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Income Inequality, Neighbourhood Effects and Product Quality *

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In this paper we analyze the effect of income inequality on market outcome and hence the welfare of the consumer in the industry which is both horizontally and vertically differentiated. The idea is that any income distribution over the spatial horizon is reflected in the demand structure and this shapes the market outcome. We consider a setting where the rich and poor live side by side on a linear city and two duopolist firms are positioned at the two ends of the city. We find that for a homogenous distribution of income or when the poor's income or density is too low, both firms offer the same quality. For a homogenous income distribution firm does not perceive much benefit from product differentiation. Given this, for a very high difference in the fixed costs, both firms offer the low quality. But when the difference in the fixed costs is low, both firms offer the high quality. For a more heterogeneous income distribution and an intermediate range of the difference in fixed costs, one firm offers the high quality and the other the low quality. Product differentiation on one hand allows firm to alleviate price competition and, on the other hand, serves consumers' demand better. We show that although in general a rise in income inequality has a spiraling negative effect on the welfare of the poor, there are situations, particularly when the poor income is very low, when an increase in the rich income could be welfare improving for the poor.

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1 Introduction

This paper explores the interaction of income inequality with the neighbourhood effect in determining market outcomes like price and quality of products and services and looks at its welfare impact. The effect of income inequality in the neighbourhood can be contrasted from the following simple illustration. The poor can easily be worse-off living in a rich neighbourhood owing to the soaring prices of products and services reflecting the higher willingness to pay of his rich neighbours. On the other hand, if he lives in a poor neighbourhood and the average income of the neighbourhood is low enough, the providers of the product or service might not enter into the neighbourhood at all as they will not be able to recover their fixed costs of production. In this scenario living with the rich might be welfare improving for the poor as at least some poor get to access the product or service since the providers could recover their fixed costs due to the higher willingness to pay of the rich.

The key idea is that with increase in income consumers value of the given quality goes up. Also wealthier individuals have preference for higher quality products. So firms may choose quality differentiation as a way to effectively reduce price competition and reach out to various sections of consumers. Some firms will concentrate on the high quality and price and hence depend on consumers with high income. The motivation is to exploit higher willingness to pay of the rich consumers. Others will offer cheaper products of lower quality in order to cater to low income groups.[1] But product differentiation makes sense only if there is enough demand for differentiated products in the market.[2] In the absence of that the firms will offer same quality product as otherwise they will not be able to break even. So the absolute values of the incomes as well as the relative size of different income groups are important in determining firm's quality choice. Another key aspect is the difference in the fixed costs of production.[3] If the cost of providing high quality is too high relative to the perceived benefits then the firm will not be in a position to offer high quality and charge a higher price to effectively exploit product differentiation to its advantage.

Consumers differ in their locations too. This imposes another constraint on the firm's choice. This is because instead of traveling all the way to buy their most preferred quality product, consumers might go for the product that is accessible relatively easily. Presence of travel costs thus inhibits firm to effectively segregate the market with respect to income.

[1] Classic works by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) provide motivation for vertical product differentiation.

[2] See Yurko (2009).

[3] For instance, Ronnen (1991), Fajgelbaum et al.(2009), Liao (2008) allow the fixed cost to be quality dependent, where high quality product costs more than a low quality one.

So the firm needs to carefully weigh its options before deciding the quality. These trade-offs have important bearing on the market outcome in terms of quality offered and price being charged and hence on the welfare of the consumers.

In order to study these trade-offs and its implication we consider a setting where the rich and poor live side by side on a linear city and two duopolist firms are positioned at the maximal distance from each other on the two ends of the city. To focus on the competition over quality choice we assume that the locations of the two firms are fixed. Competition is modeled as a two-stage game. In the first stage firms simultaneously choose between two qualities, high and low. In the second stage the firms compete in prices. We identify conditions when there is a symmetric equilibrium with each firm offering the same quality or an asymmetric equilibrium with each firm offering different quality products. For the asymmetric equilibrium we distinguish between two scenarios: vertical dominance and horizontal dominance. Vertical dominance occurs when all the rich, irrespective of the distance, buy the high quality product whereas the poor buy the low quality product. This arises when the vertical attribute, that is, the difference in the incomes and quality of the products dominate the travel cost. On the other hand horizontal dominance occurs when the travel cost is high enough to discourage consumers to buy their most preferred quality products. Instead, they end up buying the product that is available in the close neighborhood.

It turns out that the prominent factors contributing to the firms' price and quality choices are income inequality, relative proportion of rich and poor, and the cost differential between the high and low quality products relative to the perceived benefit. We find that when the income of poor is too low then both firms ignore their presence and offer the same quality. Both firms offer the low quality when difference in the fixed costs of the high and low quality products is high relative to the income level in the society. On the other hand both firms offer high quality when income of the rich is sufficiently high compared to the difference in fixed costs. For the intermediate level of the difference in fixed costs, there is an asymmetric equilibrium with one firm offering high quality and the other low quality. It follows that the welfare of the poor initially increases and then falls as there is income growth as a result of rise in income and proportion of the rich. Given the quite low income of the poor, if the rich income is also reasonably low, the firms offer only the low quality product catering to the rich and all the poor consumers are shut out of the market. For a relatively higher level of the rich income the possibility of asymmetric equilibrium emerges where poor are better-off as at least some of them can access the product that was earlier unavailable. But for a relatively high proportion of the rich, again the symmetric equilibrium prevails with both firms offering the high quality product and ignoring all the poor consumers in the process.

Similarly, when the income of the poor is relatively high, then again both firms offer low quality if the difference in the fixed cost is high. Or else both of them offer high quality when the income of rich is relatively high but the proportion of poor is low enough that product differentiation does not make sense. For the intermediate range of income gap firms choose to differentiate the market. Market differentiation allows alleviating price competition. The firm serving the rich is now able to charge a higher price to take advantage of the higher willingness to pay of the rich. Since the poor does not have a too high preference for high quality they opt for the low quality. This allows the low quality firm to charge a higher price taking advantage of his monopoly position with the poor. Poor are definitely worse-off on two accounts. First, they are priced out from buying the high quality product. In addition, they end up paying a high price for the low quality product. Thus the rise in income inequality has a spiraling negative effect on the welfare of the poor.

Our framework is close to Degryse (1996) who discusses the interaction between horizontal and vertical differentiation in determining a bank's choice whether to offer the remote access facility or not. The key difference with our work is that in order to emphasize on the market access and welfare of the poor, the possibility of non-consumption is an important aspect in our framework which he does not consider. The other works in the industrial organization literature implicitly assume that the market is either covered or uncovered. For example, Wauthy (1996) and Liao (2008) show that covered or uncovered markets are endogenous outcomes and depend on the degree of consumer heterogeneity. However, while Wauthy (1996) assumes that the costs of improving quality are zero, both papers consider only vertical product differentiation.

The chapter is organized as follows. In section 2 the basic framework is laid down. Section 3 discusses the price equilibrium in the second stage. Section 4 analyzes the quality choice in the first stage and the resulting equilibrium outcomes and welfare implications. Section 5 concludes by summarizing the main findings. The detailed proofs are developed in the Appendix.

2 Basic Model

In this section we outline the assumptions on preferences, technologies, market structure and income distribution of the economy we propose to study.

2.1 Consumer Preferences

Consumer are of two types: rich consumers with income Y_R and poor consumers with income Y_P, with $Y_R > Y_P$. At each point of the linear city of length L units, there are δ_R proportion of rich and δ_P proportion of poor, such that $\delta_R + \delta_P = 1$. Hence each individual is defined by his income level $Y \in \{Y_R, Y_P\}$ and location z on the linear city.[4] There are two firms located at the either end of the linear city offering quality θ, where $\theta \in \{\theta_H, \theta_L\}$ and $\theta_H > \theta_L$. Each consumer can buy a single unit of the product. Let $Y\theta$ be the gross utility of a consumer with income Y from consumption of the good/service of quality θ. In addition, there is a disutility from travel which enters linearly in the utility function. We denote the per-unit travel cost by t. Let p_j be the price charged and θ_j be the quality offered by firm j, $j = 1, 2$. Utility of the consumer located at the distance z from firm 1, with income Y and buying quality θ_j at price p_j is then given by

$$U(z, Y, p_j, \theta_j) = \begin{cases} Y\theta_1 - p_1 - tz & \text{if he buys from firm 1,} \\ Y\theta_2 - p_2 - t \mid L - z \mid & \text{if he buys from firm 2,} \\ Y & \text{if he does not buy.} \end{cases}$$

Y is the reservation utility of the consumer implying that $\theta_H > \theta_L > 1$.

The model incorporates the attributes of both horizontal as well as vertical differentiation with horizontal differentiation featured in the distance traveled and vertical differentiation in quality choice. We take distance literally to imply the physical distance traveled by the consumer. It is apparent from the utility function that the total price paid by the consumers (which includes the transportation cost) differs from the net market price received by the producer. Because of this difference between the actual price and delivered price there might be consumers even at the same income level who are left out of the market.

Note that the particular form of the utility function is such that everywhere it satisfies the "single-crossing" condition: $\partial\left(\frac{\partial U/\partial\theta}{\partial U/\partial Y}\right)/\partial Y > 0$. Hence any indifference curve in (θ, Y)-plane of a higher income household cuts a indifference curve of lower-income household from below. This implies that the individual with higher income has higher willingness to pay for the same marginal increase in quality. This signifies not just their ability to pay but their preference to pay for higher quality even if it comes at a higher price. The underlying assumption is that richer individuals are likely to be better informed about the benefits of quality education or health care system, and are willing to pay more for all these services.

Also, $Y\theta$ signifies that the welfare from consumption of all goods and services increases

[4] That is, this is not a model of location choice by the consumers.

if he chooses to buy the product/service under consideration. For example, if θ represents the quality level of health services, then this would imply that owing to access to the better health facility an individual is able to derive higher satisfaction from the consumption of all other goods and services. Similarly an educated person is better able to appreciate the value of other thing, because of higher degree of awareness and understanding. This enhances his utility from his overall spending.

2.2 Firms

As mentioned above, there are two firms located at the either end of the linear city. We assume a two stage game between the firms. Investment in quality is made in the first stage which can be either θ_L or θ_H. In the second stage firms simultaneously decide the price. Each firm faces two types of cost: a fixed cost denoted by $F(\theta)$ and a marginal cost given by $c(\theta)$. We assume that marginal cost of production is independent of the output level, but both fixed and marginal costs increase with the improvement in quality, that is, $F(\theta_H) > F(\theta_L)$ and $c(\theta_H) > c(\theta_L)$.

We assume that firms do not price discriminate between consumers and charge them the same price irrespective of their incomes and locations. But there is implicit price discrimination arising out of the differences in locations of individuals on the linear city. This difference in the actual cost borne by an individual has an implication on the number of consumers who finally buy the product.

After having chosen the quality in the first stage firms simultaneously choose the price in the second stage. Profit of firm j charging a price p_j and offering quality θ_j is given by

$$\pi_j = [p_j - c(\theta_j)]D_j - F(\theta_j),$$

where D_j is the demand faced by firm j and depends on its own strategic choices of price and quality and also on the strategic choices of the other firm. As mentioned above, because of the spatial aspect, there might be people who are left unserved owing to the greater distance.

3 Price Equilibrium

Given the first stage equilibrium of quality choice, there are several possible subgames: (θ_H, θ_H) - both firms offer high quality; (θ_L, θ_L) - both firms offer low quality; (θ_H, θ_L) - one firm offers high quality and the other offers low quality. In each of this subgame, given the quality choices, firms simultaneously compete in prices.

3.1 Same Quality by Both Firms

In this section we analyze a subgame where both firms offer the same quality which can be either be θ_H or θ_L. Given the income level of poor, this may imply either full market coverage where all poor are served, or partial market coverage where some poor are left unserved. The following two sub-sections characterize equilibrium under the full and partial market coverage respectively.

3.1.1 Full Market Coverage

Throughout the paper we assume that the rich income is high enough so that all rich consumers are served. In this subsection we establish conditions for an equilibrium where there is full market coverage, that is, all poor consumers are served. So each firm competes for each income type for their demand. Demand faced by each firm is determined by the distance of the marginal consumer who is indifferent between the two firms. Let the marginal consumer with income Y who is indifferent between firm 1 and firm 2, with both firms offering the same quality θ, be located at a distance z from firm 1. Then

$$U(z, Y, p_1, \theta) = U(z, Y, p_2, z) \quad \Rightarrow \quad z = \frac{p_2 - p_1 + tL}{2t}.$$

As all the rich and poor are being served, this would imply that demand faced by firm 1, is $D_1(.) = (\delta_R + \delta_P)\left[\frac{p_2 - p_1 + tL}{2t}\right]$. Profit for firm 1 is then given by

$$\pi_1 = (\delta_R + \delta_P)[p_1 - c(\theta)]\left[\frac{p_2 - p_1 + tL}{2t}\right] - F(\theta).$$

In stage 2, given the quality decision in the earlier stage, firms choose its price to maximize profit, π. For the firm 1 the first-order condition for the profit maximization with respect to price implies $D_1(.) = \frac{(\delta_R + \delta_P)[p_1 - c(\theta)]}{2t}$. After substitution and simplification this reduces to $p_2 = 2p_1 - c(\theta) - tL$. Similar exercise for firm 2 would imply that $p_1 = 2p_2 - c(\theta) - tL$. From the above two equations it follows that

$$p_1 = p_2 = tL + c(\theta).$$

So there is a unique symmetric equilibrium where both firms charge the same price given that in the initial stage they offer the same quality. Unlike the Betrand competition[5] firms

[5]See Tirole (1988), p .209 and Vives (1999), p .117.

are able to charge above the marginal cost because of horizontal differentiation. Using the first-order condition with respect to price the expression for profit reduces to

$$\pi_i = (\delta_R + \delta_P)[p_i - c(\theta)]D_i(.) - F(\theta) = (\delta_R + \delta_P)[p_i - c(\theta)]^2 \cdot \frac{1}{2t} - F(\theta),\ i = 1, 2.$$

On substituting for p in equilibrium firm's profit is given by

$$\pi_i = \frac{(\delta_R + \delta_P)[tL]^2}{2t} - F(\theta),\ i = 1, 2.$$

Clearly profits in the equilibrium where both the firms produce low quality is higher than the case where both of them produce high quality. This is because even though the mark-up over the marginal cost is the same because of competition and symmetric equilibrium but there is difference in the fixed cost of quality. As both income types are served, the density of rich and poor have equal weightage in determination of firm's profit. Higher the travel cost higher is the profit level for the firm as this raises the extent of horizontal differentiation between the two firms.

