where \( C_t^L \) is total consumption of the liquidity constrained household in period \( t \). In each period, a liquidity constrained household consumes its entire disposable income comprised of wage income.
The aggregate consumption is the weighted average of consumption by the liquidity-constrained households and the Ricardian households. The weights are the share of each type of households in the population.

\[ C_t = \lambda C^L_t + (1 - \lambda) C^R_t. \] (6)

The aggregate capital stock and investment are respectively the following

\[ K_t = (1 - \lambda) K^R_t, \quad I_t = (1 - \lambda) I^R_t, \] (7)

A representative firm produces a homogeneous good, by hiring one unit of labour from households and combining it with capital. The aggregate output is produced by Cobb Douglas technology that uses capital and unit labour as inputs,

\[ Y_t = \epsilon^{a \alpha \mu} [(1 - \lambda) K_t]^{1 - \alpha} \Gamma^a_t, \] (8)

where \( \alpha \in (0, 1) \) represents labour's share of output and \( \epsilon^{a \alpha} \) denotes the level of total factor productivity. Here \( \Gamma_t \) is the labour productivity. The two productivity processes are characterised by the following stochastic properties: total factor productivity evolves according to an AR(1) process as follows:

\[ a_t = \rho_a a_{t-1} + \epsilon^a_t \] (9)

with \( |\rho_a| < 1 \) and \( \epsilon^a_t \) represents iid draws from a normal distribution with zero mean and standard deviation \( \sigma_a^2 \).

Following (Aguiar and Gopinath, 2007), the growth rate of labour productivity \( \Gamma_t \) is defined as

\[ \Gamma_t = g_t \Gamma_{t-1} \] (10)

The growth rate of labour productivity \( g_t \) follows an AR(1) process of the form:

\[ \ln \left( \frac{g_t}{\mu_g} \right) = \rho_g \ln \left( \frac{g_{t-1}}{\mu_g} \right) + \epsilon^g_t; \quad \epsilon^g_t \sim N(0, \sigma^2_g) \] (11)

The resource constraint of the economy is given by

\[ C_t + I_t = Y_t \] (12)

In a closed economy, total output is allocated between total consumption and investment as indicated by equation (12).
Since the realisation of $g$ permanently influences $\Gamma$, output is non-stationary with a stochastic trend. We detrend output, consumption, investment and capital stock by normalising these variables with respect to the trend productivity through period $t-1$. For any variable $X$, its detrended counterpart is defined as $x_t = \frac{X_t}{\Gamma_{t-1}}$.

With the initial capital stock $K_0$, the competitive equilibrium is defined as a set of prices and quantities $(R_t, W_t, y_t, c_t, c_t^R, c_t^L, i_t, k_t)$, given the sequence of shocks to TFP and labour productivity growth, that solves the maximisation problem of the household, optimisation by the firms and satisfies the resource constraint of the economy.

### 3.2 Predictions

After normalisation of the variables by labour productivity in the previous period, the system of equations driving the dynamics of the model economy become

\begin{align*}
1 &= \beta E_{t-1} \left[ \Omega_t \frac{c_{t-1}^R}{c_t^R} g_t \right] \\
\Omega_t &= (1 - \alpha) e^{a_t} (1 - \lambda)^{1-\alpha} g_t^\alpha + (1 - \delta), \\
c_t^R &= \frac{(1 - \alpha \lambda)}{1 - \lambda} e^{a_t} [(1 - \lambda) k_t^R]^{-\alpha} g_t^\alpha + (1 - \delta) k_t^R \\
&\quad - g_t k_{t+1}^R - (\phi/2) \left( \frac{k_{t+1}^R g_t}{\kappa_t} - \mu_g \right)^2 k_t^R, \\
a_t &= \rho_a a_{t-1} + \epsilon_{t}^a, \\
\ln \left( \frac{g_t}{\mu_g} \right) &= \rho_g \ln \left( \frac{g_{t-1}}{\mu_g} \right) + \epsilon_{t}^g.
\end{align*}

The first equation in the system of equations describes intertemporal allocation of consumption by the Ricardian consumers where $\Omega_t$ is the gross return to capital. The third equation pertains to the resource constraint of the economy, after taking into account the consumption of liquidity-constrained households as in equation \(5\), total consumption, stock of capital and investment in the economy given in equations \(6\), \(7\) and making use of the fact that $W_t = \alpha e^{a_t} [(1 - \lambda) k_t^R]^{1-\alpha} g_t^\alpha$.

Log-linearisation of the above system of equations around the steady state
yields,

\[
\begin{align*}
\tilde{c}_t^R &= \left(1 - \lambda \alpha \right) \frac{y^*}{c^{Rs}} + \left[ \left(1 - \lambda \alpha \right) \frac{\mu_g k^s}{\beta c^{Rs}} - \frac{\lambda(1 - \alpha)(1 - \delta)}{1 - \lambda} \frac{k^s}{c^{Rs}} \right] \tilde{k}^R_t \\
\mu_g k^{Rs} \tilde{k}^R_{t+1} - \mu_g k^{Rs} \tilde{k}^R_t - \mu_g k^{Rs} \tilde{g}_t,
\end{align*}
\]

\[
0 = E_{t-1} \left[ \tilde{c}_{t+1}^R - \tilde{c}_t^R + A(a_t - \alpha \tilde{k}_t^R + \alpha \tilde{g}_t) \right]; \quad A = 1 - \frac{\beta(1 - \delta)}{\mu_g},
\]

\[
a_t = \rho_a a_{t-1} + \epsilon^a_t,
\]

\[
\tilde{g}_t = \rho_g \tilde{g}_{t-1} + \epsilon^g_t; \quad \tilde{g}_t = \ln \left( \frac{g_t}{\mu_g} \right),
\]

(14)

where the cyclical component of a variable \(x_t\) is defined as \(\tilde{x}_t = \ln x_t - \ln x^*\) and \(x^*\) denotes the steady state value of \(x_t\). The steady state growth rate of labour productivity is the long term average trend growth rate \(\mu_g\).

