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Abstract

This paper investigates how neighbourhood effects interacting with income inequality affect poor people's ability to access basic facilities like health care services, schooling, and so on. We model this interaction by integrating consumers' income distribution with the spatial distribution of their location and explore the consequences of an increase in income inequality on the welfare of the poor in general, and their access to market in particular. We find inverted-U shape relationships between income inequality and market access and welfare of the poor: if we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies, but, beyond a point, the aggregate market access and consumer surplus of the poor starts declining as the society becomes richer. There exist multiple equilibria: a bad equilibrium where all the poor are excluded exists simultaneously with a good equilibrium where at least some poor (if not all of them) get served by the market. We have identified the higher income gap between rich and poor as the key factor that exposes the poor to this complete exclusion possibility. Finally comparing a mixed-income neighbourhood where rich and poor live side by side with a single-income homogeneous neighbourhood we find that the poor are better-off living in the mixed neighbourhood as long as the poor income is below a certain feasibility threshold.

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1 Introduction

The key idea explored in this paper is the following: though being poor in itself is a huge disadvantage, the situation might be influenced considerably by the type of neighbourhood the poor lives in as private establishments like educational institutions, health care facilities or credit institutions take both the location and income mix of people into account while making strategic decisions like whether to enter into the neighbourhood at all, and, upon entry, what price and quality to choose for their products and services.[1] Is staying with the rich a virtue for the poor or a source of resentment? Are the poor living in poor neighbourhoods better-off because living in a richer one costs too much? Or, are they significantly worse-off as they do not even have access to many basic facilities? These are the kinds of questions we are interested in exploring in this paper.

Answers to these questions depend not just on the costs relative to income, but also on the ease of access of the facilities. The reason is that certain goods and services are required at regular intervals so that distance becomes an important factor. In the less developed countries distance from schools is an important factor leading to high drop-out rates or low school enrollment.[2] Similarly distance from the nearby health care facility is a major reason resulting in higher mortality of both mother and child during child birth in rural areas of developing countries.[3] How readily a product or service is available is thus determined by the neighbourhood an individual lives in. So it is the interaction of the two, the individual's

[1] Contrary to the conventional belief, private establishments are a huge presence in the education and health care sectors of the less developed countries. In India Dreze and Sen (2002) estimate that, even by 1994, some 30% of all 6-14 year olds in rural areas were enrolled in private schools, while 80% or more attended private schools in urban areas, including low-income families. In the poor urban, periurban and rural areas surveyed by Tooley and Dixon (2006), the vast majority of school children were found to be in private schools: 75% in Lagos State, Nigeria, 65% in Ga, Ghana and in Hyderabad, India, and roughly 50% in Mahbubnagar, rural Andhra Pradesh, India. In Lahore, Pakistan, Alderman et al. (2003) estimates 51% of children from families earning less than $1 a day attend private schools, while Andrabi et al. (2010) reports that 35% of primary enrollment in Pakistan was in the private sector by 2000. Similarly on health, World Health Organization (2011) reports the following figures on private expenditure on health as a percentage of total health expenditure in 2009: Bangladesh 68%, Brazil 54%, Chile 53%, China 50%, Egypt 59%, Ghana 47%, Guatemala 63%, India 67%, Kenya 66%, Nigeria 64%, Pakistan 67%, Sierra Leone 93%.

[2] There is strong empirical evidence showing that distance is a major predictor of school enrollment or drop-out rates in less developed countries; see, for example, Alderman et al. (2001), Andrabi et al. (2010), Colclough et al. (2000), Glick and Sahn (2006), Handa (2002), and Huisman and Smits (2009).

[3] Almost any study of health seeking behaviour in developing (and developed) countries finds some estimate of the distance or travel cost as an important and significant determinant of the choice of health care provider; see, for example, Acton (1975), Kessler and McClellan (2000), Kloos (1990), Stock (1983), and Tay (2003).

income and his postcode, that determines his welfare.