The cut-off level of Y_P which ensures full market coverage is determined as follows. As all the poor are buying, it implies that the marginal poor indifferent between the two firms is better off buying the product. Let $\overline{Y}$ be the income level at which the consumer who is indifferent between the two firms, is also indifferent between buying and not buying. We have derived above that the distance from firm 1 at which the consumer with income $\overline{Y}$ is indifferent between the two firms is $\frac{p_2 - p_1 + tL}{2t}$. Since at this distance the consumer with income $\overline{Y}$ is also indifferent in buying and not buying, it follows that $\overline{Y}\theta - p_1 - t\left(\frac{p_2 - p_1 + tL}{2t}\right) = \overline{Y}$, that is , $\overline{Y} = \frac{p_1 + p_2 + tL}{2(\theta - 1)}$. As all poor are buying, it follows that $Y_R > Y_P > \overline{Y}$. On substituting the equilibrium value of p the above inequality implies

$$Y_R \ > \ Y_P \ > \ \overline{Y} \ = \ \frac{1}{\theta - 1}\left[c(\theta) + \frac{3tL}{2}\right]$$

As is apparent from above, the cut-off is lower, lower is the level of the marginal cost as well as the travel cost. Marginal and travel costs are the prices an individual has to pay to buy the product. Higher the price, less will be the market coverage. Also observe that an increase in θ without any increase in marginal cost unambiguously reduces the cut-off. Intuitive way to understand this is to think of θ as the individuals valuation of the product. A rise in just individual's valuation without any corresponding rise in the marginal cost induces an individual to participate.[6] This is the direct artifact of the particular form of utility function

[6]WaterAid-India's rural sanitation program was making slow progress in 1995-96. A lack of demand

that we have assumed. It is interesting to see how the cut-off level varies with a change in θ. With the increase in quality, consumers gross utility increases; but now he also has to pay a higher price. Which affect dominates depends on curvature of the marginal cost curve. This is clear from the following

$$\frac{\partial \overline{Y}}{\partial \theta} = \frac{1}{(\theta - 1)} \left[c^{'}(\theta) - \frac{c(\theta)}{(\theta - 1)} - \frac{3tL}{2(\theta - 1)} \right]$$

When the marginal cost is linear in θ, $\dfrac{\partial Y}{\partial \theta}$ is strictly negative, implying that the valuation effect outweighs the cost effect. For a convex cost, it is initially negative, but for higher level of θ it might be positive. So depending on the parameter values, there might be an inverted U-shape relation between the cut-off level of income and quality. It is pertinent to observe that both the cut-off level of income and the equilibrium price level are insensitive to the income distribution. This is because once the income level of poor is high enough, the firms do not care for the income gap owing to the competitive pressure. Thus both rich and poor are treated symmetrically and their relative disparity does not matter. Above results can be summarized in the following proposition.

Proposition 1: *In a subgame where both firms offer the same quality there exists a unique equilibrium with full market coverage iff* $Y_P(\theta - 1) \geq \left[c(\theta) + \dfrac{3tL}{2} \right]$. *The equilibrium is characterized by the following properties.*

1. *All poor and rich are served.*
2. *Both firms charge the same price,* $p_1 = p_2 = tL + c(\theta) \equiv p_C$.
3. *Market price increases with increase in* t *and* $c(\theta)$, *but does not depend on* Y_P, Y_R, δ_R *or* δ_P.
4. *In equilibrium each, firms profit is given by* $\pi = \dfrac{(\delta_R + \delta_P)[tL]^2}{2t} - F(\theta)$.

We would like to look at the impact of income gap on consumer surplus. But as observed above, equilibrium price is independent of the income gap or the relative proportion of poor and rich, implying that, in the case of full market coverage, relative income gap is immaterial. The net surplus to a consumer with income Y_i and located at the distance x from the firm

from households meant that partner NGOs had constructed only 460 out of 1,100 latrines planned for the 12-month period. WaterAid-India decided that it was time to reformulate its strategy and focus on marketing sanitation. As a result of this change in approach, by the first six months of 1997-98, partner NGOs had achieved a dramatic turnaround in demand and constructed 5,000 latrines. For more on the role on information see Jalan and Somanathan (2008) and Banerjee et al. (2008).

from which he is buying is $Y_i\theta - p - tx - Y_i$, where $Y_i \in \{Y_R, Y_P\}$. Recall that, the reservation utility is given by his income level, Y_i. Since there are 2 firms each with a market coverage of $\frac{L}{2}$, the aggregate consumer surplus CS_i of the individuals with income Y_i and proportion δ_i, where $\delta_i \in \{\delta_R, \delta_P\}$ is

$$CS_i \; = \; 2\delta_i \int_0^{\frac{L}{2}} \left[Y_i\left(\theta - 1\right) - p - tx\right] dx = \delta_i L \left[Y_i\left(\theta - 1\right) - p - \frac{tL}{4}\right].$$

As expected, consumer surplus increases with income Y_i, and individual's valuation for the product given by θ, and decreases with travel cost and price. Since price is endogenous, substituting the equilibrium value of p this reduces to

$$CS \; = \; \delta_i L \left[Y_i\left(\theta - 1\right) - \frac{5tL}{4} - c(\theta)\right].$$

It is apparent that the welfare of rich is higher than poor by virtue of their higher income and the relative gap in the income level does not affect welfare. Again how does the welfare of the consumer changes with change in θ depends on the curvature of the cost curve and the parameter values. This leads us to the following proposition.

Proposition 2: *Let* $Y_P(\theta - 1) \; \geq \; \left[c(\theta) + \frac{3tL}{2}\right]$, *aggregate consumer surplus of consumers falls in* t *and* $c(\theta)$, *increases in their own income and proportion but is independent of the income gap.*

This highlights the case when income gap is not substantial, and the competitive force undermines firm's market power.

3.1.2 Partial Market Coverage

Next we consider the case where not the entire market is served: some poor consumers are left unserved owing to the greater distance from the firms. In this case the marginal poor consumer indifferent between the two firms prefers to go without buying the product. So each firm has some monopoly power over the poor since it does not compete with the other firm for the poor. In what follows we consider the situation where all rich consumers are served, some poor located closer to the firms are also served while other poor consumers are left out. It is here where the distinction between the travel cost faced by each individual becomes pronounced.

Demand faced by each firm from the rich is derived exactly the same way as above, that is, demand from the rich is given by the distance of the marginal rich indifferent between the two firms: $\frac{p_2 - p_1 + tL}{2t}$. But now each firm's demand from the poor is different: it is given by the distance from the firm where a poor becomes indifferent between buying and not buying. The distance of this indifferent poor consumer, d_p, is determined from $Y_P\theta - p_1 - td_p = Y_P$, that is, $d_p = \frac{Y_P(\theta-1) - p_1}{t}$. So the total demand faced by firm 1 is $D_1(.) = \frac{\delta_R[p_2 - p_1 + tL]}{2t} + \frac{\delta_P[Y_P(\theta-1) - p_1]}{t}$. Given this demand, firm $1's$ profit is

$$\pi_1 = [p_1 - c(\theta)]\left[\frac{\delta_R[p_2 - p_1 + tL]}{2t} + \frac{\delta_P[Y_P(\theta-1) - p_1]}{t}\right] - F(\theta).$$

The first-order condition for the profit maximization with respect to price implies

$$D_1(.) = \frac{\delta_R + 2\delta_P}{2t}[p_1 - c(\theta)].$$

After substitution and simplification this reduces to

$$p_1 = \frac{1}{2(\delta_R + 2\delta_P)}[\delta_R(p_2 + tL) + 2\delta_P Y_P(\theta - 1) + (\delta_R + 2\delta_P)c(\theta)].$$

Similar condition for firm 2 would imply a symmetric equilibrium with both firms charging the same price given by

$$p = \frac{1}{\delta_R + 4\delta_P}[\delta_R tL + 2\delta_P Y_P(\theta - 1) + (\delta_R + 2\delta_P)c(\theta)].$$

Price is independent of the income of rich, as firms are competing for them. But, since the firms have some monopoly power over the poor, equilibrium price increases with poor's income as the firms exploit their higher willingness to pay. This implies, that there could be different prices depending on income of the poor, inspite of the fact that the quality being offered is the same. Also price increases with increase in δ_R but falls with rise in δ_P. As δ_R increases firm's demand goes up causing price level to rise, whereas price level falls with increase in δ_P. With the rise in the proportion of poor, there are two opposing forces at work. Even though demand increases but, at the same time, population of poor being left out of the market also rises. It is the latter effect which prevails over the former and hence brings down the price level. This provides an interesting insight arising from the spatial nature of the model.

Using the first-order condition with respect to price the expression for profit reduces to

$$\pi_i = (\delta_R + 2\delta_P)[p_i - c(\theta)]D_i(.) - F(\theta) = (\delta_R + 2\delta_P)[p_i - c(\theta)]^2 \cdot \frac{1}{2t} - F(\theta),\ i = 1, 2.$$

On substituting for p in equilibrium firm's profit is given by

$$\pi_i = \frac{\delta_R + 2\delta_P}{2t}\left[\frac{\delta_R tL \; - \; 2\delta_P c(\theta) + 2\delta_P Y_P(\theta - 1)}{\delta_R + 4\delta_P}\right]^2 - F(\theta),\ i = 1, 2.$$

Observe that, unlike the case of full market coverage, proportions of rich and poor does not enter symmetrically in the firm's profit expression. Profit is more sensitive to the proportion of poor and also income of poor reflecting the fact that it is the poor whose coverage is partial.

In what follows we investigate the parameter values for which the above case arises. Recall that this case arises when all rich are being served, but some poor, depending on their distances from the firm, are left unserved. As above $\overline{Y} = \frac{p_1 + p_2 + tL}{2(\theta - 1)}$. Similarly for $j = 1, 2$, define $\underline{Y_j}$ to be the level of income such that the consumer even at the location of firm j is indifferent between buying and not buying, that is, $\underline{Y_j}(\theta - 1) = p_j$, implying $\underline{Y_j} = \frac{p_j}{\theta - 1}$. Clearly this above case occurs when $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$. Substituting the equilibrium values of price into the expression for $\overline{Y}$ and $\underline{Y}$ we find that $\underline{Y} < Y_P < \overline{Y}$ implies

$$\frac{\delta_R tL}{\delta_R + 2\delta_P} + c(\theta) < Y_P(\theta - 1) < \frac{(3\delta_R + 4\delta_P)tL}{2\delta_R + 4\delta_P} + c(\theta).$$

It is easy to check that both the cut-offs are increasing in δ_R and decreasing in δ_P. Increase in the lower bound with increase in δ_R simply implies that as the proportion of rich increases, some poor will be served only if Y_P is high enough. On the other end increase in the upper bound with increase in δ_R signifies that for all poor to be served Y_P should increase. The two together imply that increase in the proportion of the rich makes it less likely for all poor to be served. The intuition for this is straight forward. With increase in δ_R, firms demand increases implying that price level increases which raises the cost of consumption for poor. The opposite holds for the increase in δ_P.

Similarly, $Y_R > \overline{Y}$ implies

$$\frac{[2\delta_R Y_R + 4\delta_P(2Y_R - Y_P)](\theta - 1)}{2\delta_R + 4\delta_P} > \frac{(3\delta_R + 4\delta_P)tL}{2\delta_R + 4\delta_P} + c(\theta).$$

This implies that the partial market coverage is a possibility when the income gap is relatively higher. The following proposition, summarizes the above discussion.

Proposition 3: *A unique equilibrium in a subgame where both firms offer same quality, with the partial market coverage exists iff*

$$\frac{\delta_R tL}{\delta_R + 2\delta_P} + c(\theta) < Y_P(\theta - 1) < \frac{(3\delta_R + 4\delta_P)tL}{2\delta_R + 4\delta_P} + c(\theta)$$

and

$$\frac{[2\delta_R Y_R + 4\delta_P(2Y_R - Y_P)](\theta - 1)}{2\delta_R + 4\delta_P} > \frac{(3\delta_R + 4\delta_P)tL}{2\delta_R + 4\delta_P} + c(\theta)$$

and is characterized by the following properties.

1. *Poor at relatively higher distance from the firm are left unserved.*
2. *Both firms charge the same price,* $p_1 = p_2 = \dfrac{1}{\delta_R + 4\delta_P}[\delta_R tL + 2\delta_P Y_P(\theta - 1) + (\delta_R + 2\delta_P)c(\theta)] \equiv p_M$.
3. p_M *increases with increase in* Y_P *and* δ_R *but falls with increase in* δ_P.
4. *Profit for each firm is given by* $\pi = \dfrac{\delta_R + 2\delta_P}{2t}\left[\dfrac{\delta_R tL \; - \; 2\delta_P c(\theta) + 2\delta_P Y_P(\theta - 1)}{\delta_R + 4\delta_P}\right]^2 - F(\theta)$.
5. $p_C \; > \; p_M$, *that is price in the case of full market coverage is higher than the case of partial market coverage.*

As discussed above, this case arises when some poor are left unserved. It has been proved in Appendix A.1.1 and A.1.2 that p_M increases with increase in δ_R but falls with rise in δ_P. This has a direct implication on the welfare of rich. Rich are better-off staying in relatively poor neighborhood. Also, it warrants a mention that the price level under the partial market coverage is lower than the price under full market coverage. This is because price is sensitive to income of the poor, which is relatively low in the case of partial market coverage. This has been formally proved in Appendix A.1.3.

As above, the net surplus of the rich consumer located at a distance x from the firm from which it buys is given by $Y_R\theta - p - tx - Y_R$. As all rich are being served, the consumer surplus of rich is given by

$$CS_R \; = \; 2\delta_R \int_0^{\frac{L}{2}} \left[Y_R\left(\theta - 1\right) - p - tx\right] dx = \delta_R L \left[Y_R\left(\theta - 1\right) - p - \frac{tL}{4}\right].$$

As expected, consumer surplus increases with income Y_R and falls with the rise in price and travel cost. Because p falls with rise in δ_P and fall in Y_P, this implies that rich are better-off in a relatively poor neighborhood when quality level is fixed. Rise in the welfare with increase in Y_R and δ_R is obvious.