The solution of the system of equations (14) takes the form

\[
\begin{align*}
\tilde{k}^R_{t+1} &= a_1 \tilde{k}^R_t + b_1 a_t + d_1 \tilde{g}_t \\
\tilde{c}_t^R &= a_2 \tilde{k}_t^R + b_2 a_t + d_2 \tilde{g}_t.
\end{align*}
\]

(15)

The unknown parameters \((a_1, b_1, d_1, a_2, b_2, d_2)\) are functions of \((\beta, \alpha, \delta, \lambda, \mu_g)\) and are solved using the method of undetermined coefficients. The solution (15) of the dynamic system indicates that each endogenous variable in time \(t\) is a linear function of the state variables \((\tilde{k}^R_t, a_t, g_t)\). For the system to satisfy transversality condition, i.e., convergence of the system to the steady state over time, \(\tilde{k}^R_t\) must converge to zero following a shock. That is, \(a_1\) must satisfy the condition \(a_1 < 1\).

Given the total consumption of the economy as in equation (6), and making use of the equation (5) and the fact that \(W_t = \alpha Y_t\) implying \(\tilde{c}_t^R = \tilde{y}_t\), one can arrive at the volatility of consumption relative to output as,

\[
\frac{\sigma_{\tilde{c}}^2}{\sigma_{\tilde{y}}^2} = \left( \frac{c^{Rs}}{c^*} \right)^2 \left( 1 - \lambda \right)^2 \frac{\sigma_{\tilde{c}}^2}{\sigma_{\tilde{y}}^2} + \left( \frac{c^{L*}}{c^*} \right)^2 \lambda^2.
\]

(16)

Here the fluctuations in a Ricardian household’s consumption and total out-

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2We use the approximation \(x_t = e^{\tilde{x}_t} x^* \approx (1 + \tilde{x}_t) x^* \) to log-linearise the equations of the model.

3Appendix II describes the solution method in details.
put

\[ \sigma^2_{cR} = \left[ \frac{a^2b_l^2}{1-a_1^2} + b_2^2 \right] \sigma_a^2 + \left[ \frac{a^2d_1^2}{1-a_1^2} + d_2^2 \right] \sigma_g^2, \]

\[ \sigma^2_y = \left[ 1 + \frac{(1-\alpha)b_2^2}{1-a_1^2} \right] \sigma_a^2 + \left[ \alpha^2 + \frac{(1-\alpha)d_2^2}{1-a_1^2} \right] \sigma_g^2, \]

are derived from the solution system (15) and the log-linearised expression of output in equation (8). The last two equations in the system of equations (14) yield volatility of transitory and permanent income shocks as \( \sigma^2_a = \frac{\sigma^2_{\epsilon a}}{1-\rho_a^2} \) and \( \sigma^2_g = \frac{\sigma^2_{\epsilon g}}{1-\rho_g^2} \). The effects of transitory and permanent income shocks on the volatility of consumption in the economy relative to volatility of output can be summarised as follows.

**Proposition 1** With everything else remaining unchanged,

(i) Volatility of consumption of a liquidity constrained household relative to output volatility is always unity, i.e., \( \frac{\sigma_{cL}}{\sigma_y} = 1 \), when \( \sigma_{\epsilon a} > 0; \sigma_{\epsilon g} > 0 \).

(ii) Due to a transitory shock in income, both volatility of consumption of a Ricardian household relative to output volatility and the volatility of total consumption relative to output volatility are lower than 1, irrespective of the share of liquidity constrained households in the population, i.e., \( \frac{\sigma_{cR}}{\sigma_y} < 1 \) and \( \frac{\sigma_c}{\sigma_y} < 1 \) for \( \lambda \in [0,1) \), when \( \sigma_{\epsilon a} > 0; \sigma_{\epsilon g} = 0 \).

(iii) Due to a shock to the trend growth of income, volatility of consumption of a Ricardian household relative to volatility of output always exceeds 1, irrespective of the share of liquidity constrained households in the economy, while the volatility of total consumption relative to output volatility depends on the share of liquidity constrained households in the economy, i.e., \( \frac{\sigma_{cR}}{\sigma_y} > 1 \), and \( \frac{\sigma_c}{\sigma_y} \gtrless 1 \), for \( \lambda \in [0,1) \), when \( \sigma_{\epsilon a} = 0; \sigma_{\epsilon g} > 0 \).

(iv) In the presence of shock to the trend growth rate, both volatility of consumption of a Ricardian household relative to output volatility and the volatility of total consumption relative to volatility of output increases when the share of liquidity-constrained households in the economy decreases, i.e., \( \frac{\partial}{\partial \lambda} \left( \frac{\sigma_{cR}}{\sigma_y} \right) < 0 \), and \( \frac{\partial}{\partial \lambda} \left( \frac{\sigma_c}{\sigma_y} \right) < 0 \), for \( \lambda \in [0,1) \), when \( \sigma_{\epsilon a} = 0; \sigma_{\epsilon g} > 0 \).