There is a substantial body of evidence showing how neighbourhood poverty affects poor people's ability to access facilities such as health care and schooling. Consider health care first. An established body of studies has demonstrated that neighbourhood indicators of socioeconomic status predict individual mortality. For example, Stafford and Marmot (2003) and Yen and Kaplan (1999) find that low-income adults in advantaged neighbourhoods might experience a lower mortality risk than low-income adults in disadvantaged neighbourhoods because they benefit from the collective resources in their neighbourhoods. On the other hand, Roos et al. (2004), Veugelers et al. (2001) and Winkleby et al. (2006) show that low-income adults in advantaged neighbourhoods experience a higher risk of dying because of relative deprivation and/or low relative social standing. Analyzing a set of 85 developing country Demographic and Health Surveys, Montgomery and Hewett (2005) find that both household and neighbourhood living standards make a significantly important difference to health in the cities and towns of developing countries. They report striking differentials in health depending on the region: poor city dwellers often face health risks that are nearly as bad as what is seen in the countryside and, sometimes, the risks are decidedly worse.[4] For Rio de Janeiro, Brazil, Szwarcwald et al. (2002) find higher neighbourhood mean poverty and higher variance both act to increase infant mortality and adolescent fertility rates at the census tract level. In Delhi, India, Das and Hammer (2005) find that doctors located in the poorest neighbourhoods are one full standard deviation worse than doctors located in the richest neighbourhoods. In India, while the rural poor are underserved, at least they can access the limited number of government-supported medical facilities that are available to them; the urban poor fares even worse because they cannot afford to visit the private facilities that thrive in India's cities (PriceWaterhouse Coopers, 2007).

Similarly on education, based on observed spatial variations in school performance and drop-out rates, an extensive amount of research has identified that neighbourhood socioeconomic characteristics affect various aspects of educational outcomes. Compared to adults from wealthier neighbourhoods, those from relatively disadvantaged neighbourhoods tend to have lower test scores and grades (Dornbusch et al., 1991; Gonzales et al., 1996; Turley, 2003), a higher risk of dropping out of school (Aaronson, 1998; Brooks-Gunn et al., 1993; Connell et al., 1995; Crane, 1991; Ensminger et al., 1996), a lower likelihood of post-secondary education (Duncan, 1994), and complete fewer years of schooling (Corcoran et al.,

[4] For instance, they find that in the slums of Nairobi rates of child mortality substantially exceed those found elsewhere in Nairobi; on the other hand, the slum residents are better shielded from risk than rural dwellers with respect to births attended by doctors, nurses and trained midwives.

1992). Using data on rural residential neighbourhoods from Bangladesh, Asadullah (2009) identifies positive and significant neighbourhood effects on school completion of children. Montgomery et al. (2005) find that educational attainment of poor children in urban Egypt and in the slums of Allahabad, India, depend not only on the standards of living of their own families, but also on the economic composition of their local surroundings. For a sample of rural households in Ethiopia, Weir (2007) finds that children's schooling benefit significantly from the education of women in their neighbourhood.

Although the evidence is compelling, there seems to be very little analytical research to understand how neighbourhood effects interacting with income inequality affect poor people's ability to access basic facilities like health care services, schooling, and so on. This paper makes an early attempt to model this interaction by integrating consumers' income distribution with the spatial distribution of their location and explores the consequences of an increase in income inequality on the welfare of the poor in general, and their access to market in particular.

We consider a homogeneous product or service in a competitive framework with free entry and exit. It is very interesting to investigate the interaction of inequality and neighbourhood effect in such an ideal market structure. The inequality-neighbourhood interaction is captured by the spatial structure where the neighbourhood is a circular city across which the consumers are uniformly distributed with rich and poor consumers living side by side. The preference structure reflects the higher willingness to pay of the richer consumers and the consumers' reluctance to travel farther to access the product or service under consideration. The industrial structure is characterized by the presence of a fixed cost of production. The set-up is a two-stage game. In the first stage, the potential providers of the product or service decide whether to enter into the neighbourhood or not; in the second stage, the entering firms choose their prices simultaneously. In this set-up we explore the interaction of income inequality with the neighbourhood effect in determining the market outcomes and its consequences on the market access and welfare of the poor.