Finally consider the aggregate consumer surplus of the poor. Since the poor in between the distance $\dfrac{Y_P\left(\theta - 1\right) - p}{t}$ and $\dfrac{L}{2}$ does not buy the product from any firm, their consumer surplus is zero. Hence the aggregate consumer surplus of the poor is

$$CS_P = 2\delta_P \left[\int_0^{\frac{Y_P(\theta-1)-p}{t}} \left[Y_P\left(\theta - 1\right) - p - tx\right] dx\right] = \frac{\delta_P\left[Y_P\left(\theta - 1\right) - p\right]^2}{t}.$$

It is apparent from above that welfare of the poor is negatively related to the price level; which falls with the increase in the proportion of poor and rises with the increase in the proportion of rich. On substituting the equilibrium value of p welfare of the poor is given by

$$CS_P = \frac{\delta_P \left[(\delta_R + 2\delta_P)[Y_P(\theta-1) - c(\theta)] - \delta_R tL\right]^2}{t(\delta_R + 4\delta_P)^2}.$$

Note that even though p also rises with the rise in Y_P but the income effect dominates the price effect. This results in the increase in the welfare of the poor with rise in Y_P. This leads us to the following proposition.

Proposition 4: *For the case of partial market coverage, with both firms offering the same quality, welfare of rich consumers falls with increase in Y_P but rises with increase in δ_P, whereas the welfare of poor consumers rises with increase in both the income level Y_P and δ_P but falls with rise in δ_R.*

3.1.3 No Poor Being Served

At the other extreme is the scenario where no poor is served. This holds when Y_P is so low that firm does not find it worthwhile to serve them. On the other hand, all the rich consumers are served and firms are competing for them. In this case, firm $1's$ demand is given by

$$D_1(.) = \frac{\delta_R[p_2 - p_1 + tL]}{2t}.$$

Working exactly the same way as in the case of full market coverage, this implies that in equilibrium price is

$$p = tL + c(\theta).$$

This case arises when $Y_R > \overline{Y} = \frac{1}{\theta-1}\left[c(\theta) + \frac{3tL}{2}\right]$, and $\underline{Y} = \frac{p}{\theta-1} > Y_P$ implying that $Y_P < \frac{tL + c(\theta)}{\theta - 1}$. Above result can be summarized in the following proposition.

Proposition 5: *In a subgame where both firms offer the same quality, there exists a unique equilibrium with no poor being served iff $Y_P(\theta-1) < tL + c(\theta)$ and $Y_R > \overline{Y} = \frac{1}{\theta-1}\left[c(\theta) + \frac{3tL}{2}\right]$. The equilibrium characterized by the following properties.*

1. *No poor is served.*
2. *Both firms charge the same price, $p_1 = p_2 = tL + c(\theta) \equiv p_C$.*
3. *Market price increases with increase in t and $c(\theta)$, but does not depend on Y_P, Y_R, δ_R or δ_P.*

4. Each firm's profit is given by $\pi = \dfrac{\delta_R[tL]^2}{2t} - F(\theta)$.

It merits a mention that in this case the equilibrium price is the same as the first case of full market coverage. As income of poor is low, the firms completely ignore their presence and cater only to the rich. As a result the market size of each firm is smaller, affecting firm's profit adversely.

3.2 High quality by One Firm and Low by the Other

In this section we consider a subgame where the duopolistic firms operate with different quality levels so that one firm offers high quality, θ_H, and the other low quality, θ_L. Depending on the relative dominance of either travel cost (horizontal attribute) or income gap (vertical attribute) this may lead to the following subcases.

3.2.1 Vertical Dominance

Vertical Dominance arises when there is complete market segregation. A rich consumer, even at the location of the firm producing low quality, has a preference for high quality over low quality, that is, $Y_R\theta_H - p_H - tL > Y_R\theta_L - p_L$. Similarly, a poor consumer at the location of the firm offering high quality prefers low quality over high quality, that is, $Y_P\theta_L - p_L - tL > Y_P\theta_H - p_H$. Combining these two inequalities we get

$$(Y_R - Y_P)(\theta_H - \theta_L) > 2tL.$$

As is apparent from the equation, this case arises when the income and quality difference, that is, the vertical attribute, outweighs the travel cost, the horizontal attribute. We call this the *vertical dominance*. So the two forces, quality and income differences reinforce each other leading to this outcome. The rationale is that income and quality gaps are so high that rich are willing to travel all the way to access the high quality product, whereas poor, even at the location of high quality producing firm, find it beyond their means. Similarly, given the income and the quality gaps, each firm finds it more profitable to serve either type exclusively.

To investigate further, we determine the demand faced by each firm. As above, depending on the income of the poor, there can be either full market coverage or partial market coverage where some poor are left out. The two cases are discussed below.

3.2.2 Partial Market Coverage

We first analyze the case when some poor consumers are left unserved. Recall that the poor even at the location of the firm offering high quality prefers low quality over high quality. What follows is that only the poor located closer to the firm offering low quality are served, others are left out. Demand from the poor is given by the distance at which the marginal poor is indifferent between buying and not buying. As derived in the context of partial market coverage when both firms were offering same quality, this distance is $\frac{Y_P(\theta_L - 1) - p_L}{t}$, where p_L denotes the price charged by the firm offering low quality. Thus the firm has some monopoly power over the poor. Since no rich buys the low quality product, total demand faced by the firm offering low quality is $D_L(.) = \frac{\delta_P[Y_P(\theta_L - 1) - p_L}{t}$. Hence the profit of the firm offering low quality is

$$\pi_L = [p_L - c(\theta_L)]\frac{\delta_P[Y_P(\theta_L - 1) - p_L]}{t} - F(\theta_L).$$

The first-order condition for profit maximization with respect to price implies that $D_L(.) = \frac{\delta_P[p_L - c(\theta_L)]}{t}$. After simplification it follows that in equilibrium price charged by the firm offering low quality is

$$p_L = \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2}.$$

As the firm has some monopoly power over the poor, price charged increases in the income of the poor. Price also increases with increase in θ_L. On substituting for p_L, implied profit of the firm is

$$\pi_L = \frac{\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L).$$

There is partial market coverage when $L > \frac{Y_P(\theta_L - 1) - p_L}{t}$, that is, when the poor at the other end of the city is not willing to buy. Substituting for the equilibrium price, it follows that not all poor consumers will be served when $Y_P(\theta_L - 1) < c(\theta_L) + 2tL$. But there are some poor, located relatively closer to the firm offering low quality, who are willing to buy. Specifically, the poor at the location of the firm is better-off buying, which implies that $Y_P\theta_L - p_L > Y_P$. On substituting for p_L, the condition reduces to $Y_P > \frac{c(\theta_L)}{\theta_L - 1}$. Putting the two inequalities together, we get that there is partial market coverage when

$$\frac{2tL + c(\theta_L)}{\theta_L - 1} > Y_P > \frac{c(\theta_L)}{\theta_L - 1}.$$

Similarly we can evaluate the demand for the firm offering high quality. We assume that Y_R is high enough that all rich consumers are buying. For the case of vertical dominance rich buys only the high quality product. So the demand faced by the firm offering high quality is $\delta_R L$. It follows that the profit of the firm is

$$\pi_H = [p_H - c(\theta_H)]\delta_R L - F(\theta_H).$$

Pricing strategy of the firm is the following: the firm sets its price such that the marginal rich consumer, that is, the rich consumer at the location of the firm offering low quality, is indifferent between buying high quality and low quality products, that is $Y_R\theta_H - p_H - tL = Y_R\theta_L - p_L$. This implies that $p_H = Y_R(\theta_H - \theta_L) + p_L - tL$. This allows the firm to extract the maximum possible surplus from the consumers. The pricing strategy above is plausible as the competition between the firms has been relaxed for two reasons. First, the two firms now offer two distinct qualities. Secondly, as the income gap is substantial, there is a market segregation. Price and profit level for the firm offering high quality is obtained by a simple substitution for p_L. So the price level of the firm offering high quality is given by

$$p_H = Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL.$$

It is easy to check that p_H increases with increase in Y_R and θ_H. With the rise in income and quality consumer's willingness to pay increases and this results in the higher price. With rise in θ_L, there are three forces at work. The first is the valuation effect which increases p_L. The second is the cost effect. These two together put an upward pressure on p_H. Finally, there is the competition effect: as the quality differentiation between the firms falls, competition increases. This lowers p_H. Which effect dominates depends of the curvature of the cost curve. Similarly, an increase in Y_P has positive spill over effect on p_H. As poor constitutes captive market of the firm offering low quality so an increase in Y_P results in rise in p_L. Because of the higher p_L, the high quality firm can charge a higher price that makes the marginal rich indifferent between the two quality products. The fact that p_H falls with increase in the travel cost may appear counterintuitive. But higher the travel cost lower the surplus that the firm can take away from the rich consumer to make him indifferent between the two firms. Therefore the price falls. Finally the high quality firm's profit is given by

$$\pi_H = \left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right]\delta_R L - F(\theta_H).$$

The equilibrium above has been worked out assuming that all rich are served and at least some poor are buying. This implies that Y_R has to be high enough so that the rich even at the location of the firm offering low quality is willing to buy, that is, $Y_R\theta_H - p_H - tL > Y_R$.

After substituting for p_H, the condition reduces to: $[2Y_R - Y_P](\theta_L - 1) > c(\theta_L)$. Above results are summarized in the following proposition.

Proposition 6: *An equilibrium with vertical dominance exists when* $(Y_R - Y_P)(\theta_H - \theta_L) > 2tL$. *When* $\dfrac{2tL + c(\theta_L)}{\theta_L - 1} > Y_P > \dfrac{c(\theta_L)}{\theta_L - 1}$ *and* $[2Y_R - Y_P](\theta_L - 1) > c(\theta_L)$ *there is partial market coverage. For this equilibrium we have*

1. $p_H = Y_R(\theta_H - \theta_L) + \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL,$
2. $p_L = \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2},$
3. $\pi_L(.) = \dfrac{\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L),$
4. $\pi_H(.) = \left[Y_R(\theta_H - \theta_L) + \dfrac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right]\delta_R L - F(\theta_H).$
5. *p_H increase with the increase in Y_R, Y_P and θ_H. p_L increases with increase in θ_L and Y_P and is independent of Y_R. Both prices are unaffected by the change in the relative proportions.*

For this to qualify as an equilibrium, neither of the firms should have any incentive to deviate. Conditions for this have been laid down in Appendix A.2. It is shown that the proposed price strategy will qualify to be an equilibrium only when the income gap is substantial. This is intuitive: only for the relatively larger income gap each producer can benefit by serving the segregated market.

To calculate the net consumer surplus, we proceed as in the previous sections. In the case of vertical dominance there is complete market segregation. So the net consumer surplus of a rich consumer located at a distance x from the firm offering high quality is $Y_R\theta_H - p_H - tx - Y_R$. Since all the rich consumers buy from the firm offering high quality, the aggregate consumer surplus for the rich is given by

$$CS_R = \delta_R \int_0^L [Y_R(\theta_H - 1) - p_H - tx]\, dx = \delta_R L\left[Y_R(\theta_H - 1) - p_H - \frac{tL}{2}\right].$$

Surplus increases with Y_R and falls with p_H. After substituting for p_H consumer surplus for the rich reduces to

$$\delta_R L\left[\frac{(2Y_R - Y_P)(\theta_L - 1)}{2} - \frac{c(\theta_L)}{2} + \frac{tL}{2}\right].$$

Following observations warrant a mention. First, surplus for the rich is independent of θ_H. With increase in θ_H, utility of rich goes up because of the valuation effect. But increase in θ_H further raises the intensity of vertical dominance. This allows the firm to extract the entire surplus from the rich consumers. For the reason outlined above, surplus increases with increase in tL. Change in surplus of the rich consumers with increase in θ_L depends on how price changes with change in θ_L.

Next we evaluate the consumer surplus of poor. Surplus of poor when not all them are served is given by

$$CS_P = \delta_P \int_0^{\frac{Y_P(\theta_L - 1) - p_L}{t}} \left[Y_P\left(\theta_L - 1\right) - p_L - tx\right] dx = \frac{\delta_P \left[Y_P\left(\theta_L - 1\right) - p_L\right]^2}{t}.$$

On substituting for p_L this reduces to

$$CS_P = \frac{\delta_P [Y_P\left(\theta_L - 1\right) - c(\theta_L)]^2}{4t}.$$

As expected, consumer surplus increases with the increase in the income level of poor. Although increase in Y_P allows the firm to increase price, the valuation effect dominates this price effect, leading to an increase in the net consumer surplus. As discussed earlier, change in the consumer surplus with a change in quality depends on the assumption on the marginal cost curve. When the marginal cost is linear in θ, consumer surplus goes up with increase in quality. Observe that the consumer surplus is independent of Y_R or θ_H. The following proposition summarizes the results on consumer surplus.

Proposition 7. *Suppose that, under vertical dominance, there exists an equilibrium with partial market coverage. Then*

1. *the consumer surplus of rich increases with increase in Y_R and t but falls with increase in Y_P, and*
2. *the consumer surplus of poor is unaffected with the changes in Y_R or θ_H, increasing in Y_P, but decreasing in transportation cost.*

3.2.3 Full Market Coverage

There is full market coverage when the poor at the other end of the city is willing to travel all the way to buy low quality product. Demand for the firm offering low quality is $\delta_P L$. Given that the firm is a monopolist with respect to the poor, it is able to extract the maximum possible surplus from them. It charges the price such that the marginal poor is indifferent between buying and not buying that is, $p_L = Y_P(\theta_L - 1) - tL$. Following the similar analysis as above it follows that there is full market coverage when $Y_P > \dfrac{2tL + c(\theta)}{\theta_L - 1}$. Demand and the pricing strategy for the firm offering high quality remains the same as under partial market coverage. Results are summarized in the following proposition.