The proof of the Proposition 1 is presented in the Appendix II in detail.

Liquidity constrained households who have no access to savings instruments can respond to any change in income by changing consumption by the amount of changed income. Hence volatility of consumption of a liquidity constrained household relative to output volatility is always one irrespective of the nature of shock.
In response to a transitory income shock, a Ricardian household smooths consumption by re-allocating changed income between consumption and savings. Hence consumption fluctuates by a lesser amount compared to income fluctuation. Hence consumption volatility of a Ricardian household relative to output volatility is always less than one irrespective of the level of financial development. Relative volatility of total consumption when total consumption is a weighted average of the relative consumption volatility of a Ricardian household and that of a liquidity constrained household is also less than one in all states of financial development.

Ricardian households perceive a rise in income in the future following a permanent income shock. They respond to it by raising current consumption more than the rise in current income by borrowing against future income or reducing current savings. Thus relative volatility of consumption of a Ricardian household with respect to output volatility is greater than one. Relative volatility of total consumption when total consumption is a weighted average of the relative consumption volatility of a Ricardian household and that of a liquidity constrained household may be smaller or higher than 1 depending on the size of $\lambda$.

Financial development reduces the share of liquidity constrained households in the economy and hence allows more people to respond to the permanent income shock by raising current consumption more than the rise in current income. As a result, volatility of total consumption relative to output volatility increases with financial development.

Combining these observations, the main theoretical prediction of our model can be stated as follows.

**Main prediction:** Other things unchanged, under the occurrence of permanent income shock, financial development leads to a rise in the volatility of consumption in the economy relative to output volatility.

We test this prediction by calibrating the model economy to Indian data. We test our hypothesis for an emerging economy where relative consumption volatility shows an increase after witnessing of financial sector development.

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4The weights corresponds to a combination of the share of consumption of the respective household type in total consumption and the share of such households in total population.
Table 3 Business cycle stylised facts for the Indian economy in the pre and post reform period

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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Std. dev. Rel. dev. Cont. cor.</td>
<td>First ord. auto corr.</td>
</tr>
<tr>
<td>Real GDP</td>
<td>2.25 1.00 1.00</td>
<td>1.87 1.00 1.00 0.678</td>
</tr>
<tr>
<td>Pvt. Cons.</td>
<td>1.88 0.83 0.70</td>
<td>2.14 1.14 0.90 0.598</td>
</tr>
<tr>
<td>Investment</td>
<td>5.42 2.41 0.14 0.479</td>
<td>5.58 2.98 0.75 0.425</td>
</tr>
</tbody>
</table>

4 Case study

4.1 Evidence for India

We calibrate the model for India, an emerging economy which has witnessed financial sector reform. Ang (2011) finds that financial liberalisation increases fluctuations in consumption in India during 1950-2005. Also, relative to income volatility, consumption volatility in India increased after reform (Ghate et al., 2011).

India has witnessed development of its domestic financial sector in the post reform period, while remaining fairly closed in terms of capital account openness even after the reform. Thus India serves as an example of an emerging economy, with a low level of financial integration and a moderate expansion of domestic financial services. Figure 2 shows the expansion of financial services in India from the pre to post-reform periods. Interestingly, the country witnessed a small decline in banking services before witnessing a sharp increase. This period is included in the post reform sample to achieve reasonable sample size.

We simulate this model for the pre and post-reform periods, keeping all deep parameters, except the share of non-Ricardian households the same for both periods. Expansion of the financial services is captured by a lower value of the share of liquidity-constrained households in the post reform period. In this way we seek to identify one of the key factors which may explain the differences in relative consumption volatility between pre and post financial reform periods. We simulate the model for two different values of the share of liquidity constrained households and compare the simulated business cycle moments with business cycle stylized facts observed in pre and post reform India.

We calculate key business cycle moments for per capita output, consumption
Figure 2 Financial development in India

This figure shows the behaviour of some financial development indicators in India. The ratio of total number of bank accounts in the economy to total population, the number of bank branches in 100,000 population, bank deposit to GDP ratio and private credit to GDP are all seen to rise. The dashed lines show the mean values before and after financial reforms.
and investment at annual frequency. Output, consumption and investment are measured by real GDP at factor cost, private consumption expenditure and gross fixed capital formation for the period 1951-2012. To examine the transition in the business cycle stylised facts, the sample is divided into pre (1951-1991), and post reform periods (1992-2009). Key business cycle moments are obtained from the HP-filtered cyclical components of per capita output, consumption and investment.

The change in business cycle facts for the Indian economy from 1951-2009 are depicted in Table 3. Per capita Real GDP has become less volatile in the post-reform period in India. The level of volatility is still high and comparable to emerging economies. The absolute per capita consumption volatility as well as the relative consumption volatility with respect to output increased in the post-reform period. Per capita investment volatility, both absolute, and relative to output volatility show a small increase in the post-reform period. Contemporaneous correlation of consumption and investment with output has increased in the post-reform period. No significant persistence in the output and consumption cycle is seen in the pre-reform period. In the post-reform period, output and consumption cycle are observed to have higher persistence. Persistence in the investment cycle shows a small decline in the post-reform period.

There has been a sharp increase in access to finance after reforms. The ratio of bank account in total population was merely 20% in 1980, it has jumped to above 70% in 2010, except for a period of decline in the trend during 1990-2005. Similarly bank branches per 100,000 population in 2010 have more than doubled the value in 1970.