We find an *inverted-U* shape relationship between income inequality and the welfare of the poor: if we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies, but, beyond a point, the aggregate consumer surplus of the poor starts declining as the society becomes richer. Interestingly the same inverted-U shape relationship is also observed between income inequality and market access of the poor. The reason for this inverted-U shape relationships can be traced to the opposing welfare impacts of income inequality working through equilibrium price and number of firms.

Consumers benefit from the increase in number of firms as it increases their market access, but lose from the increase in price. As the neighbourhood of the poor becomes richer, both price and number of firms increase steadily. For the poor community as a whole the number of firms effect dominates initially: the poor residing closer to the firms get to access the product as the number of firms increases. But, beyond a point, the adverse price effect takes over. Studies by Feng and Yu (2007) and Li and Zhu (2006) lend strong empirical support to our theoretical results. They find an inverted-U association between self-reported health status and neighbourhood level inequality using individual data from the China Health and Nutrition Survey (CHNS).

In order to examine the role of inequality in its purest form, we also analyze the effect of mean-preserving spread: increase the rich income together with a decrease in the proportion of rich keeping fixed the poor income and the average income of the society. We find that the effect depends on the initial proportion of poor in the neighbourhood. If the initial proportion of poor is to the left of the peaks of the inverted-U relationships, then both market access and consumer surplus of poor increase in the beginning, reach a maximum and then fall as the spread increases. If the initial proportion of poor is to the right of the peaks of the inverted-U relationships, then both market access and consumer surplus of poor decrease steadily as the spread increases. As the spread increases through a decrease in the proportion of rich, firms are forced to lower price and some firms leave as they find it unattractive to serve the neighbourhood. Fewer number of firms reduces the poor people's market access and hence their welfare, whereas lower price increases market access and welfare. To the left of the inverted-U price effect dominates while the number of firms effect dominates to the right of the peak.

There exists a substantial body of literature addressing the effects of income inequality on a variety of socioeconomic outcomes.[5] For example, higher inequality is found to be positively correlated with higher infant mortality (Waldman, 1992), lower economic growth (Alesina and Rodrik, 1994; Persson and Tabellini, 1994), violent crime (Fajnzylber et al., 2002), subversion of institutions (Glaeser et al., 2003), and so on. This paper complements this literature by exploring the impact of income inequality working through price and number of firms. Atkinson (1995) is the only work that we are aware of which investigates the implications of inequality operating through industrial structure. But Atkinson (1995) considers only a monopolist firm and does not allow free entry. The tension between price and number of firms effects is the key feature that gets highlighted in our paper.

[5] For an extensive review of this literature see Atkinson and Bourguignon (2000).

We find that the nature of equilibrium depends on two thresholds of the poor income. We identify an *upper income threshold* for the poor income such that all poor consumers get served by the market only if the poor income is above this upper threshold. On the other hand there exists a *lower income threshold* for the poor income such that no poor consumer is served if the poor income is below this lower threshold. When the poor income is in between the upper and lower income thresholds, there are pockets of the neighbourhood where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

We have also identified the possibility of *multiple equilibria*. There exists a wide range of parameter values such that a good equilibrium and a bad equilibrium exist side by side for the same parameter configurations. Under the good equilibrium at least some poor (if not all of them) gets served, those who are located closer to the firms. Whereas under the bad equilibrium all the poor are excluded; the firms completely ignore their presence and choose the price and quality as if there were only rich individuals residing in the city. We have isolated the *higher income gap* between the rich and poor as the key factor that exposes the poor to the complete exclusion possibility. We have also found that poor are more likely to be completely excluded when they are a *minority*: firms may completely ignore the poor even when the rich are not ultra rich just because the rich are more in number.