Proposition 8: *An equilibrium with Vertical Dominance exists when* $(Y_R - Y_P)(\theta_H - \theta_L) > 2tL$. *There will be full market coverage when* $\dfrac{2tL + c(\theta_L)}{\theta_L - 1} < Y_P$. *For the case of full market coverage we have*

1. $p_H \;=\; Y_R(\theta_H - \theta_L) + Y_P(\theta_L - 1) - 2tL,$
2. $p_L \;=\; Y_P(\theta_L - 1) - tL,$
3. $\pi_L(.) = \delta_P L[Y_P(\theta_L - 1) - tL - c(\theta_L)] - F(\theta_L),$
4. $\pi_H(.) = \left[Y_R(\theta_H - \theta_L) + Y_P(\theta_L - 1) - 2tL - c(\theta_H) \right] \delta_R L - F(\theta_H).$

It has been shown in the Appendix A.3 that the equilibrium with the full market coverage will not exists as the firm offering low quality will have an incentive to deviate. This is because income of both poor and rich is high enough that increase profit due to increase in the market coverage outweighs the loss coming from the reduced price.

3.2.4 Horizontal Dominance

As opposed to vertical dominance, horizontal dominance arises when both the firms serve both income groups. The rich consumer at the location of the firm producing low quality prefers low quality rather than traveling all the way to the other end of the city for high quality implying that $Y_R\theta_L - p_L > Y_R\theta_H - p_H - tL$. Given their income, the rich consumers do not perceive quality difference to be high enough to refrain from buying the low quality product. Similarly, the poor consumer at the location of the firm producing high quality prefers high quality over low quality available at the other end implying that $Y_P\theta_H - p_H > Y_P\theta_L - p_L - tL$. Together the two inequalities imply

$$2tL \;>\; (Y_R - Y_P)(\theta_H - \theta_L).$$

This inequality lends important insight to understand the mechanism. This case arises when travel cost is significantly large as compared to the income differences. Because of this consumers prefer to settle for the quality available close by to avoid the high travel cost. When the relative income gap is not substantial and services are available at relatively larger distance individuals prefer to buy whatever quality is easily accessible. To characterize the nature of equilibrium, we again look at two situations: full market coverage and partial market coverage.

3.2.5 Full Market Coverage

We first determine demand for the high quality product arising from the poor consumers. Let the distance of the marginal poor consumer from the firm offering high quality be x. x is determined from $Y_P\theta_H - p_H - tx = Y_P\theta_L - p_L - t(L - x)$. Thus the demand from poor consumers for the high quality product is

$$D_P = \frac{\delta_P\{Y_P(\theta_H - \theta_L) - (p_H - p_L) + tL\}}{2t}.$$

Demand from rich consumers is determined in a similar way. So total demand faced by the firm offering high quality is

$$D_H(.) = \frac{(Y_R\delta_R + Y_P\delta_P)(\theta_H - \theta_L) - (\delta_R + \delta_P)[p_H - p_L + tL]}{2t}.$$

Observe that the firm offering high quality serves relatively larger proportion of rich than poor. This is because the rich, by the virtue of their higher income, have relatively higher preference for better quality. This makes them more willing to travel greater distance for the better quality product. Profit for the firm is

$$\pi_H(.) = [p_H - c(\theta_H)]\frac{(Y_R\delta_R + Y_P\delta_P)(\theta_H - \theta_L) - (\delta_R + \delta_P)[p_H - p_L + tL]}{2t} - F(\theta_H).$$

The first-order condition for profit maximization with respect to price implies $D_H(.) = \frac{(\delta_R + \delta_P)[p_H - c(\theta_H)]}{2t}$. On substitution and simplification it follows that $2(\delta_R + \delta_P)p_H = (\delta_R Y_R + \delta_P Y_P)(\theta_H - \theta_L) + (\delta_R + \delta_P)[p_L + tL + c(\theta_H)]$. Similar exercise for the firm offering low quality implies that in equilibrium

$$p_L = \frac{1}{3(\delta_R + \delta_P)}\left\{ (\delta_R Y_R + \delta_P Y_P)(\theta_L - \theta_H) + (\delta_R + \delta_P)[3tL + 2c(\theta_L) + c(\theta_H)]\right\}$$

and

$$p_H = \frac{1}{3(\delta_R + \delta_P)}\left\{ (\delta_R Y_R + \delta_P Y_P)(\theta_H - \theta_L) + (\delta_R + \delta_P)[3tL + 2c(\theta_H) + c(\theta_L)]\right\}.$$

It is worth observing that p_L falls whereas p_H increases with the rise in general income level and the relative proportion of the rich. The intuition for this comes from the preference structure. With the rise in the general income level, firm offering high quality attracts more consumers at the expense of the firm offering low quality. An increase in the market demand leads to higher price for the high quality product. Opposite is true for the firm offering low quality. Profit for each firm is obtained by simple substitution of the price level.

The cut-off level of Y_P that ensures full market coverage under horizontal dominance is determined as follows. Utility of the marginal poor individual buying from firm offering high quality is

$$U(Y_P, p_H, \theta_H) = Y_P\theta_H - p_H - \frac{t[Y_P(\theta_H - \theta_L) - (p_H - p_L) + tL]}{2t}.$$

As the marginal poor consumer is better-off buying, his utility from consumption is greater than his reservation utility Y_P. This implies that there will be full market coverage if

$$Y_P \geq \frac{3tL + c(\theta_H) + c(\theta_L)}{(\theta_H - 1) + (\theta_L - 1)}.$$

Above results can be summarized in the following proposition.

Proposition 9: *An equilibrium with horizontal dominance exists iff* $(Y_R - Y_P)(\theta_H - \theta_L) < 2tL$. *There will be full market coverage iff* $Y_P \geq \dfrac{3tL + c(\theta_H) + c(\theta_L)}{(\theta_H - 1) + (\theta_L - 1)}$. *The equilibrium is characterized by the following properties.*

1. *All poor are served.*
2. $p_H = \dfrac{1}{3(\delta_R + \delta_P)}\Big\{ (\delta_R Y_R + \delta_P Y_P)(\theta_H - \theta_L) + (\delta_R + \delta_P)[3tL + 2c(\theta_H) + c(\theta_L)]\Big\}.$
3. $p_L = \dfrac{1}{3(\delta_R + \delta_P)}\Big\{ (\delta_R Y_R + \delta_P Y_P)(\theta_L - \theta_H) + (\delta_R + \delta_P)[3tL + 2c(\theta_L) + c(\theta_H)]\Big\}.$
4. p_H *increases with the increase in* Y_R, Y_P, δ_R *and* δ_P *where as* p_L *falls with the increase in* Y_R, Y_P, δ_R *and* δ_P.
5. $\pi_L(.) = \dfrac{\Big[\{\delta_R Y_R + \delta_P Y_P\}(\theta_L - \theta_H) + (\delta_R + \delta_P)[3tL + c(\theta_H) - c(\theta_L)]\Big]^2}{18t(\delta_R + \delta_P)^2} - F(\theta_L).$
6. $\pi_H(.) = \dfrac{\Big[\{\delta_R Y_R + \delta_P Y_P\}(\theta_H - \theta_L) + (\delta_R + \delta_P)[3tL + c(\theta_L) - c(\theta_H)]\Big]^2}{18t(\delta_R + \delta_P)^2} - F(\theta_H).$

To calculate the aggregate consumer surplus for the case of horizontal dominance we proceed as follows. Observe that both firms serve both rich and poor. So the surplus of the consumers also depends on, which firm they buy from. The net consumer surplus to the rich buying quality θ_i, where $\theta_i \in \{\theta_H, \theta_L\}$, located at the distance x from the firm from which he is buying is $Y_R\theta_i - p_i - tx - Y_R$. The rich consumers upto the distance $\dfrac{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL}{2t}$ buy from the firm offering high quality, whereas the remaining rich consumers buy the low quality product. The aggregate consumer surplus for rich is given by

$$CS_R = \delta_R \int_0^{X_1} \left[Y_R\left(\theta_H - 1\right) - p_H - tx\right] dx + \delta_R \int_0^{X_2} \left[Y_R\left(\theta_L - 1\right) - p_L - tx\right] dx,$$

where $X_1 = \dfrac{\{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL\}}{2t}$ and $X_2 = \dfrac{\{Y_R(\theta_L - \theta_H) - (p_L - p_H) + tL\}}{2t}$.

As there are many opposing forces at work, it is difficult to say anything conclusive about the change in aggregate consumer surplus with changes in the general income level or relative proportions. To get some idea let us first look at the consumer surplus of the rich already buying from the firm offering high quality. With the rise in Y_R there are two opposite effects that influence this surplus. First, utility of the rich increases because of the valuation effect. This raises the surplus. But, with increase in Y_R, p_H also increases leading to a fall in the surplus. For $\theta_H - 1 > \dfrac{\delta_R(\theta_H - \theta_L)}{3}$, the former effect dominates, resulting in the rise in the surplus. We also need to check how the fraction of rich consumers served by the firm offering θ_H changes with the rise in Y_R. It is easy to calculate that the ratio of the rich served increases if $\dfrac{3}{2} > \delta_R$. So it implies that the surplus of the rich being served by the firm offering high quality increases with increase in Y_R.[7] Surplus of the rich buying low quality increases unambiguously. This is because p_L falls with the increase in Y_R. Thus the two effects, income and price effects, reinforce each other. But there is fall in the fraction of rich served by the firm offering low quality as $\dfrac{3}{2} > \delta_R$.

Similarly one can evaluate change in the surplus of rich with increase in Y_P. Intuition spelt out in the last paragraph continues to help. With an increase in Y_P there only price affect that determines the surplus of the rich continuing to buy from the same firm. Clearly p_H increases with increase in Y_P leading to a fall in the surplus of the consumers continuing to buy from the firm offering θ_H. Also the number of rich consumers buying from the firm offering high quality falls. On the other hand both the surplus and fraction of rich buying from the firm offering low quality increases. Because of the opposing forces at work it is difficult to conclude about the overall impact on the surplus of the rich with a rise in Y_P.

With an increase in δ_R and δ_P, p_H increases while p_L falls implying a corresponding change in the surplus of the consumers continuing to buy from the respective firms. Also, there is redistribution of consumers between firms with change in price.

[7] Note $\delta_R < \dfrac{3}{2}$ implies $\theta_H - 1 > \dfrac{\delta_R(\theta_H - \theta_L)}{3}$.

Consumer surplus of the poor is given by

$$CS_P = \left[\int_0^{Z_1} \left[Y_P\left(\theta_L - 1\right) - p_H - tx\right] dx\right] + \left[\int_0^{Z_2} \left[Y_P\left(\theta_L - 1\right) - p_L - tx\right] dx\right],$$

where $Z_1 = \dfrac{\{Y_P(\theta_H - \theta_L) - (p_H - p_L) + tL\}}{2t}$ and $Z_2 = \dfrac{\{Y_P(\theta_L - \theta_H) - (p_L - p_H) + tL\}}{2t}$.

Similar logic works if one looks at the changes in consumer surplus of poor owing to the changes in Y_P, Y_R, δ_P and δ_R. With the rise in Y_P market share of the firm offering high quality increases at the expense of the market share of the firm offering low quality. Also, with an increase in Y_P price of θ_H rises but that of θ_L falls. So the overall result is ambiguous. What one can infer conclusively from the above analysis is that with the increase in income there is a redistribution of market shares in favor of the firm offering high quality.

3.2.6 Partial Market Coverage

When the income level of the poor is low then some poor, those who are farther away from either firm, might be left unserved. The firms compete for the rich but have some monopoly power over the poor. Demand faced by each firm from the rich is derived exactly the same way as above, that is, the demand from the rich consumer for the firm offering high quality is given by the distance of the marginal consumer indifferent between the two firms. So the demand from the rich is given by $\dfrac{\delta_R[Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL]}{2t}$. Firm's demand from the poor is determined by the distance such that poor becomes indifferent between buying and not buying, that is, $\dfrac{\delta_P[Y_P(\theta_H - 1) - p_H]}{t}$. Total demand for the firm producing high quality is then given by

$$D_H(.) = \frac{\delta_R\{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL\} + 2\delta_P\{Y_P(\theta_H - 1) - p_H\}}{2t}.$$

The first order condition with respect to price implies $D_H(.) = \dfrac{(\delta_R + 2\delta_P)[p_H - c(\theta_H)]}{2t}$. Similarly we derive the price response for the firm offering low quality.

This case is worked out in the similar way as the case of partial market coverage with both firms offering the same quality. Once the equilibrium price level is solved, we investigate the parameter values for which the above case arises. As above this possibility arises when $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$. As the algebra is quite involved, details of the analysis are given in the Appendix A.4. The main results are summarized in the following proposition.

Proposition 10: *An equilibrium with the partial market coverage exists if*

$$\frac{2tL\delta_R + \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P)}{[\delta_R + 2\delta_P][(\theta_L - 1) + (\theta_H - 1)]} < Y_P < \frac{[3\delta_R + 4\delta_P]tL + (\delta_R + 2\delta_P)[(c(\theta_L) + c(\theta_H)]}{[\delta_R + 2\delta_P][(\theta_L - 1) + (\theta_H - 1)]}$$

and

$$\{Y_R[\delta_R + 4\delta_P] - 2\delta_P Y_P\}[(\theta_H - 1) + (\theta_L - 1)] > \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P) + tL[3\delta_R + 4\delta_P],$$

and is characterized by the following properties.

1. *All rich are served. Poor at relatively higher distance from either firm are left unserved.*
2. *Both p_H and p_L increase with the increase in Y_P. But, when Y_R increases, p_H increases while p_L falls.*

As poor constitutes the captive market for the firms both the prices charged increase with their income. As income of the rich goes up, market demand for the firm offering high quality goes up at the expense of the firm offering low quality. As a result it is optimal for the firm offering high quality to raise its price, whereas for the firm offering low quality to reduce its price.

It is difficult to calculate the change in the consumer surplus with change in relative proportions of rich and poor as the algebra is quite involved. What is unambiguous though is that price charged by both the firms increase with the increase in income of the poor. This implies that welfare of the poor already buying high or low quality products decreases due to the price effect.

The two cases, vertical dominance and horizontal dominance, illustrate how income disparity interacts with travel cost to determine the pattern of equilibrium. Even with apparently the same outcome with one firm offering high quality and the other low, there are differences with respect to the size and the type of customers each firm serves depending on the level and extent of income inequality. This has its bearing on the welfare of the consumers.