As seen in Table 3, relative consumption volatility in India has risen from 0.83 during 1951-1991 to 1.14 during 1992-2012. Thus after improved access to savings instruments and credit, fluctuations in consumption relative to fluctuations in income has increased.

Figure 3 shows the trend in one of the key variables of our analysis, namely, relative consumption volatility. The figure shows that the mean of relative consumption volatility has increased in the post reform period.

### 4.2 Calibration

Table 4 summarises the benchmark parameter values used in our calibration exercise. The access of households to banking is captured by the number
This figure shows five year rolling relative consumption volatility in India during 1956-2009.

Figure 3 Trend in relative consumption volatility

Table 4 Benchmark parameter values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$ 0.98</td>
</tr>
<tr>
<td>Rate of Depreciation</td>
<td>$\delta$ 5%</td>
</tr>
<tr>
<td>Share of labour</td>
<td>$\alpha$ 0.7</td>
</tr>
<tr>
<td>Adjustment cost parameter</td>
<td>$\phi$ 2.82</td>
</tr>
<tr>
<td>Mean trend growth rate of labour productivity</td>
<td>$\mu_g - 1$ 4.7%</td>
</tr>
<tr>
<td>Persistence in TFP shock process</td>
<td>$\rho$ 0.495</td>
</tr>
<tr>
<td>Volatility in TFP</td>
<td>$\sigma_a$ 0.015</td>
</tr>
<tr>
<td>Persistence in labour productivity growth shock</td>
<td>$\rho_g$ 0.261</td>
</tr>
<tr>
<td>Volatility in labour productivity growth shock</td>
<td>$\sigma_g$ 0.020</td>
</tr>
</tbody>
</table>

of bank accounts to population. Hence the proxy for $\lambda$, i.e., the share of liquidity constrained households is derived from this ratio.

The number of bank accounts to population ratios in 1980 and 2010 are used to calibrate the share of liquidity constrained households in the pre and post reform periods. In 1980 21.4% of the population had access to banking. Thus the share of households without access to finance, i.e., $\lambda$ was 0.786 in the pre reform period. In 2010 66.9% of the population had access to banking services, or $\lambda$ rose to 0.331 in the post reform period.

We follow the existing literature in choosing some of the other parameter values. A period is a year. The discount rate $\beta$ is set to 0.98 as in Aguiar and Gopinath (2007). The share of labour $\alpha$ for India is 0.7 as in Verma (2008), while the rate of depreciation is 5% as in Virmani (2004).
Table 5 Sectoral shares of factors of production

<table>
<thead>
<tr>
<th></th>
<th>Land</th>
<th>Physical capital</th>
<th>Labour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>0.2</td>
<td>0.24</td>
<td>0.56</td>
</tr>
<tr>
<td>Industry</td>
<td>0.3</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>0.3</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>

We estimate $\mu_g - 1$, the mean trend growth rate of labour productivity and find it to be 4.7%. We also estimate the shock processes of TFP and growth in labour productivity for India. This analysis is described in the following sections.

4.2.1 Shock process in the total factor productivity series

In order to obtain the amplitude and persistence of the shock process, we obtain a measure for the Total Factor Productivity (TFP) for the Indian economy over the span of years 1980-2009. We compute the aggregate TFP series as the weighted average of sectoral TFP series using sectoral GDP and the net fixed capital stock data and a time series for employment using NSSO data. \footnote{The distribution of labour force (per 1000 households, male/female, rural/urban) is reported for each sector in the NSSO’s quinquennial Employment Unemployment Survey as well as in the annual surveys based on a thin sample. We generate a time series of the distribution of sectoral employment based on these reports. National labour force data published by the World Bank are available from 1980 at annual frequency. Using the sectoral distribution of labour force and the total labour force data, we obtain sectoral employment series. Finally, we measure the sectoral TFP series for India using sectoral real GDP, net fixed capital stock and employment data. Given the availability of employment data, our TFP series spans 1980-2009.}

Using the sectoral shares ($w^j_s$) of capital, labour and land in agriculture, industry and services from Verma (2008), shown in Table 5, we measure the sectoral TFP series as

$$\log(A^s_t) = \log(Y^s_t) - \sum_{j=1}^{n_s} w^j_s \log(X^{sj}_t), \quad \sum_{j=1}^{n_s} w^j_s = 1,$$

(17)

where $n_s$ is the number of inputs used in sector $s$. Here $s$ denotes major sectors constituting the economy namely, agriculture, industry and services, $Y$ represents real GDP and $X^j$ denotes factors of production in the respective sector. For example when, $s = \text{agriculture}$, $j = \text{land, physical capital, labour}$. When $s = \text{industry, services}$, $j = \text{physical capital, labour.}$
Aggregate TFP is measured as a weighted average of the sectoral TFPs as following:

$$\log(A_t) = \sum_s \gamma_s^t \log(A^s_t), \quad s = \text{Agriculture, Industry, Services}$$  \hspace{1cm} (18)

where $\gamma_s^t$ denotes the share of sector $s$ in total GDP. Next, we de-trend the TFP series using a quadratic trend regression. Finally, we fit an AR(1) model on the cyclical component of TFP to obtain the persistence parameter and the standard deviation of the residual.