Finally we compare a mixed-income neighbourhood where rich and poor live side by side with a single-income neighbourhood inhabited only by a single income group. We have identified a *feasibility income threshold* in a single-income neighbourhood such that it is not feasible for any firm to operate if the common income is below this feasibility threshold. Comparing mixed versus single-income neighbourhoods we show that the poor are better-off staying in the mixed-income neighbourhood as long as the poor income is below this feasibility threshold. At least some poor get to enjoy the product or service in the mixed-income neighbourhood as the firms recover their fixed costs due to the higher willingness to pay of the rich. This is not possible in a single-income poor neighbourhood.

The idea that people with higher income generally have higher willingness to pay and that firms do take this into account while making strategic decisions was developed by Gabszewicz and Thisse (1979) and extended by Shaked and Sutton (1982, 1983). Our specification allows consumers to differ with respect to both their income and location. The basic horizontal product differentiation model was introduced by Hotelling (1929) and later developed by Salop (1979). The literature on industrial organization that follows these seminal works (for example, Economides, 1993; Neven and Thisse, 1990) looks at product specifications combining both the vertical and horizontal characteristics. But, understandably, the industrial organization literature does not explore the implications of income inequality.

The paper is organized as follows. Section 2 outlines the model with the spatial structure capturing the inequality-neighbourhood interaction. Section 3 analyzes the generic scenario where the poor has partial market access while the rich has complete access. The effect of inequality on market access and welfare of the poor is investigated in section 4. In section 5 we characterize all the equilibrium possibilities highlighting the role of inequality in generating the possibility of multiple equilibria. The comparison with the single-income neighbourhood is also discussed in this section. Finally we conclude in section 6.

2 The Model

Our model adapts the framework of Salop (1979).[6] There is a circular city of circumference 1 unit. Two types of consumers, rich and poor, are uniformly distributed along the circumference of the city: there are f proportion of poor with income Y_P and $(1-f)$ proportion of rich with income Y_R. Obviously $Y_R > Y_P$. The total number of consumers is normalized to 1.

There are n private establishments in the city providing a homogeneous product or service. Examples of such establishments are private schools, hospitals, banks, and so on. For the sake of brevity let us refer to them as firms. These n firms are located equidistant to each other around the circle so that the distance between adjacent firms is $\frac{1}{n}$. The number of firms is not fixed; it is determined endogenously from free entry and exit condition.[7]

Each consumer buys either one unit of the homogeneous product from his most preferred firm, or does not buy the product at all. Let θY be the *gross* utility a consumer with income Y enjoys from consuming the product. Here θ is a preference parameter indicating consumers' valuation of the product. Since $\theta Y_R > \theta Y_P$, this formulation of gross utility captures the feature that willingness to pay is higher for the rich. Let us use the notations x_j for location of firm j and p_j for the price it charges, $j = 1, 2, ..., n$. A consumer at location

[6] Our adaptation of the Salop (1979) framework is similar to Bhaskar and To (1999, 2003) and Brekke et al. (2008).

[7] In this paper we are not modeling firms' location choice, rather our interest is to analyze the extent of entry. It is the extent of entry that determines the market access of the poor and hence their welfare. Our justification of this modeling structure is similar to Tirole (1988): " Omitting the choice of location allows us to study the entry issue in a simple and tractable way" (page 283).

z has to travel a distance $|x_j - z|$ to access the product or service from firm j and he incurs a travel or transportation cost of $t|x_j - z|$. Of course he has to pay the price p_j. Hence the *net* utility of a consumer at location z with income Y and purchasing from firm j is given by

$$U(z, Y, j) = \theta Y - p_j - t|x_j - z|.$$

If a consumer does not buy the product, he still has his income Y to spend on other goods and services implying that his reservation utility is Y.[8]

This formulation of the utility function helps to model the interaction of neighbourhood effects with income inequality in a simple and tractable way. While the gross utility captures the higher willingness to pay of the rich, the presence of travel cost reflects the disutility if the facility is not available nearby in the neighbourhood. Unlike the industrial organization literature where distance reflects horizontal product differentiation, we treat the distance literally as physical distance from the facility. For facilities like schools or hospitals the importance of distance or accessibility is undeniable.