3.2.7 Intermediate Case

An intermediate case arises when one firm serves both the income types whereas the other firm serves just one income group. The first possibility we consider is where the firm producing low quality θ_L serves both the income types, but the firm producing θ_H serves only the rich. This implies $Y_R\theta_L - p_L > Y_R\theta_H - p_H - tL$ and $Y_P\theta_L - p_L - tL > Y_P\theta_H - p_H$. The first inequality says that the firm offering low quality serves some rich consumers as well. The second inequality says that poor even at the extreme prefers low quality to high quality. Together these two conditions imply

$$p_H - p_L > \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2}.$$

It has been shown in the Appendix A.5 that in equilibrium when there is full market coverage this condition implies

$$\frac{c(\theta_H) - c(\theta_L)}{3} > \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2} - \frac{2Y_R(\theta_H - \theta_L)}{3} + \frac{2tL\delta_P}{3\delta_R}$$

$$\Rightarrow \frac{3\delta_R}{\delta_P}\left[\frac{c(\theta_H) - c(\theta_L)}{3} - \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2} + \frac{2Y_R(\theta_H - \theta_L)}{3}\right] > 2tL.$$

This highlights the scenario when the high marginal cost limits the access of the poor to the high quality product. For example in the case of the medical services, high-tech assistance might only exasperate the cost of services, making it unviable for the poor. This case may coincide with either vertical or horizontal dominance thus giving rise to the possibility of multiple equilibria. This is clear from the following inequalities. Define $\Omega = \left[\frac{c(\theta_H) - c(\theta_L)}{3} - \frac{[Y_R + Y_P](\theta_H - \theta_L)}{2} + \frac{2Y_R(\theta_H - \theta_L)}{3}\right]$. It will be the case of vertical dominance if either

$$[Y_R - Y_P](\theta_H - \theta_L) \geq \frac{3\delta_R\Omega}{\delta_P} > 2tL$$

or

$$\frac{3\delta_R\Omega}{\delta_P} > [Y_R - Y_P](\theta_H - \theta_L) \geq 2tL$$

holds, whereas horizontal dominance occurs if we have

$$\frac{3\delta_R\Omega}{\delta_P} > 2tL > [Y_R - Y_P](\theta_H - \theta_L).$$

Given horizontal dominance, an increase in the proportion of rich raises the possibility of the intermediate case: firm offering high quality serves only the rich, whereas firm offering low quality serves both income groups. Given horizontal dominance, as proportion of rich increases, price charged by the firm producing high quality also rises. Higher prices forces the poor to go away without consuming, raising the possibility of intermediate equilibrium.

The poor are worse-off as they now have access to only the low quality product. Similarly given vertical dominance, an increases in the proportion of rich again raises the possibility of intermediate case. This is because with the increase in the price of the high quality product, rich may now be forced to buy the low quality product as well.

The condition under which this equilibrium can be sustained is stated in Appendix A.5. The opposite case, that is, when firm producing high quality serves both income types but the firm producing low quality serves only poor, cannot be sustained in an equilibrium. The fact that the high quality firm serves both income groups implies that income difference is not substantial. In this case the low quality firm can strategically deviate and be better-off.

4 Quality Stage and Equilibrium Outcomes

In the last section we have discussed the various possibilities that will arise in the second stage of the game, the price stage. In this section we consider the first stage of the game, the quality stage where each firm decides which quality to offer, θ_H or θ_L. The two stages combined together gives us all the possible equilibrium outcomes that may arise in this game under consideration.

To convey the main message of the analysis in a cleaner way we introduce two simplifications in this section. The first one is a simplifying assumption on the structure of the marginal cost, $c(\theta)$. The simplest possibility is to assume that the marginal cost is linear in quality. Even if the marginal cost is convex in quality, our presentation of the quality stage of the game will be simplified if we assume that the increase in the gross utility owing to an improvement in quality more than outweighs the loss due to a higher marginal cost. For example, consider the lower bound of the poor income in Proposition 5 below which no poor is served when both firms offer the same quality, $\frac{tL + c(\theta)}{\theta - 1}$. Our simplifying assumption would imply that $\frac{tL + c(\theta_H)}{\theta_H - 1} < \frac{tL + c(\theta_L)}{\theta_L - 1}$, that is, if no poor is served when both firms offer the high quality, then same will happen to the poor when both firms offer the low quality also. The second simplification is just for the purpose of exposition: we discuss only the case of vertical dominance for the case of the partial market coverage. As will be clear from the following analysis, the nature of equilibrium outcomes is very similar under the case of horizontal dominance but involves a lot of tedious algebraic expressions without adding much to our understanding.

From the analysis in the last section let us first summarize the relevant thresholds for Y_P under which different equilibrium possibilities arise. Proposition 5 defines the lower bound for Y_P below which no poor is served when both firms offer the same quality, $\dfrac{tL + c(\theta)}{\theta - 1}$. Define

$$Y_P^1 \equiv \frac{tL + c(\theta_H)}{\theta_H - 1}.$$

Since our simplifying assumption above implies that $\dfrac{tL + c(\theta_H)}{\theta_H - 1} < \dfrac{tL + c(\theta_L)}{\theta_L - 1}$, it follows that if $Y_P < Y_P^1$ then poor consumers are completely excluded when the firms offer the same quality, no matter whether that common quality is high or low.

Similarly Proposition 1 identifies the upper bound for Y_P such that when both firms offer the same quality all the poor consumers are served only if $Y_P > \dfrac{1}{\theta - 1}\left[c(\theta) + \dfrac{3tL}{2}\right]$. Define

$$Y_P^2 \equiv \frac{1}{\theta_H - 1}\left[\frac{3tL}{2} + c(\theta_H)\right].$$

Since our simplifying assumption implies that $\dfrac{1}{\theta_H - 1}\left[c(\theta_H) + \dfrac{3tL}{2}\right] < \dfrac{1}{\theta_L - 1}\left[c(\theta_L) + \dfrac{3tL}{2}\right]$, it follows that if $Y_P^1 < Y_P < Y_P^2$ then there could only be partial market coverage of the poor consumers when the firms offer the same quality.

Finally, Proposition 6 defines the upper cut-off level for Y_P above which all poor consumers are served when there is vertical dominance. Let us define this upper cut-off as

$$Y_P^3 \equiv \frac{3tL + c(\theta_L) + c(\theta_H)}{(\theta_H - 1) + (\theta_L - 1)}.$$

Observe that our assumption on the marginal cost structure guarantees that $Y_P^2 < Y_P^3$.

These income thresholds and the fact that fixed cost depends on product quality ($F(\theta_H) > F(\theta_L)$) impose restrictions on the quality that the firms can offer. There are various forces that influence firms' decisions. In the price stage we have observed the trade-off in terms of market coverage: a firm can charge a higher price to take advantage of the higher willingness to pay of the rich consumers, but, in the process, it will lose its market size by losing the poor consumers. In the quality stage, the opportunity to offer different quality products, can relax the price competition. But here too it has to weigh its choices given the existence of different pressures. First, market share of the firm offering low quality might shrink since individuals prefer high quality over low quality. Second, the high-quality firm also may lose some market share since a poor consumer with not too intense preference over quality (due to relatively lower income) can now opt for the lower quality product. Third, even though higher quality induces higher willingness to pay, to provide the higher quality firm has to bear a higher fixed cost also.[8] In what follows we use the thresholds for Y_P, the trade-off over market shares in the price stage and the trade-off over market shares and fixed costs in the quality stage to characterize the possible equilibrium outcomes.

4.1 $Y_P < Y_P^1$

When $Y_P < Y_P^1$, if both firms offer the same quality, the poor consumer even at the location of the firm cannot afford to buy the product, no matter what the common quality is. On the other hand since $\frac{c(\theta_L)}{\theta_L - 1} < Y_P^1$, it follows from Proposition 6 that when the two firms offer two different quality products, some of the poor consumers residing closer to the firm offering the low quality product can buy the low quality product. So the question is to investigate what quality profile will prevail in equilibrium.

Consider first whether both firms offering low quality, that is, the quality profile (θ_L, θ_L), is an equilibrium outcome. As stated in Proposition 5, profit of each firm is $\pi(\theta_L, \theta_L) = \frac{\delta_R [tL]^2}{2t} - F(\theta_L)$. Suppose one firm deviates and offers the high quality, θ_H. To find out the profit under this deviation we have to consider the price equilibrium followed by the subgame (θ_H, θ_L) under vertical dominance. Since $Y_P < Y_P^1 < \frac{2tL + c(\theta_L)}{\theta_L - 1}$, there can only be partial market coverage of the poor as long as $\frac{c(\theta_L)}{\theta_L - 1} < Y_P$. Assuming this it follows from Proposition 6 that the profit of the deviating firm will be $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) = \left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] \delta_R L - F(\theta_H)$. Clearly the firm will deviate if $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) > \pi(\theta_L, \theta_L)$. So we can conclude that when $Y_P < Y_P^1$, (θ_L, θ_L) is an equilibrium outcome if $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) \leq \pi(\theta_L, \theta_L)$, that is, if

$$\left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right] \delta_R L - \frac{\delta_R [tL]^2}{2t} \leq F(\theta_H) - F(\theta_L). \quad (1)$$

Consider next whether (θ_H, θ_H) could be an equilibrium outcome. Proposition 5 says that each firm's profit is $\pi(\theta_H, \theta_H) = \frac{f_R [tL]^2}{2t} - F(\theta_H)$. Suppose one firm deviates and offers the low quality, θ_L, so that the relevant subgame is once again (θ_L, θ_H) under vertical dominance. By the same logic given above, there could only be partial market coverage of the poor so that, following Proposition 6, the profit of the deviating firm is $\pi_L|_{\text{deviation}}(\theta_L, \theta_H) =$

[8] See Coibon and Hallack (2007) for the theoretical determinants of substantial differences in demand elasticities and associated markups among products of heterogeneous quality. He also refers to empirical studies like Bresnahan (1987), which have reported substantial variation in estimated price elasticities of demand and associated markups across products for oligopolistic industries.

$\dfrac{\delta_P[Y_P(\theta_L-1)-c(\theta_L)]^2}{4t}-F(\theta_L)$. Hence we conclude that (θ_H,θ_H) is an equilibrium outcome if $\pi_L|_{\text{deviation}}(\theta_L,\theta_H)\le\pi(\theta_H,\theta_H)$, that is, if

$$F(\theta_H)-F(\theta_L)\le\frac{\delta_R[tL]^2}{2t}-\frac{\delta_P[Y_P(\theta_L-1)-c(\theta_L)]^2}{4t}. \tag{2}$$

Finally consider whether firms could offer different qualities in equilibrium, that is, whether (θ_H,θ_L) or (θ_L,θ_H) could be equilibrium outcomes. We consider only (θ_H,θ_L) since the other case is just symmetric. From the above analysis it is clear that the high quality producing firm will have no incentive to deviate if $\pi_H(\theta_H,\theta_L)\ge\pi(\theta_L,\theta_L)$, that is, if $F(\theta_H)-F(\theta_L)\le\left[Y_R(\theta_H-\theta_L)+\dfrac{Y_P(\theta_L-1)+c(\theta_L)}{2}-tL-c(\theta_H)\right]\delta_RL-\dfrac{\delta_R[tL]^2}{2t}$, whereas the low quality producing firm will have no incentive to deviate if $\pi_L(\theta_H,\theta_L)\ge\pi(\theta_H,\theta_H)$, that is, if $\dfrac{\delta_R[tL]^2}{2t}-\dfrac{\delta_P[Y_P(\theta_L-1)-c(\theta_L)]^2}{4t}\le F(\theta_H)-F(\theta_L)$. Combining the two inequalities we conclude that (θ_H,θ_L) is an equilibrium outcome if

$$\begin{aligned}\frac{\delta_R[tL]^2}{2t}-\frac{\delta_P[Y_P(\theta_L-1)-c(\theta_L)]^2}{4t}&\le F(\theta_H)-F(\theta_L) \qquad (3)\\ &\le\left[Y_R(\theta_H-\theta_L)+\frac{Y_P(\theta_L-1)+c(\theta_L)}{2}-tL-c(\theta_H)\right]\delta_RL-\frac{\delta_R[tL]^2}{2t}.\end{aligned}$$

We would like to highlight two points that that emerge from inequalities (1), (2) and (3) above. The first observation relates to the difference in the fixed costs of producing the two quality products, $F(\theta_H)-F(\theta_L)$. Given all the other parameter values, both the firms offer the low quality if the difference in fixed costs is too high (inequality (1)) and offer the high quality if the difference in fixed costs is low enough (inequality (2)), whereas, for an intermediate difference in fixed costs, one firm offers high quality while the other goes for low quality.

The second observation is on the income of the rich, Y_R, and their relative proportion in the city population, f_R, given, of course, all other parameter values, in particular, given the poor income and the difference in fixed costs. Given the quite low income of the poor, if either the rich income is reasonably low (inequality (1)) or the proportion of rich is very high (inequality (2)), the poor consumers are completely shut out of the market. Both the firms completely ignore their presence and offer the low quality product if the rich income is low and the high quality product if the density of the rich is high. Interestingly, for a moderate level of rich income and the proportion of rich when (inequality 3) is satisfied, poor are better off as at least some poor consumers residing closer to the firm producing low quality product gets access to it.

The purpose of our analysis is to investigate how the level of income and the inequality in its distribution affect the equilibrium outcome in terms of the quality offered and price charged by each firm and then to see its implications on the welfare of the consumers. Of particular interest is the welfare of the people in the lowest income category fallen deeper into poverty with long-term consequences on their health and education and thus on their future earning potential. When the poor income is so low, the poor consumer even at the location of the firm is not able to afford the product. Only the rich constitutes the market for the firms. Utility of the poor consumers is their reservation utility given by their income level Y_P. For a relatively higher level of Y_R, poor are better-off as few of them can access the product that was earlier unavailable. For a relatively high proportion of the rich, there is again a paradigm shift with both firms offering the high quality product and ignoring the poor consumers in the process. This analysis implies that in case of extreme deprivation the poor might be better-off being with the rich as at least few of them can access the product or service which was earlier beyond their reach. But as the income gap widens further, their welfare reduces to the same level as again all of them are priced out of the market. It is the combination of both the factors, low proportion and low income level of poor, that leads to this kind of outcome. From the above analysis it follows that the welfare of the poor initially increases and then falls as there is income growth (arising due to rise in income and density of rich). It seems that the welfare of the poor shows an inverted U-shaped pattern in the income of the rich.