### 4.2.2 Shock process in the growth of labour productivity

Using sectoral real GDP and labour force data, we obtain the annual time series of sectoral and aggregate labour productivity for India based on the following:

$$\log(\Gamma^{s}_t) = \log(Y^{s}_t) - \log(X^{sj}_t), \quad j = \text{labour}$$

$$\log(\Gamma^{s}_t) = \sum_s \gamma^{s}_t \log(\Gamma^{s}_t), \quad s = \text{Agriculture, Industry, Services}$$  \hspace{1cm} (19)

The gross growth rate of labour productivity $\frac{\Gamma^{t}_t}{\Gamma^{t}_{t-1}}$ is

$$g_t = 1 + \Delta \log(\Gamma^{t}_t)$$  \hspace{1cm} (20)

Next, we de-trend the growth of labour productivity using a trend regression. The mean trend net growth rate over the period 1980-2009 turns out to be $\mu_g - 1 = 4.7\%$. Finally, we fit an AR(1) model on the de-trended component of labour productivity growth to obtain the persistence parameter and the standard deviation of the residual.

### 4.3 Effect of financial development on relative consumption volatility

Our model predicts that a decline in the share of liquidity-constrained households in the population would allow more people to respond to permanent income shocks. They can increase current consumption more than the rise in current income. This is predicted to result in a rise in the relative consumption volatility.
Table 6 Business cycle volatilities from the simulated model

This table presents absolute and relative business cycle volatilities from the simulated model for the pre and post-reform period.

<table>
<thead>
<tr>
<th></th>
<th>Std. dev.</th>
<th>Rel. std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Y</td>
<td>C</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>2.25</td>
<td>1.88</td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.87</td>
<td>2.14</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>3.25</td>
<td>3.21</td>
</tr>
<tr>
<td>Post-reform</td>
<td>3.22</td>
<td>3.23</td>
</tr>
</tbody>
</table>

Table 7 Business cycle correlation and persistence from the simulated model

This table presents absolute and relative business cycle volatilities from the simulated model for the pre and post-reform period.

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
<th>Auto-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>I</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.70</td>
<td>0.14</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.90</td>
<td>0.75</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.99</td>
<td>0.69</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.98</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 6 presents our main findings. Relative consumption volatility shows a rise in the post reform period. This result supports our key prediction. Since financial development allows more people to access savings instruments, when households perceive a permanent income shock which raises both current and future income, more people can respond to the shock by reducing current savings and raising current consumption more than the rise in current income. As a result of financial development, the volatility of consumption relative to volatility of output rises.

Our model also successfully replicates the pattern of changes in absolute output and consumption volatility. Table 7 shows that the simulated correlation of consumption and investment cycles with the output cycle and their persistence matches broadly with the pattern of correlation and persistence observed in the data. Finally, Figure 4 shows that our model replicates the cyclical pattern in output and consumption fairly well.
Figure 4 Actual and simulated cycles

This figure compares cyclical movements in per capita GDP, consumption expenditure and investment in the pre and post-reform periods with simulated output, consumption and investment cycles for these two periods.
Table 8 Sensitivity analysis with respect to the financial development parameter

This table presents business cycle moments from the simulated model for the pre and post-reform period using an alternative measure of $\lambda$. The measure used in this analysis is based on deposit to GDP ratio.

<table>
<thead>
<tr>
<th></th>
<th>Std. dev.</th>
<th>Rel. std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Y</td>
<td>C</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>2.25</td>
<td>1.88</td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.87</td>
<td>2.14</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>3.24</td>
<td>3.21</td>
</tr>
<tr>
<td>Post-reform</td>
<td>3.22</td>
<td>3.23</td>
</tr>
</tbody>
</table>

Table 9 Sensitivity analysis with respect to the financial development parameter

This table shows that business cycle moments from the simulated model for the pre and post-reform period using the alternative measure of $\lambda$ based on deposit to GDP ratio are broadly unchanged.

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
<th>Auto-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>I</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.70</td>
<td>0.14</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.90</td>
<td>0.75</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.99</td>
<td>0.70</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.98</td>
<td>0.73</td>
</tr>
</tbody>
</table>

4.4 Sensitivity to the measure of financial development

In the above analysis we measured financial development as the share of the population with bank accounts. As a robustness check we now use another measure of financial development, the bank deposit to GDP ratio to obtain the fraction of liquidity constrained households in the economy. By this measure, $\lambda = 0.687$ is in the pre-reform period and 0.305 in the post liberalisation period.

Table 8 and 9 shows that key moments from the business cycle model for the pre and post-reform periods based on an alternative measure of $\lambda$ are similar to those of the benchmark model.
5 Conclusion

Emerging economies have been seen to witness an increase in consumption volatility relative to output volatility after financial development. This behaviour appears puzzling since traditional models and evidence from advanced economies suggests that consumption should become smoother after credit constraints are reduced.

A distinguishing feature of the developing economies is that a large share of the population does not have access to finance. In the last two decades, these economies have experienced reforms in the financial sector giving greater access to financial services for households and firms. Yet, these economies experienced an increase in consumption volatility relative to output volatility in the post-reform period. This paper addresses this empirical puzzle. This puzzle can be explained in a model featuring credit constraints and shocks to trend growth of productivity. The model predicts that relative consumption volatility will rise when more consumers can smooth consumption.

The model, when simulated for India before and after an increase in financial development is fairly able to replicate the rise in the relative consumption volatility observed in the data. Other empirical regularities observed from the data are also replicated by this model.

Our framework represents a closed economy, as the concept of financial development is limited to domestic financial development. There exists a significant strand of literature exploring the role of financial liberalization, financial integration, inter-linkage of financial openness and domestic financial development on macroeconomic fluctuations. It will be interesting to examine how increase in households’ access to foreign financial instruments affects consumption-smoothing behaviour in our framework for an emerging economy.