Production requires fixed costs; in order to produce any output at all, each firm must incur a fixed cost F. Further, there is a marginal cost of production, c, which is independent of output. Profit of firm j charging a price p_j is then given by

$$\pi_j = [p_j - c]D_j - F,$$

where D_j denotes demand faced by firm j. Given the spatial structure, we elaborate in the next subsection how D_j depends on firm j's own price, p_j, and on the prices of the two adjacent firms, p_{j-1} and p_{j+1}.

The set-up is a two-stage game. In the first stage, firms decide whether to enter or not, and the entering firms locate equidistantly around the circumference of the city. In the second stage, firms choose their prices simultaneously.

2.1 Demand Structure

Consider firm j located between the two adjacent firms $j-1$ and $j+1$. Let $\delta_{j,j+1}$ denote the distance from firm j of the marginal consumer with income Y who is indifferent between firms j and $j+1$, that is, $U(x_j + \delta_{j,j+1}, Y, j) = U(x_j + \delta_{j,j+1}, Y, j+1)$. It follows that

$$\delta_{j,j+1} = \frac{1}{2t}\left(p_{j+1} - p_j + \frac{t}{n}\right).$$

[8]Since the gross utility is θY while the reservation utility is Y, we must have $\theta > 1$.

Utility of this marginal consumer is $\frac{1}{2}\left[2\theta Y - (p_{j+1} + p_j) - \frac{t}{n}\right]$.

Let $\overline{Y}_{j,j+1}$ denote the income level such that the consumer with income $\overline{Y}_{j,j+1}$ who is indifferent between firms j and $j+1$ at a distance $\delta_{j,j+1}$ is also indifferent between buying and not buying, that is, $U\left(x_j + \delta_{j,j+1}, \overline{Y}_{j,j+1}, j\right) = \overline{Y}_{j,j+1}$. It follows that

$$\overline{Y}_{j,j+1} = \frac{p_{j+1} + p_j + \frac{t}{n}}{2(\theta - 1)}.$$

Clearly $U(x_j + \delta_{j,j+1}, Y, j) \geq Y$ for all $Y \geq \overline{Y}_{j,j+1}$, and the marginal consumer with income Y (at a distance $\delta_{j,j+1}$ from firm j) will buy from firm j. But $U(x_j + \delta_{j,j+1}, Y, j) < Y$ for all $Y < \overline{Y}_{j,j+1}$, and the marginal consumer with income Y (at a distance $\delta_{j,j+1}$ from firm j) will *not* buy from firm j. The implication for demand is that for all $Y \geq \overline{Y}_{j,j+1}$, the measure of consumers located between x_j and x_{j+1} and buying from firm j is $\delta_{j,j+1}$.

Now consider the consumers with income $Y < \overline{Y}_{j,j+1}$. These consumers are surely not buying the product or service from firm $j+1$. Whether they will buy it from firm j depends on whether they are better off from buying or not buying. Let $\eta_{j,j+1}(Y)$ denote the distance from firm j of the consumer with income $Y < \overline{Y}_{j,j+1}$ who is indifferent between buying and not buying from firm j, that is, $U\left(x_j + \eta_{j,j+1}, Y, j\right) = Y$. It follows that

$$\eta_{j,j+1}(Y) = \frac{1}{t}\left[Y(\theta - 1) - p_j\right].$$

But note that $\eta_{j,j+1}(Y) < 0$ for

$$Y < \frac{p_j}{\theta - 1} \equiv \underline{Y}_j,$$

that is, consumers with income $Y < \underline{Y}_j$ are not buying from firm j even when they are located at the same location as firm j. The implication for demand is that the measure of consumers located between x_j and x_{j+1} and buying from firm j is $\eta_{j,j+1}(Y)$ for all $\underline{Y}_j \leq Y < \overline{Y}_{j,j+1}$, and 0 for $Y < \underline{Y}_j$.