This pattern of growth initially helps but later penalizes the poor, especially if the products or services under consideration are the merit goods like health or education, as the accessibility and quality of these services determines individual's earning capacity in future as well. So the already marginalized section of the society finds itself trapped into the vicious circle of poverty. The extreme case that one may consider is to look at the mortality rate of poor. By one estimate, in India, the infant mortality rate is 2.5 times higher among the poorest 20% of the society than among the richest 20% (Deogaonkar, 2004).

4.2 $Y_P^1 < Y_P < Y_P^2$

When $Y_P^1 < Y_P < Y_P^2$, it follows from Proposition 1 that there could only be partial market coverage of the poor consumers when the firms offer the same quality. Similarly since $Y_P^2 < Y_P^3$, Proposition 6 implies that there will be partial market coverage of the poor consumers when the firms offer different qualities too.

Consider first whether (θ_L, θ_L) is an equilibrium outcome. Since there is partial market coverage of the poor, it follows from Proposition 3 that profit of each firm is $\pi(\theta_L, \theta_L) = \frac{\delta_R + 2\delta_P}{2t}\left[\frac{\delta_R tL - 2\delta_P c(\theta_L) + 2\delta_P Y_P(\theta_L - 1)}{\delta_R + 4\delta_P}\right]^2 - F(\theta_L)$. If any firm deviates to offer quality θ_H, Proposition 6 implies that profit of the deviating firm is $\pi_H|_{\text{deviation}}(\theta_H, \theta_L) = \left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right]\delta_R L - F(\theta_H)$. It follows that (θ_L, θ_L) is an equilibrium outcome if

$$\begin{aligned}\left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right]\delta_R L \\ -\frac{\delta_R + 2\delta_P}{2t}\left[\frac{\delta_R tL - 2\delta_P c(\theta_L) + 2\delta_P Y_P(\theta_L - 1)}{\delta_R + 4\delta_P}\right]^2 \leq F(\theta_H) - F(\theta_L).\end{aligned} \tag{4}$$

Next consider whether (θ_H, θ_H) is an equilibrium outcome. When each firm offers high quality it follows from Proposition 3 that profit of each firm is $\pi(\theta_H, \theta_H) = \frac{\delta_R + 2\delta_P}{2t}\left[\frac{\delta_R tL - 2\delta_P c(\theta_H) + 2\delta_P Y_P(\theta_H - 1)}{\delta_R + 4\delta_P}\right]^2 - F(\theta_H)$. If any firm deviates to offer quality θ_L, Proposition 6 implies that profit of the deviating firm is $\pi_L|_{\text{deviation}}(\theta_L, \theta_H) = \frac{\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L)$. Hence (θ_H, θ_H) is an equilibrium outcome if

$$F(\theta_H) - F(\theta_L) \leq \frac{\delta_R + 2\delta_P}{2t}\left[\frac{\delta_R tL - 2\delta_P c(\theta_H) + 2\delta_P Y_P(\theta_H - 1)}{\delta_R + 4\delta_P}\right]^2 - \frac{\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t}. \tag{5}$$

Finally consider whether (θ_H, θ_L) could be an equilibrium outcome. From the above analysis it is clear that the high quality producing firm will have no incentive to deviate if $\pi_H(\theta_H, \theta_L) \geq \pi(\theta_L, \theta_L)$, that is, if $\left[Y_R(\theta_H - \theta_L) + \frac{Y_P(\theta_L - 1) + c(\theta_L)}{2} - tL - c(\theta_H)\right]\delta_R L - \frac{\delta_R + 2\delta_P}{2t}\left[\frac{\delta_R tL - 2\delta_P c(\theta_L) + 2\delta_P Y_P(\theta_L - 1)}{\delta_R + 4\delta_P}\right]^2 \geq F(\theta_H) - F(\theta_L)$, whereas the low quality producing firm will have no incentive to deviate if $\pi_L(\theta_H, \theta_L) \geq \pi(\theta_H, \theta_H)$, that is, if $F(\theta_H) - F(\theta_L) \geq \frac{\delta_R + 2\delta_P}{2t}\left[\frac{\delta_R tL - 2\delta_P c(\theta_H) + 2\delta_P Y_P(\theta_H - 1)}{\delta_R + 4\delta_P}\right]^2 - \frac{\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t}$. Combining the two inequalities we conclude that (θ_H, θ_L) is an equilibrium outcome if

$$
\begin{aligned}
&\frac{\delta_R+2\delta_P}{2t}\left[\frac{\delta_R tL-2\delta_P c(\theta_H)+2\delta_P Y_P(\theta_H-1)}{\delta_R+4\delta_P}\right]^2-\frac{\delta_P[Y_P(\theta_L-1)-c(\theta_L)]^2}{4t} \\
&\leq F(\theta_H)-F(\theta_L) \\
&\leq \left[Y_R(\theta_H-\theta_L)+\frac{Y_P(\theta_L-1)+c(\theta_L)}{2}-tL-c(\theta_H)\right]\delta_R L \\
&\qquad\qquad -\frac{\delta_R+2\delta_P}{2t}\left[\frac{\delta_R tL-2\delta_P c(\theta_L)+2\delta_P Y_P(\theta_L-1)}{\delta_R+4\delta_P}\right]^2.
\end{aligned}
\tag{6}
$$

Inequalities (4), (5) and (6) reiterate a similar observation made in the last subsection: given all the other parameter values, both the firms offer the low quality if the difference in fixed costs is too high (inequality (4)) and offer the high quality if the difference in fixed costs is low enough (inequality (5)), whereas, for an intermediate difference in fixed costs, one firm offers high quality while the other goes for low quality.

Now consider varying the rich income, Y_R. For this intermediate level of Y_P, when income level in the economy in general is not so high (relative to the difference in fixed costs), then both firms offer low quality. This might be more relevant for rural India where although facilities are available but the products and services are of substandard quality. Owing to low income some poor are left out.

As Y_R increases there is a transition in the market to favor the rich with the outcome being an asymmetric one where one firm offers high quality and the other low quality. Richer is an individual higher is his preference for better quality and more is his willingness to pay for it. This gives an incentive to the firm to offer high quality. Earlier when both firms were offering the low quality, it follows from Proposition 3 that the poor with market access were paying a price $\frac{1}{\delta_R+4f_P}[\delta_R tL+2\delta_P Y_P(\theta_L-1)+(\delta_R+2\delta_P)c(\theta_L)]$. With the rise in Y_R there is a shift in equilibrium with the price charged by the firm producing low quality being given by $\frac{Y_P(\theta_L-1)+c(\theta_L)}{2}$ (see Proposition 6). Parameter values for which this holds is such that the price for the latter case is higher than the former. That is, the price the firm charges as a monopolist is higher than the case when it is competing for the rich. This has cascading effects on the welfare of poor. The price rise pushes more poor people out of the reach of the market. In addition since one firm switches to offering high quality which is beyond the reach of the poor, the poor consumers residing closer to this firm lose market access too. For services like education or health care, this leads to a poverty spiral. This is especially true for the developing countries where the absolute poverty levels are relatively very high. So these countries must make investment in social safety nets a development priority.

4.3 $Y_P^3 < Y_P$

Since $Y_P^2 < Y_P^3$, it follows from Propositions 1 and 9 that when $Y_P^3 < Y_P$ all the poor consumers are served no matter whether the firms offer the same quality or different qualities. Following the same methodology discussed in the last two subsections we can come to the following conclusion under this case when the poor are rich enough then in equilibrium both the firms will serve both income groups:

(θ_L, θ_L) is an equilibrium outcome if

$$\frac{\Big[\{\delta_R Y_R + \delta_P Y_P\}(\theta_H - \theta_L) + (\delta_R + \delta_P)[3tL + c(\theta_L) - c(\theta_H)]\Big]^2}{18t(\delta_R + \delta_P)^2} - \frac{(\delta_R + \delta_P)\,[tL]^2}{2t} \leq F(\theta_H) - F(\theta_L);$$

(θ_H, θ_H) is an equilibrium outcome if

$$F(\theta_H) - F(\theta_L) \leq \frac{\Big[\{\delta_R Y_R + \delta_P Y_P\}(\theta_H - \theta_L) + (\delta_R + \delta_P)[3tL + c(\theta_L) - c(\theta_H)]\Big]^2}{18t(\delta_R + \delta_P)^2};$$

and (θ_H, θ_L) is an equilibrium outcome if

$$\begin{aligned}\frac{\Big[\{\delta_R Y_R + \delta_P Y_P\}(\theta_H - \theta_L) + (\delta_R + \delta_P)[3tL + c(\theta_L) - c(\theta_H)]\Big]^2}{18t(\delta_R + \delta_P)^2} &\leq F(\theta_H) - F(\theta_L) \\ &\leq \frac{\Big[\{\delta_R Y_R + \delta_P Y_P\}(\theta_H - \theta_L) + (\delta_R + \delta_P)[3tL + c(\theta_L) - c(\theta_H)]\Big]^2}{18t(\delta_R + \delta_P)^2}\end{aligned}$$

Once again, which quality will be offered in equilibrium depends on the difference in fixed costs relative to the general income level.

Implications of an increase in rich income on the welfare of the poor is similar to the last subsection. Other parameters remaining the same as Y_R increases equilibrium outcome switches from (θ_L, θ_L) to (θ_H, θ_L) and the low quality firm makes the poor worse off taking advantage of the relatively higher monopoly power over the poor. Still further increase in Y_R makes it more attractive for the low quality firm to reap the benefit of the higher willingness of the rich leading to the outcome where both firms produce the high quality product. The high quality comes at the higher price. But, at the same time, competition between the firms may mellow down the price level.

5 Conclusion

In this paper, we are interested in functional inequality. We look at the existing level of inequality through prism of market. We characterize situations where on one hand income disparity might exacerbate existing levels of inequality or on the other hand act as catalyst in improving the welfare of those at the lowest rung. We demonstrate that for a homogenous distribution of income or when the poor's income or density is too low, both firms offer the same quality. For a homogenous income distribution firm does not perceive much benefit from product differentiation. Similarly when income and density of the poor is low, it implies a low demand for a different variety. In these scenarios the poor are either left completely unserved, or they end up buying whatever the market has to offer. Given this, for a very high difference in the fixed costs, both firms offer the low quality. But when the difference in the fixed costs is low, both firms offer the high quality.

For a more heterogeneous income distribution and intermediate range of the difference in fixed costs, one firm offers the high quality and the other the low quality. Product differentiation on one hand allows firm to alleviate price competition and, on the other hand, serves consumers' demand better. Within this there can either be horizontal dominance - both firms serving either income groups, or vertical dominance - all the rich buying the high quality product and the poor buying the low quality product. Horizontal dominance arises when the travel cost outweighs the income and quality difference. When the income and quality difference is substantial compared to the travel cost, it makes the case for vertical dominance. We show that although in general a rise in income inequality has a spiraling negative effect on the welfare of the poor, there are situations, particularly when the poor income is very low, when an increase in the rich income could be welfare improving for the poor.

6 Appendix

A.1 Proof of Proposition 3

A.1.1 p_M increases in δ_R

From Proposition 3, it follows that for the case of partial market coverage with both firms offering same quality, equilibrium price is given by

$$p_M = \frac{1}{\delta_R + 4\delta_P}[\delta_R tL + 2\delta_P Y_P(\theta - 1) + (\delta_R + 2\delta_P)c(\theta)].$$

This implies that

$$\begin{aligned} \frac{\partial p_M}{\partial \delta_R} &= \frac{[tL + c(\theta)](\delta_R + 4\delta_P) - [\delta_R tL + 2\delta_P Y_P(\theta - 1) + (\delta_R + 2\delta_P)c(\theta)]}{(\delta_R + 4\delta_P)^2} \\ &= \frac{2\delta_P[2tL - [Y_P(\theta - 1) - c(\theta)]]}{(\delta_R + 4\delta_P)^2}. \end{aligned} \tag{A.1.1.a}$$

From Proposition 3, it follows that there is a partial market coverage with both firms offering same quality, when

$$\frac{\delta_R tL}{\delta_R + 2\delta_P} + c(\theta) < Y_P(\theta - 1) < \frac{(3\delta_R + 4\delta_P)tL}{2\delta_R + 4\delta_P} + c(\theta). \tag{A.1.1.b}$$

Therefore, in this case the maximum value that $Y_P(\theta - 1) - c(\theta)$ can take is $\frac{[3\delta_R + 4\delta_P]tL}{2\delta_R + 4\delta_P}$. Substituting for this in equation (A.1.1.a) we get that the corresponding minimum value that $\frac{\partial p_M}{\partial \delta_R}$ can take is

$$\frac{2\delta_P\left[2tL - \frac{[3\delta_R + 4\delta_P]tL}{2\delta_R + 4\delta_P}\right]}{(\delta_R + 4\delta_P)^2} = \frac{\delta_P[\delta_R + 4\delta_P]tL}{(\delta_R + 4\delta_P)^2(\delta_R + 2\delta_P)} \; > \; 0.$$

It follows that p_M increases as the proportion of the rich increases.

A.1.2 p_M falls in δ_P

Proof of this has been worked out in the similar lines as above. Again we have

$$p_M = \frac{1}{\delta_R + 4\delta_P}[\delta_R tL + 2\delta_P Y_P(\theta - 1) + (\delta_R + 2\delta_P)c(\theta)].$$

This implies that

$$\frac{\partial p_M}{\partial \delta_P} = \frac{2[Y_P(\theta - 1) + c(\theta)](\delta_R + 4\delta_P) - 4[\delta_R tL + 2\delta_P Y_P(\theta - 1) + (\delta_R + 2\delta_P)c(\theta)]}{(\delta_R + 4\delta_P)^2}$$

$$= \frac{2\delta_R[Y_P(\theta-1) - c(\theta) - 2tL]}{(\delta_R + 4\delta_P)^2}.$$

From the inequality (A.1.1.b) it follows that $Y_P(\theta-1) - c(\theta)$ has the minimum value $\dfrac{\delta_R tL}{\delta_R + 2\delta_P}$; consequently the maximum value $\dfrac{\partial p_M}{\partial \delta_P}$ can take is

$$\frac{2\delta_R\left[\dfrac{\delta_R tL}{\delta_R + 2\delta_P} - 2tL\right]}{(\delta_R + 4\delta_P)^2} = -\frac{2\delta_R tL}{(\delta_R + 4\delta_P)(\delta_R + 2\delta_P)} \;<\; 0.$$

Hence p_M falls as the proportion of the poor increases.