Finally, our model assumes that the household sector is the sole channel for the financial development to work. This is one plausible reason for our model’s weak performance in replicating the business cycle patterns with respect to investment. By including credit-constrained firms in our framework, one can examine the role of financial development further.
## Appendix I

### Table 10 Data span for GDP and consumption expenditure: Emerging economies

<table>
<thead>
<tr>
<th>Region &amp; reform date</th>
<th>Start date</th>
<th>End date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latin America: 1990</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td>1974</td>
<td>2010</td>
</tr>
<tr>
<td>Colombia</td>
<td>1968</td>
<td>2010</td>
</tr>
<tr>
<td>Mexico</td>
<td>1978</td>
<td>2011</td>
</tr>
<tr>
<td>Peru</td>
<td>1989</td>
<td>2011</td>
</tr>
<tr>
<td>East Asia: 1996</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>1978</td>
<td>2011</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1970</td>
<td>2011</td>
</tr>
<tr>
<td>Philippines</td>
<td>1958</td>
<td>2011</td>
</tr>
<tr>
<td>Korea</td>
<td>1953</td>
<td>2010</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1981</td>
<td>2011</td>
</tr>
<tr>
<td>Thailand</td>
<td>1950</td>
<td>2011</td>
</tr>
<tr>
<td>East Europe: 1990</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>1989</td>
<td>2010</td>
</tr>
<tr>
<td>Poland</td>
<td>1981</td>
<td>2011</td>
</tr>
<tr>
<td>Hungary</td>
<td>1971</td>
<td>2011</td>
</tr>
<tr>
<td>South Asia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>India: 1992</td>
<td>1951</td>
<td>2012</td>
</tr>
<tr>
<td>Africa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>South Africa: 1994</td>
<td>1950</td>
<td>2011</td>
</tr>
</tbody>
</table>

Given the availability of data for Peru, we compare relative consumption volatility before and after 2000.
B Appendix II

B.1 Solution of the log-linearised system of equations using method of undetermined coefficients

The log-linearised system of equations spanning the dynamics of the model economy is as follows:

\[
\begin{align*}
\tilde{c}_R^t &= \left(1 - \frac{\lambda \alpha}{\beta c^{R*}}\right) \frac{y^*}{c^{R*}} + \left(1 - \frac{\lambda \alpha}{1 - \lambda}\right) \left[\mu_g k^{R*} - \frac{\lambda(1 - \alpha)(1 - \delta)}{1 - \lambda} k^{R*}\right] \tilde{k}_t^R \\
&\quad - \frac{\mu_g k^{R*}}{c^{R*}} \tilde{k}_t^R - \frac{\mu_g k^{R*}}{c^{R*}} \tilde{g}_t,
\end{align*}
\]

\[0 = E_{t-1} \left[\tilde{c}_t^R + A(a_t - \alpha \tilde{k}_t^R + \alpha \tilde{g}_t)\right]; \quad A = 1 - \frac{\beta(1 - \delta)}{\mu_g},\]

\[a_t = \rho a_{t-1} + \epsilon_t,\]

\[\tilde{g}_t = \rho g \tilde{g}_{t-1} + \epsilon_t^g; \quad \tilde{g}_t = \ln \left(\frac{g_t}{\mu_g}\right)\]

\[(B.1)\]

The solution of the system of equations (B.1) takes the form

\[
\begin{align*}
\tilde{k}_{t+1}^R &= a_1 \tilde{k}_t^R + b_1 a_t + d_1 \tilde{g}_t \\
\tilde{c}_t^R &= a_2 \tilde{k}_t^R + b_2 a_t + d_2 \tilde{g}_t.
\end{align*}
\]

\[(B.2)\]

The solution of the system of equations (B.1) takes the form

\[
\begin{align*}
\tilde{k}_{t+1}^R &= a_1 \tilde{k}_t^R + b_1 a_t + d_1 \tilde{g}_t \\
\tilde{c}_t^R &= a_2 \tilde{k}_t^R + b_2 a_t + d_2 \tilde{g}_t.
\end{align*}
\]

The unknown parameters \((a_1, b_1, d_1, a_2, b_2, d_2)\) are functions of \((\beta, \alpha, \delta, \lambda, \mu_g)\) are solved using the method of undetermined coefficients where the condition for convergence requires \(a_1 < 1\). Substituting solution (B.2) in the first equation in the system of equations (B.1), we have

\[
\begin{align*}
a_2 \tilde{k}_t^R + b_2 a_t + d_2 \tilde{g}_t &= \left(1 - \frac{\lambda \alpha}{1 - \lambda}\right) \frac{y^*}{c^{R*}} + \left(1 - \frac{\lambda \alpha}{1 - \lambda}\right) \left[\mu_g k^{R*} - \frac{\lambda(1 - \alpha)(1 - \delta)}{1 - \lambda} k^{R*}\right] \tilde{k}_t^R \\
&\quad - \frac{\mu_g k^{R*}}{c^{R*}} [a_1 \tilde{k}_t^R + b_1 a_t + d_1 \tilde{g}_t] - \frac{\mu_g k^{R*}}{c^{R*}} \tilde{g}_t
\end{align*}
\]
Re-arranging the terms yields

\[
\begin{align*}
    a_2 &= \left[ \frac{1 - \lambda \alpha}{1 - \lambda} \right] \mu_g k^{R*} - \frac{\lambda(1 - \delta)(1 - \alpha) k^{R*}}{1 - \lambda} \frac{a_1}{c^{R*}} \\
    b_2 &= \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) y^* - \frac{\mu_g k^{R*} c^{R*}}{c^{R*}} b_1 \\
    d_2 &= -\left[ \frac{\mu_g k^{R*}}{c^{R*}} d_1 + \frac{\mu_g k^{R*}}{c^{R*}} b_1 \right]
\end{align*}
\]