Proceeding in the same way we can define $\overline{Y}_{j,j-1}$, $\underline{Y}_j$, $\delta_{j,j-1}$ and $\eta_{j,j-1}(Y)$ symmetrically replacing $j+1$ with $j-1$ in the corresponding expressions and conclude that the measure of consumers located between x_j and x_{j-1} and buying from firm j is $\delta_{j,j-1}$ for $Y \geq \overline{Y}_{j,j-1}$, $\eta_{j,j-1}(Y)$ for $\underline{Y}_j \leq Y < \overline{Y}_{j,j-1}$, and 0 for $Y < \underline{Y}_j$.

To sum up, demand for firm j's product generated from the consumers located between

x_j and x_{j+1} is

$$D_{j,j+1} = \begin{cases} \frac{1}{2t}\left(p_{j+1} - p_j + \frac{t}{n}\right) & \text{for } Y \geq \overline{Y}_{j,j+1} \\ \frac{Y(\theta - 1) - p_j}{t} & \text{for } \underline{Y}_j \leq Y < \overline{Y}_{j,j+1} \\ 0 & \text{for } Y < \underline{Y}_j. \end{cases}$$

Similarly, demand for firm j's product generated from the consumers located between x_j and x_{j-1} is

$$D_{j,j-1} = \begin{cases} \frac{1}{2t}\left(p_{j-1} - p_j + \frac{t}{n}\right) & \text{for } Y \geq \overline{Y}_{j,j-1} \\ \frac{Y(\theta - 1) - p_j}{t} & \text{for } \underline{Y}_j \leq Y < \overline{Y}_{j,j-1} \\ 0 & \text{for } Y < \underline{Y}_j. \end{cases}$$

Clearly, $D_j = D_{j,j+1} + D_{j,j-1}$.

It is interesting to note the difference in demand patterns arising from the relatively rich and poor. For the relatively rich consumers (with $Y \geq \overline{Y}_{j,j-1}$ or $Y \geq \overline{Y}_{j,j+1}$) firm j has to compete with the two adjacent firms, and the demand reflects that: $\delta_{j,j+1}$ and $\delta_{j,j-1}$ does depend on the strategic choices of the two adjacent firms, p_{j+1} and p_{j-1}, respectively. In contrast, firm j does not compete with its adjacent firms for the relatively poor consumers (with $\underline{Y}_j \leq Y < \overline{Y}_{j,j-1}$ and $\underline{Y}_j \leq Y < \overline{Y}_{j,j+1}$); they form a captive market for firm j over which it exercises some monopoly power.

Difference between the rich and poor gets reflected in the price response to demand also. Price response for the part of demand arising from the rich, $\frac{\partial \delta_{j,j+1}}{\partial p_j} = \frac{\partial \delta_{j,j-1}}{\partial p_j} = -\frac{1}{2t}$, is clearly lower than that arising from the poor, $\frac{\partial \eta_{j,j+1}(Y)}{\partial p_j} = \frac{\partial \eta_{j,j-1}(Y)}{\partial p_j} = -\frac{1}{t}$, because of the presence of competitive pressure.

2.2 The Symmetric Equilibrium

Given the symmetric model structure, in what follows we characterize the symmetric equilibrium where each of the n entering firms chooses the same price in stage 2, that is, $p_j = p$, for all j. Then in stage 1 entry (that is, the number of operating firms) is determined by the zero-profit condition.