A.1.3 $\quad p_C > p_M$

From Proposition 1, it follows that the equilibrium price p_C when there is full market coverage, is $tL + c(\theta)$. From Proposition 3 it follows that the equilibrium price when there is partial market coverage, is $p_M = \dfrac{1}{\delta_R + 4\delta_P}[\delta_R tL + 2\delta_P Y_P(\theta-1) + (\delta_R + 2\delta_P)c(\theta)]$. In order to prove that $p_C > p_M$, we need to show the following:

$$tL + c(\theta) \;>\; \frac{1}{\delta_R + 4\delta_P}[\delta_R tL + 2\delta_P Y_P(\theta-1) + (\delta_R + 2\delta_P)c(\theta)].$$

That is,

$$[tL + c(\theta)][\delta_R + 4\delta_P] \;>\; [\delta_R tL + 2\delta_P Y_P(\theta-1) + (\delta_R + 2\delta_P)c(\theta)].$$

This simplifies to

$$2tL + c(\theta) \;>\; Y_P(\theta-1).$$

If the above inequality is true for the maximum value that $Y_P(\theta-1)$ can take, then it will hold for all values of $Y_P(\theta-1)$ for the case implied. From the inequality (A.1.1.b) it follows that the upper threshold for $Y_P(\theta-1)$ is $\dfrac{(3\delta_R + 4\delta_P)tL}{2\delta_R + 4\delta_P} + c(\theta)$. Substituting for this value of $Y_P(\theta-1)$, this condition becomes

$$2tL + c(\theta) \;>\; \frac{(3\delta_R + 4\delta_P)tL + 2(\delta_R + 2\delta_P)c(\theta)}{2\delta_R + 4\delta_P}.$$

Or equivalently

$$(\delta_R + 4\delta_P)tL \;>\; 0,$$

which is always true. Thus it follows that $p_C > p_M$.

A.2 Existence of Equilibrium with Partial Market Coverage under Vertical Dominance

Here we will derive the conditions for an equilibrium where no firm has any incentive for deviation. First, let us consider the firm producing high quality product. It can consider lowering its price. This will make sense only if it is able to attract poor consumers as well. At a lower price it will retain its market for the rich. So the possible deviation is to lower price such that it starts competing for the poor. Let μ denote the distance from the firm offering high quality product at which a poor consumer is indifferent between buying high quality product at new price p_H^* and low quality product at p_L. This implies

$$Y_P\theta_H - p_H^* - t\mu = Y_P\theta_L - p_L - t(L-\mu).$$

So the demand from the poor is given by

$$D_P = \frac{\delta_P[Y_P(\theta_H-\theta_L) - (p_{H^*}-p_L) + tL]}{2t}.$$

Market demand from rich is again given by $\delta_R L$. The expression for the profit of the firm is

$$\pi_H = [p_H^* - c(\theta_H)]\left\{\delta_R L + \delta_P\left[\frac{Y_P(\theta_H-\theta_L) - (p_{H^*}-p_L) + tL}{2t}\right]\right\} - F(\theta_H),$$

where from Proposition 6, $p_L = \dfrac{Y_P(\theta_L-1)+c(\theta_L)}{2}$. The first-order condition of profit maximization with respect to price implies

$$D_H = \frac{[p_H^* - c(\theta_H)]\delta_P}{2t} \text{ which implies } \pi_H = \frac{\delta_P[p_H^* - c(\theta_H)]^2}{2t} - F(\theta_H).$$

It follows that in equilibrium

$$p_H* = \frac{4tL\delta_R + 2tL\delta_P + 2\delta_P Y_P(\theta_H-\theta_L) + \delta_P Y_P(\theta_L-1) + 2\delta_P c(\theta_H) + \delta_P c(\theta_L)}{4\delta_P}$$

and

$$\pi_H* = \frac{\left[4tL\delta_R + 2tL\delta_P + 2\delta_P Y_P(\theta_H-\theta_L) + \delta_P Y_P(\theta_L-1) - 2\delta_P c(\theta_H) + \delta_P c(\theta_L)\right]^2}{32t\delta_P} - F(\theta_H).$$

We can compare this to the corresponding expression given in Proposition 6:

$$\pi_H = \left[Y_R(\theta_H-\theta_L) + Y_P(\theta_L-1) - 2tL - c(\theta_H)\right]\delta_R L - F(\theta_H).$$

Firm will not deviate if $\pi_H > \pi_H*$, that is, if

$$\left[4tL\delta_R + 2tL\delta_P + 2\delta_P Y_P(\theta_H-\theta_L) + \delta_P Y_P(\theta_L-1) - 2\delta_P c(\theta_H) + \delta_P c(\theta_L)\right]^2 < 32t\delta_R\delta_P L\left[Y_R(\theta_H-\theta_L) + Y_P(\theta_L-1) - 2tL - c(\theta_H)\right].$$

It is clear that above inequality is likely to hold when Y_R is high enough, implying that there will be Vertical Dominance only when the income gap is substantial.

Next we consider the possibility of deviation by the firm offering low quality. This firm might consider to charge a low price so that some rich are also willing to buy. Firm will continue to be a monopolist with respect to the poor. Demand by the poor at the new price p_L^* is given by D_p where

$$D_p \;=\; \frac{\delta_P[Y_P(\theta_L-1)-p_L^*]}{t}.$$

Since this firm competes for the rich, so demand from the rich is given by $\mu\delta_R$ where μ is determined from

$$\begin{aligned} Y_R\theta_L - p_L^* - t\mu &= Y_R\theta_H - p_H - t[L-\mu] \\ \Rightarrow \mu &= \frac{Y_R(\theta_L-\theta_H)-(p_{L^*}-p_H)+tL}{2t}. \end{aligned}$$

Firm's profit is given by

$$\pi_L*(.) = [p_L^* - c(\theta_L)]\left\{\delta_P \times \frac{Y_P(\theta_L-1)-p_L^*}{t} + \delta_R\left[\frac{Y_R(\theta_L-\theta_H)-(p_{L^*}-p_H)+tL}{2t}\right]\right\} - F(\theta_L),$$

where from Proposition 6 it follows that $p_H = Y_R(\theta_H-\theta_L) + \dfrac{Y_P(\theta_L-1)+c(\theta_L)}{2} - tL$. The first-order condition for profit maximization with respect to price implies

$$D_L(.) \;=\; \frac{[p_L^*-c(\theta_L)][\delta_R+2\delta_P]}{2t}$$

that is,

$$\left\{\delta_P \times \frac{Y_P(\theta_L-1)-p_L^*}{t} + \delta_R\left[\frac{Y_R(\theta_L-\theta_H)-(p_{L^*}-p_H)+tL}{2t}\right]\right\} \;=\; \frac{[p_L^*-c(\theta_L)][\delta_R+2\delta_P]}{2t},$$

which implies

$$p_L* = \frac{(4\delta_P+\delta_R)Y_P(\theta_L-1)+c(\theta_L)[3\delta_R+4\delta_P]}{2(\delta_R+2\delta_P)}.$$

Also from the first-order condition expression for profit simplifies to

$$\pi_L*(.) \;=\; \frac{[\delta_R+2\delta_P][p_L^*-c(\theta_L)]^2}{2t} \;-\; F(\theta_L).$$

On substitution for price it becomes

$$\pi_L*(.) \;=\; \frac{\Big[(4\delta_P+\delta_R)Y_P(\theta_L-1)+\delta_R c(\theta_L)\Big]^2}{8t[\delta_R+2\delta_P]} \;-\; F(\theta_L).$$

The corresponding expression for the profit of the firm from Proposition 6 is

$$\pi_L(.) = \frac{\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2}{4t} - F(\theta_L).$$

Firm will have no incentive to deviate if $\pi_L(.) > \pi_L * (.)$, that is, if

$$2(\delta_R + 4\delta_P)\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2 \; > \; \Big[(4\delta_P + \delta_R)Y_P(\theta_L - 1) + \delta_R c(\theta_L)\Big]^2.$$

Or if

$$2(\delta_R + 4\delta_P)\delta_P[Y_P(\theta_L - 1) - c(\theta_L)]^2 \; > \; (4\delta_P + \delta_R)^2\left[Y_P(\theta_L - 1) + \frac{\delta_R}{4\delta_P + \delta_R}c(\theta_L)\right]^2.$$

Above inequality reduces to

$$2[Y_P(\theta_L - 1) - c(\theta_L)]^2 \; > \; \left(4 + \frac{\delta_R}{\delta_P}\right)\left[Y_P(\theta_L - 1) + \frac{\delta_R}{4\delta_P + \delta_R}c(\theta_L)\right]^2.$$

Clearly above inequality is more likely to hold when relative density of poor is high.

A.3 Existence of Equilibrium with Full Market Coverage under Vertical Dominance

Here again, we will derive the conditions for an equilibrium where no firm has any incentive to deviate. As before, we first consider the firm offering high quality product. The possible deviation that firm can consider is to lower its price. At a lower price it will retain its share of rich consumers. But the deviation will be profitable only if the firm is able to serve poor consumers as well. So we assume that at a lower price this firm starts competing for poor consumers with the firm offering low quality product. Hence the demand from the poor at lower price p_H^* is given by

$$D_P = \delta_P\left[\frac{Y_P(\theta_H - \theta_L) - (p_{H^*} - p_L) + tL}{2t}\right].$$

So profit of the firm at the reduced price p_H^* is given by

$$\pi_H^*(.) = [p_H^* - c(\theta_H)]\left\{\delta_R L + \delta_P\left[\frac{Y_P(\theta_H - \theta_L) - (p_{H^*} - p_L) + tL}{2t}\right]\right\} - F(\theta_H).$$

Where, it follows from Proposition 8, that $p_L = Y_P(\theta_L - 1) - tL$. This implies

$$\pi_H^*(.) \;=\; [p_H^* - c(\theta_H)]\left\{\delta_R L + \delta_P\left[\frac{Y_P(\theta_H - \theta_L) - (p_{H^*} - [Y_P(\theta_L - 1) - tL]) + tL}{2t}\right]\right\} - F(\theta_H).$$

The first-order condition of profit maximization with respect to price implies

$$D_H \;=\; \frac{[p_H^* - c(\theta_H)]\delta_P}{2t}.$$

This implies

$$p_H* = \frac{2tL\delta_R + \delta_P Y_P(\theta_H - 1) + \delta_P c(\theta_H)}{2\delta_P}.$$

and the profit of the firm is given by

$$\pi_H^*(.) \;=\; \frac{\delta_P[p_H^* - c(\theta_H)]^2}{2t} \;-\; F(\theta_H).$$

Substituting for price, expression for profit becomes

$$\pi_H^*(.) \;=\; \frac{\Big[2tL\delta_R + \delta_P Y_P(\theta_H - 1) - \delta_P c(\theta_H)\Big]^2}{8t\delta_P} \;-\; F(\theta_H).$$

We can compare this to the corresponding expression given in Proposition 8:

$$\pi_H(.) = [Y_R(\theta_H - \theta_L) \;+\; Y_P(\theta_L - 1) - 2tL - c(\theta_H)]\delta_R L - F(\theta_H).$$

Firm will not deviate if $\pi_H^*(.) < \pi_H(.)$, that is, if

$$\Big[2tL\delta_R + \delta_P Y_P(\theta_H - 1) - \delta_P c(\theta_H)\Big]^2 < \; 8t\delta_R\delta_P L\Big[Y_R(\theta_H - \theta_L) \; + Y_P(\theta_L - 1) - 2tL - c(\theta_H)\Big].$$

It is clear from above that above inequality will hold when Y_R is high enough. It is intuitive that, it is only for the high difference in the income level that the equilibrium with *Vertical Dominance* will qualify.

Next we consider a similar possibility for the firm producing low quality. Deviation will be profitable only if it implies that the firm reduces its price to the extent that some rich are also willing to buy the low quality product. With the reduced price, firm will continue to serve all poor. Demand from the poor at this new price p_L^* is given by

$$D_L \;= \delta_P L.$$

Demand from rich is given by $\mu\delta_R$, such that

$$Y_R\theta_L - p_L^* - t\mu = Y_R\theta_H - p_H - t[L - \mu]$$

$$\Rightarrow \mu = \frac{Y_R(\theta_L - \theta_H) - (p_{L^*} - p_H) + tL}{2t}.$$

Firm's profit at the new price is

$$\pi_L(.) = [p_L^* - c(\theta_L)]\left\{\delta_P L + \delta_R\left[\frac{Y_R(\theta_L - \theta_H) - (p_L^* - p_H) + tL}{2t}\right]\right\} - F(\theta_L),$$

where, from Proposition 8, it follows that,

$$p_H = Y_R(\theta_H - \theta_L) + Y_P(\theta_L - 1) - 2tL.$$

The first-order condition of profit maximization with respect to price implies

$$D_L(.) = \frac{[p_L^* - c(\theta_L)]\delta_R}{2t}.$$

$$\Rightarrow p_L* = \frac{2\delta_P tL + \delta_R Y_R(\theta_L - \theta_H) + \delta_R p_H + \delta_R tL + \delta_R c(\theta_L)}{2\delta_R}.$$

Substituting for p_H

$$p_L^* = \frac{2\delta_P tL + \delta_R Y_P(\theta_L - 1) + \delta_R c(\theta_L) - tL\delta_R}{2\delta_R}.$$

Also, from the first-order condition of profit maximization, it follows that

$$\pi_L^*(.) = \frac{\delta_R[p_L^* - c(\theta_L)]^2}{2t} - F(\theta_L).$$

On substituting for p_L^*, it reduces to

$$\pi_L^*(.) = \frac{\left[\delta_R[Y_P(\theta_L - 1) - c(\theta_L)] - tL\delta_R + 2\delta_P tL\right]^2}{8t\delta_R} - F(\theta_L).$$

Whereas corresponding expression for profit given in Proposition 8 is

$$\pi_L(.) = \delta_P L[Y_P(\theta_L - 1) - tL - c(\theta_L)] - F(\theta_L).$$

Firm will have no incentive to deviate only if

$$\delta_P L[Y_P(\theta_L - 1) - c(\theta_L) - tL] > \frac{\left[\delta_R[Y_P(\theta_L - 1) - c(\theta_L)] - tL\delta_R + 2\delta_P tL\right]^2}{8t\delta_R}.$$

That is, if

$$8\delta_P\delta_R tL[Y_P(\theta_L - 1) - c(\theta_L) - tL] > \left[\delta_R[Y_P(\theta_L - 1) - c(\theta_L)] - tL\delta_R + 2\delta_P tL\right]^2.$$

Again, above inequality can be written as

$$\frac{8\delta_P tL}{\delta_R}[Y_P(\theta_L-1)-c(\theta_L)-tL] > \left[[Y_P(\theta_L-1)-c(\theta_L)]-tL+\frac{2\delta_P tL}{\delta_R}\right]^2.$$

Which will never be true, implying that the firm will always have an incentive to deviate.