(B.3) (B.4) (B.5)

where the steady state value of consumption of a Ricardian household, and the steady state value of the output are respectively

\[
c^{R*} = \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) \left( \frac{1 - \delta(1 - \alpha)}{1 - \lambda} \frac{k^{R*}}{k^{R*}} - \alpha \mu_g \beta \right)
\]

(B.6) (B.7) (B.8)

(B.2)

The steady state expression of stock of capital of a Ricardian household is derived from the Euler equation and the expression of gross return to capital given in the equation system (13) assuming that in the steady state, all variables normalised by the labour productivity of the previous period remain constant along the steady state, i.e., \( k_t = k_{t+1} = k^{R*} \) and \( c_t = c_{t+1} = c^{R*} \). The steady state growth rate of labour productivity is the long-run average trend growth rate \( \mu_g \). Also, the steady state is free of any transitory movement in the total factor productivity, hence, \( a_t = a_{t+1} = 0 \). Given the value of \( k^{R*} \), the steady state value of consumption \( c^{R*} \) is derived from the resource constraint equation in (13). It then follows from equations (B.2), (B.3), (B.5) that the steady state consumption of the liquidity constrained households and the total consumption are respectively \( c^{L*} = \alpha y^* \) and \( c^* = (1 - \lambda) c^{R*} + \lambda c^{L*} \).

Substituting the solution (B.2) in the second equation of (B.1), and making the use of \( E_{t-1} a_t = \rho_a a_{t-1} \) and \( E_{t-1} \tilde{g}_t = \rho_g \tilde{g}_{t-1} \) yields

\[
\begin{align*}
    a_2 - a_2 a_1 - A \alpha a_1 &= 0 \\
    b_2(1 - \rho_g) - a_2 b_1 + A \rho_a - A \alpha b_1 &= 0 \\
    d_2(1 - \rho_g) - a_2 d_1 + A \alpha d_1 + A \alpha \rho_g &= 0
\end{align*}
\]

(B.6) (B.7) (B.8)

Again, substituting expression (B.4) in the expression (B.8), and rearranging the terms yields a quadratic equation in \( a_1 \),

\[
a_1^2 - \gamma_1 a_1 + \gamma_2 = 0
\]

(B.9)
where

\[ \gamma_1 = 1 + \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) \frac{1}{\beta} - \frac{\lambda(1 - \alpha)(1 - \delta)}{(1 - \lambda)\mu_g} + A\alpha \frac{c^{Rs}}{\mu_g k^{Rs}} \]
\[ \gamma_2 = \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) \frac{1}{\beta} - \frac{\lambda(1 - \alpha)(1 - \delta)}{(1 - \lambda)\mu_g} \]

(B.10) (B.11)

The solution of the above quadratic equation is

\[ a_1 = \frac{\gamma_1}{2} \pm \sqrt{\left( \frac{\gamma_1}{2} \right)^2 - \gamma_2} \]

(B.12)

If there is no friction in the credit market so that the entire population can access financial services and hence smooth consumption, then \( \lambda = 0 \). Then the product of the two roots in expression (B.12) is \( \gamma_2 = 1/\beta > 1 \), where the value of \( \gamma_2 \) is obtained by evaluating expression (B.11) at \( \lambda = 0 \). Again, as \( \lim_{\lambda \to 0} \gamma_2 \to \infty \). Hence, the product of the two roots as in the expression (B.12) \( \gamma_2 \) is always greater that 1, irrespective of the value of \( \lambda \). Therefore it follows that the larger one must exceed 1 and only the smaller one can possibly satisfy the convergence condition \( a_1 < 1 \). Hence, the solution of the quadratic equation (B.9) is

\[ \hat{a}_1 = \frac{\gamma_1}{2} - \sqrt{\left( \frac{\gamma_1}{2} \right)^2 - \gamma_2} \]

(B.13)

Given this solution of \( a_1 \), from equations (B.4), (B.5), (B.6), (B.7), (B.8), (B.9) and making use of the steady state values \( (k^{Rs}, c^{Rs}, y^*) \), one can solve for \( (a_2, b_1, b_2, d_1, d_2) \) as

\[ \hat{a}_2 = \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) \frac{\mu_g k^{Rs}}{\beta c^{Rs}} - \frac{\lambda(1 - \delta)(1 - \alpha) k^{Rs}}{1 - \lambda} \frac{c^{Rs}}{c^{Rs}} - \hat{a}_1 \]
\[ \hat{b}_1 = \frac{(1 - \rho_\alpha) \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) y^*}{c^{Rs} + \hat{\alpha}_2 + \alpha A} \]
\[ \hat{b}_2 = \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) \frac{y^*}{c^{Rs}} - \frac{\mu_g k^{Rs}}{c^{Rs}} \hat{b}_1 \]
\[ \hat{d}_1 = \frac{A\alpha \rho_g - (1 - \rho_g) \frac{\mu_g k^{Rs}}{c^{Rs}}}{1 - \rho_g \frac{\mu_g k^{Rs}}{c^{Rs}}} + \hat{a}_2 + A\alpha \]
\[ \hat{d}_2 = -\frac{\mu_g k^{Rs}}{c^{Rs}} (1 + \hat{d}_1) \]

(B.14)
We demonstrate a proof of Proposition 1 through a numerical analysis. The volatility of consumption and output are computed using the expressions in (B.10), (B.11), (B.13) and (B.14) evaluated with values of deep parameters, the steady state growth rate and the persistence parameters of technology and growth shock structures used in the simulation exercise in section 4. The values of the parameters in the numerical exercise are \( \beta = 0.98 \), \( \alpha = 0.7 \), \( \delta = 0.05 \), \( \mu_g = 1.047 \), \( \rho_a = 0.495 \), and \( \rho_g = 0.261 \).