In a symmetric equilibrium the income thresholds relevant to define the demand structure become

$$\overline{Y}_{j,j+1} = \overline{Y}_{j,j-1} = \frac{2p + \frac{t}{n}}{2(\theta - 1)} = \frac{p}{\theta - 1} + \frac{t}{2n(\theta - 1)} \equiv \overline{Y}, \tag{1}$$

and

$$\underline{Y}_{j,j+1} = \underline{Y}_{j,j-1} = \frac{p_j}{\theta - 1} = \frac{p}{\theta - 1} \equiv \underline{Y}. \tag{2}$$

We always consider the scenario where the rich has complete market coverage, that is, $Y_R \geq \overline{Y}$. Then, depending on whether the poor has complete or partial coverages, that is, depending on the position of Y_P vis-a-vis $\overline{Y}$ and $\underline{Y}$, we have the following cases to consider:

(1) $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$: both rich and poor have full market coverage;

(2) $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$: complete market coverage for rich, but only partial coverage for poor;

(3) $Y_R > \overline{Y}$ and $Y_P < \underline{Y}$: complete market coverage for rich, but no coverage for poor.

In what follows we analyze in detail case (2), the generic case where all the rich consumers are served, whereas, for the poor, some are served while others are left out. Analysis of the other cases is similar, and we summarize and discuss the relevant results in section 5.[9]

3 Partial Market Access for Poor and Complete Access for Rich

For case (2), $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$, let us first derive the expression for demand faced by firm j. It follows from the demand structure discussed in section 2.1 that

$$D_j = (1 - f) \cdot \left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + \dfrac{2t}{n}}{2t} \right] + f \cdot 2 \left[\frac{Y_P (\theta - 1) - p_j}{t} \right]. \tag{3}$$

The first segment of demand comes from the rich, the second segment from the poor. Notice that the rich segment of demand is independent of the rich income since firm j is competing with its adjacent firms for the rich consumers. On the other hand the size of the poor segment is determined by the poor income. An increase in the number of firms reduces the size of the rich segment while the poor segment remains unaffected. The own price effect dominates the cross price effect within the rich segment, and the presence of the poor segment reinforces this domination.

[9] In section 5 we also discuss two other cases exemplifying the 'kinked equilibrium' possibilities as in Salop (1979): (4) $Y_R > \overline{Y}$ and $Y_P = \overline{Y}$, and (5) $Y_R = \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$.

The price response to demand is given by $\dfrac{\partial D_j}{\partial p_j} = -\left(\dfrac{1+f}{t}\right)$. Note that since the demand loss due to increased price is larger in the poor segment, an increase in the proportion of poor increases the price response to demand. On the other hand an increase in travel cost makes it costlier to access the facilities which in turn reduces the price response to demand.

To determine the equilibrium price and number of firms we proceed in the standard backward fashion. In stage 2, given the entry decision in stage 1, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$2(1+f)p_j - \left(\tfrac{1-f}{2}\right)p_{j-1} - \left(\tfrac{1-f}{2}\right)p_{j+1} = (1-f)\frac{t}{n} + 2fY_P(\theta-1) + c(1+f),\ \ j = 1,2,...,n.$$

It is easy to see that this linear system has a unique solution,[10]

$$p_1 = p_2 = ... = p_n = \left(\tfrac{1-f}{1+3f}\right)\frac{t}{n} + \left(\tfrac{2f}{1+3f}\right)Y_P(\theta-1) + \left(\tfrac{1+f}{1+3f}\right)c \equiv p. \tag{4}$$

In stage 1, firms' entry decision is determined by the zero-profit condition. Using (3) and (4) the common expression for profit becomes

$$\pi_j = (p_j - c)D_j - F = \left[\left(\tfrac{1-f}{1+3f}\right)\frac{t}{n} + \left(\tfrac{2f}{1+3f}\right)[Y_P(\theta-1)-c]\right]^2 \cdot \left(\frac{1+f}{t}\right) - F,\ \ j = 1,2,...,n.$$