A.4 Partial Market Coverage with Horizontal Dominance

Here we derive the equilibrium when there is partial market coverage under the case of horizontal dominance. We first consider the case of the firm offering high quality product. As firm is competing for the rich and some poor are left unserved, it follows that the firm's demand is given by

$$D_H(.) = \frac{\delta_R\{Y_R(\theta_H-\theta_L)-(p_H-p_L)+tL\}+2\delta_P\{Y_P(\theta_H-1)-p_H\}}{2t}.$$

Profit of the firm producing high quality is given by

$$\pi_H(.) = [p_H-c(\theta_H)]D_H(.)-F(\theta_H).$$

The first-order condition of profit maximization with respect to price implies

$$D_H(.) = \left[\frac{[p_H-c(\theta_H)](\delta_R+2\delta_P)}{2t}\right].$$

On substitution and simplification this reduces to

$$2p_H(\delta_R+2\delta_P) = \delta_R\{Y_R(\theta_H-\theta_L)+p_L)+tL\}+2\delta_P Y_P(\theta_H-1) \ + \ c(\theta_H)(\delta_R+2\delta_P). \quad \text{(A.4.1)}$$

$$\Rightarrow p_H = \frac{\delta_R\{Y_R(\theta_H-\theta_L)+p_L)+tL\}+2\delta_P Y_P(\theta_H-1) \ + \ c(\theta_H)(\delta_R+2\delta_P)}{2(\delta_R+2\delta_P)}.$$

A similar exercise for the firm offering low quality results in the following

$$2p_L(\delta_R+2\delta_P) = \delta_R\{Y_R(\theta_L-\theta_H)+p_H)+tL\}+2\delta_P Y_P(\theta_L-1) \ + \ c(\theta_L)(\delta_R+2\delta_P). \quad \text{(A.4.2)}$$

From equations (A.4.1) and (A.4.2) it follows that in equilibrium

$$\begin{aligned}&[4(\delta_R+2\delta_P)^2-\delta_R^2]p_L \\ &= \Big[\delta_R(\delta_R+4\delta_P)Y_R(\theta_L-\theta_H)+\delta_R(3\delta_R+4\delta_P)tL+2\delta_R\delta_P Y_P(\theta_H-1) \ + \ \delta_R(\delta_R+2\delta_P)c(\theta_H) \\ &\quad + 4\delta_P(\delta_R+2\delta_P)Y_P(\theta_L-1) \ + \ 2(\delta_R+2\delta_P)^2 c(\theta_L)\Big]\end{aligned}$$

and

$$[4(\delta_R + 2\delta_P)^2 - \delta_R^2]p_H$$
$$= \Bigg[\delta_R(\delta_R + 4\delta_P)Y_R(\theta_H - \theta_L) + \delta_R(3\delta_R + 4\delta_P)tL + 2\delta_P\delta_R Y_P(\theta_L - 1) + \ \delta_R(\delta_R + 2\delta_P)c(\theta_L)$$
$$+ 4\delta_P(\delta_R + 2\delta_P)Y_P(\theta_H - 1) \ + \ 2(\delta_R + 2\delta_P)^2 c(\theta_H)\Bigg].$$

This case arises when the poor consumer who is indifferent between the two adjacent firms is better-off by not buying. This implies that the distance at which a poor consumer is indifferent between buying and not buying high quality product is lower than that corresponding to the poor consumer, that is

$$\frac{Y_P(\theta_H - \theta_L) - (p_H - p_L) + tL}{2t} > \frac{Y_P(\theta_H - 1) - p_H}{t}.$$
$$\Rightarrow tL > Y_P(\theta_H - 1) + Y_P(\theta_L - 1) - p_H - p_L. \tag{A.4.3}$$

From equations (A.4.1) and (A.4.2) it follows

$$p_L + p_H = \frac{2tL\delta_R + 2\delta_P Y_P(\theta_L - 1) + 2\delta_P Y_P(\theta_H - 1) + \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P)}{\delta_R + 4\delta_P} \tag{A.4.4}$$

Substituting for $p_L + p_H$ from equation (A.4.4) in equation (A.4.3) it follows that

$$tL > Y_P(\theta_H - 1) + Y_P(\theta_L - 1)$$
$$-\left\{\frac{2tL\delta_R + 2\delta_P Y_P(\theta_L - 1) + 2\delta_P Y_P(\theta_H - 1) + \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P)}{\delta_R + 4\delta_P}\right\}.$$

It follows that there will be partial market coverage if

$$\frac{[3\delta_R + 4\delta_P]tL + (\delta_R + 2\delta_P)[(c(\theta_L) + c(\theta_H)]}{[\delta_R + 2\delta_P][(\theta_L - 1) + (\theta_H - 1)]} > Y_P.$$

Given that few poor are buying from either firm it follows

$$Y_P(\theta_H - 1) - p_H \ > 0 \text{ and } \ Y_P(\theta_L - 1) - p_L \ > 0$$

The two inequalities imply

$$Y_P[(\theta_H - 1) + (\theta_L - 1)] \ > \ p_L + p_H.$$

Substituting for $p_L + p_H$ from equation (A.4.4) above inequality reduces to

$$Y_P[(\theta_H - 1) + (\theta_L - 1)] \ > \ \frac{2tL\delta_R + 2\delta_P Y_P(\theta_L - 1) + 2\delta_P Y_P(\theta_H - 1) + \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P)}{\delta_R + 4\delta_P}$$

$$\Rightarrow Y_P \ > \ \frac{2tL\delta_R + \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P)}{[\delta_R + 2\delta_P][(\theta_H - 1) + (\theta_L - 1)]}.$$

Also, we need a condition on Y_R to ensure that all rich are served. Therefore, the distance at which the rich is indifferent in buying and not buying, say the high quality product, is higher than the distance at which he indifferent between the two adjacent firms.

$$\Rightarrow \frac{Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL}{2t} < \frac{Y_R(\theta_H - 1) - p_H}{t}.$$

Above equation together with equation (A.4.4) implies

$$\begin{aligned} & Y_R[(\theta_H - 1) + (\theta_L - 1)] \\ > \ & \frac{2tL\delta_R + 2\delta_P Y_P(\theta_L - 1) + 2\delta_P Y_P(\theta_H - 1) + \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P)}{\delta_R + 4\delta_P} + tL. \end{aligned}$$

That is,

$$[Y_R(\delta_R + 4\delta_P) - 2\delta_P Y_P][(\theta_H - 1) + (\theta_L - 1)] \ > \ \{c(\theta_L) + c(\theta_H)\}(\delta_R + 2\delta_P) + tL(3\delta_R + 4\delta_P).$$

A.5 Full Market Coverage for Intermediate Case

We initially look at the equilibrium when firm producing θ_L serves both rich and poor, whereas firm producing θ_H serves only rich. The firm offering low quality has to compete for rich, implying that the demand from rich is given by

$$D_{L_R}(.) \ = \ \frac{\delta_R[Y_R(\theta_L - \theta_H) - (p_L - p_H) + tL]}{2t}. \qquad \text{(A.5.1)}$$

As there is full market coverage, and poor buy only the low quality product, it implies that demand from poor is $\delta_P \times L$. So the profit of the firm offering low quality product is given by the following expression

$$\pi_L(.) = [p_L - c(\theta_L)]\left\{\delta_P L + \frac{\delta_R[Y_R(\theta_L - \theta_H) - (p_L - p_H) + tL]}{2t}\right\} - F(\theta_L).$$

The first-order condition of profit maximization with respect to price implies

$$D_L(.) \ = \ \frac{\delta_R[p_L - c(\theta_L)]}{2t} \qquad \text{(A.5.2)}$$

hence

$$\pi_L(.) = \frac{\delta_R}{2t}[p_L - c(\theta_L)]^2 - F(\theta_L).$$

From equation (A.5.1) and (A.5.2) it follows that

$$p_L = \frac{2tL\delta_P + \delta_R[Y_R(\theta_L - \theta_H) + p_H + tL + c(\theta_L)]}{2\delta_R}.$$

Next we consider the firm offering high quality. Given the case that we consider here, firm producing high quality serves only rich. So profit of the firm offering high quality product is given by

$$\pi_H(.) = [p_H - c(\theta_H)]\left[\frac{\delta_R[Y_R(\theta_H - \theta_L) - (p_H - p_L) + tL]}{2t}\right] - F(\theta_H).$$

The first-order condition of profit maximization with respect to price implies

$$D_H(.) = \frac{\delta_R[p_H - c(\theta_H)]}{2t} \tag{A.5.3}$$

and

$$\pi_H(.) = \frac{\delta_R}{2t}[p_H - c(\theta_H)]^2 - F(\theta_H)$$

From equation (A.5.3) it follows that

$$p_H = \frac{Y_R(\theta_H - \theta_L) + p_L + tL + c(\theta_H)}{2}.$$

Substituting for p_H in the expression for profit implies that firm's profit is given by

$$\pi_H(.) = \frac{\delta_R}{2t}\left[\frac{Y_R(\theta_H - \theta_L)}{2} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2 - F(\theta_H). \tag{A.5.4}$$

The two first-order conditions imply that in equilibrium

$$p_L = tL + \frac{c(\theta_H)}{3} + \frac{2c(\theta_L)}{3} + \frac{Y_R(\theta_L - \theta_H)}{3} + \frac{4tL\delta_P}{3\delta_R}.$$

and

$$p_H = tL + \frac{Y_R(\theta_H - \theta_L)}{3} + \frac{2c(\theta_H)}{3} + \frac{c(\theta_L)}{3} + \frac{2tL\delta_P}{3\delta_R}.$$

From the above two equations it follows that

$$p_H - p_L = \frac{2Y_R(\theta_H - \theta_L)}{3} + \frac{c(\theta_H) - c(\theta_L)}{3} - \frac{2tL\delta_P}{3\delta_R}.$$

Let us now consider the condition under which the above can be sustained as an equilibrium. Firm producing low quality already serves both the income types. So the possible deviation can be from the firm producing high quality. Firm may consider reducing its price to the extent so that it is able to serve some consumers with low income as well. In which case profit of the firm at the new price p_H^* is given by

$$\pi_H^* = [p_H^* - c(\theta_H)]\Bigg\{\delta_P\left[\frac{Y_P(\theta_H-\theta_L)-(p_H^*-p_L)+tL}{2t}\right] + \delta_R\left[\frac{Y_R(\theta_H-\theta_L)-(p_H^*-p_L)+tL}{2t}\right]\Bigg\} - F(\theta_H).$$

The first-order condition with respect to price implies

$$D_H(.) = \frac{(\delta_P + \delta_R)[p_H^* - c(\theta_H)]}{2t}.$$

It follows that

$$p_H^* \quad = \quad \frac{(\delta_P Y_P + \delta_R Y_R)(\theta_H - \theta_L)}{2(\delta_P + \delta_R)} \quad + \quad \frac{p_L + tL + c(\theta_H)}{2}.$$

So in equilibrium, profit of the firm offering high quality is given by

$$\pi_H^*(.) = \frac{1}{2t}\left[\frac{(\delta_P Y_P + \delta_R Y_R)(\theta_H - \theta_L)}{2(\delta_P + \delta_R)} \quad + \quad \frac{p_L + tL - c(\theta_H)}{2}\right]^2 - F(\theta_H).$$

From equation (A.5.4) it follows that the corresponding expression for profit of the firm offering high quality is

$$\pi_H(.) \; = \; \frac{\delta_R}{2t}\left[\frac{Y_R(\theta_H - \theta_L)}{2} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2 \; - \; F(\theta_H).$$

So the firm will not deviate if $\pi_H(.) > \pi_H^*(.)$, that is, if

$$\frac{\delta_R}{2t}\left[\frac{Y_R(\theta_H - \theta_L)}{2} + \frac{p_L + tL - c(\theta_H)}{2}\right]^2 \; > \; \frac{1}{2t}\left[\frac{(\delta_P Y_P + \delta_R Y_R)(\theta_H - \theta_L)}{2(\delta_P + \delta_R)} \quad + \quad \frac{p_L + tL - c(\theta_H)}{2}\right]^2.$$

Above inequality is more likely to hold for high income of the rich consumers. This is because high income of rich consumers allows the firm offering high quality to charge high price, that more than offsets for the lost demand from the poor section of the society. Similarly we can work out the case, when firm producing θ_H serves both income types, but firm producing θ_L serves only poor. For this case firm offering low quality has an incentive to deviate. Similar calculations as above imply that firm will not deviate if

$$\begin{aligned} &\frac{\delta_P}{2t}\left[\frac{Y_P(\theta_L - \theta_H)}{2} + \frac{p_H + tL - c(\theta_L)}{2}\right]^2 \\ &> \; \frac{(\delta_P + \delta_R)}{2t}\left[\frac{(\delta_P Y_P + \delta_R Y_R)(\theta_L - \theta_H)}{2(\delta_P + \delta_R)} \quad + \quad \frac{p_H + tL - c(\theta_L)}{2}\right]^2. \end{aligned}$$

Clearly, this will never be true. So there can never be an equilibrium when firm offering high quality product serves both rich and poor, whereas firm offering low quality product serves only the poor.

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