B.2 Proof of Proposition 1 (i)

Log-linearisation of \( c^L_t = W_t = \alpha y_t \) yields \( \tilde{c}^L_t = \tilde{y}_t \). Hence \( \sigma^2_{\tilde{c}^L_t} = \sigma^2_{\tilde{y}} \) proves Proposition 1 (i).

B.3 Proof of Proposition 1 (ii)

Under the assumption of the absence of growth shock, \( \sigma^2_{\tilde{c}^R} = 0 \), \( \sigma^2_{\tilde{c}^a} > 0 \), from expressions in (17), the relative consumption volatility of a Ricardian household with respect to output volatility is obtained as

\[
\frac{\sigma^2_{\tilde{c}^R}}{\sigma^2_{\tilde{y}}} = \left[ \frac{a^2 \hat{c}_t^2}{1-a_t^2} + b^2 \right] \frac{1}{1 + \left( 1 - \alpha \right)^2 b^2}.
\]

We evaluate this ratio at two values of \( \lambda \). In one scenario, \( \lambda = 0.9 \) that is the share of liquidity-constrained households is very high. In the second scenario, the economy is populated only by the Ricardian households, i.e., \( \lambda = 0 \). Given the value of relative volatility of consumption of a Ricardian household with respect to output volatility, the volatility of total consumption with respect to output volatility \( \frac{\sigma^2_{\tilde{c}}}{\sigma^2_{\tilde{y}}} \) can be computed from equation (16) under two alternative state of financial development. The relative consumption volatilities are shown in Table 11.

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \frac{\sigma^2_{\tilde{c}^R}}{\sigma^2_{\tilde{y}}} )</th>
<th>( \frac{\sigma^2_{\tilde{c}}}{\sigma^2_{\tilde{y}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.900</td>
<td>0.745</td>
<td>0.807</td>
</tr>
<tr>
<td>0</td>
<td>0.406</td>
<td>0.406</td>
</tr>
</tbody>
</table>

The results indicate that as long as the economy contains a share of population of Ricardian type who can smooth consumption, the consumption
volatility relative to output volatility is less than one under a transitory income shock. Also note that when $\lambda = 0$, that is the economy is populated by only Ricardian households, relative volatility of total consumption with respect to output is same as the relative consumption volatility of the Ricardian household.

B.4 Proof of Proposition 1 (iii)

Under the assumption of the absence of transitory income shock, $\sigma_{\epsilon a}^2 = 0$, $\sigma_{\epsilon g}^2 > 0$, from expressions in (17), the relative consumption volatility of a Ricardian household with respect to output volatility is obtained as

$$\frac{\sigma_{\tilde{c}R}^2}{\sigma_{\tilde{y}}^2} = \frac{\left[\frac{a^2d_1^2}{1-a_1^2} + d_2^2\right]}{\alpha^2 + \frac{(1-\alpha)^2d_1^2}{1-a_1^2}}.$$ 

We evaluate this ratio for a range of values of $\lambda$. The highest value of $\lambda = 0.9$ corresponds to the scenario when the share of liquidity-constrained households is very high. The lowest value that $\lambda$ takes is 0. This scenario represents an economy with a matured financial system so that it is populated only by the Ricardian households. We also consider the values of $\lambda$ used in the simulation exercise in section 4 to evaluate relative consumption volatility of Ricardian households and that of the entire economy. Given the value of relative volatility of consumption of a Ricardian household with respect to output volatility, the volatility of total consumption with respect to output volatility $\frac{\sigma_{\tilde{c}}^2}{\sigma_{\tilde{y}}^2}$ can be computed from equation (16) under alternative states of financial development. The relative consumption volatilities are shown in Table 12.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\frac{\sigma_{\tilde{c}R}^2}{\sigma_{\tilde{y}}^2}$</th>
<th>$\frac{\sigma_{\tilde{c}}^2}{\sigma_{\tilde{y}}^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.900</td>
<td>1.493</td>
<td>0.834</td>
</tr>
<tr>
<td>0.786</td>
<td>1.992</td>
<td>1.018</td>
</tr>
<tr>
<td>0.500</td>
<td>3.086</td>
<td>1.897</td>
</tr>
<tr>
<td>0.331</td>
<td>3.556</td>
<td>2.585</td>
</tr>
<tr>
<td>0</td>
<td>4.223</td>
<td>4.223</td>
</tr>
</tbody>
</table>

The results indicate that the relative consumption volatility with respect to output volatility of a Ricardian household always exceeds 1. The relative volatility of total consumption with respect to output volatility depends on
the level of financial development. The relative consumption volatility may fall below 1 if the economy consists of a large fraction of liquidity-constrained households.

B.5 Proof of Proposition 1 \(^{(iv)}\)

From Table 12, it is evident that the volatility of consumption of a Ricardian household and that of the total consumption with respect to output volatility increases as share of liquidity constrained households in the economy declines.
References