Hence the zero-profit condition implies

$$\left[\left(\tfrac{1-f}{1+3f}\right)\frac{t}{n} + \left(\tfrac{2f}{1+3f}\right)[Y_P(\theta-1)-c]\right]^2 \cdot \left(\frac{1+f}{t}\right) - F = 0. \tag{5}$$

Using (4) and (5) we derive the equilibrium price and number of firms:

$$p = c + \sqrt{\frac{tF}{1+f}}, \tag{6}$$

$$\frac{1}{n} = \frac{1}{t(1-f)}\left[(1+3f)\sqrt{\frac{tF}{1+f}} - 2f[Y_P(\theta-1)-c]\right]. \tag{7}$$

Note that since the firms are competing for rich consumers, both price and number of firms are independent of the rich income. Price is independent of the poor income also. But, since the poor forms a captive market for the firms the size of which is restricted by their income, number of firms increases with the poor income. As poor income increases, demand

[10] The coefficient matrix of this system of equations forms a *circulant* matrix (in a circulant matrix each row vector is rotated one element to the right relative to the preceding row vector). The solution is unique since the determinant of a circulant matrix is non-zero if the sum of the elements of a row is non-zero.

size of each firm increases, and, price remaining the same, each firm makes more than normal profit. This super-normal profit attracts fresh entry of firms into the neighbourhood.

Before we investigate this case any further, it is important to identify parameter values, in particular the income ranges of rich and poor, under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$, where the income thresholds $\overline{Y}$ and $\underline{Y}$ are endogenous (as expressed in equations (1) and (2) above). Substituting the equilibrium values of price and number of firms into the expressions for $\overline{Y}$ and $\underline{Y}$ we find that $Y_P < \overline{Y}$ implies $Y_P(\theta - 1) - c < \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}$, whereas $Y_P > \underline{Y}$ implies $Y_P(\theta - 1) - c > \sqrt{\frac{tF}{1+f}}$. Combining the two we get $\sqrt{\frac{tF}{1+f}} < Y_P(\theta - 1) - c < \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}$. Similarly, $Y_R > \overline{Y}$ implies $[(1-f)Y_R + fY_P](\theta - 1) - c > \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}$. Thus we conclude that case (2) arises when the poor and rich incomes are such that

$$\begin{aligned} \frac{c + \sqrt{\frac{tF}{1+f}}}{(\theta - 1)} \quad < \quad & Y_P < \frac{c + \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}}{(\theta - 1)} \\ & \text{and} \\ (1-f)Y_R + fY_P \quad > \quad & \frac{c + \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}}{(\theta - 1)}. \end{aligned} \tag{8}$$

So we have identified an upper income threshold and a lower income threshold for the poor income such that if the poor income is in between these two thresholds whereas the rich income is high enough so that the average income is higher than the upper income threshold, then the firms do not compete with the adjacent firms for the poor consumers but do so only for the rich consumers. All the rich consumers are served by the market, but some poor are left out – only those poor who are located closer to the firms get served. These two income thresholds are shown in Figure 3.

With the help of these two income thresholds we can now see how equilibrium price and number of firms respond to changes in the proportion of poor. This will be useful to understand the mechanism of the impact of income inequality on the welfare of the poor analyzed in the next section. We find that both equilibrium price and number of firms increases steadily as the proportion of poor (f) decreases from 1 to 0.[11] Y_P and Y_R remaining

[11] While it is obvious from equation (6) that $\frac{\partial p}{\partial f} < 0$, from equation (7) we derive

$$\frac{\partial}{\partial f}\left(\frac{1}{n}\right) = \frac{\left(3f^2 + 6f + 7\right)\sqrt{\frac{tF}{1+f}} - 4(1+f)\left[Y_P(\theta - 1) - c\right]}{2t(1-f)^2(1+f)} > 0$$

the same as f decreases the society or the neighbourhood becomes richer and the average willingness to pay of the society increases. This induces the existing firms to charge a higher price, and, at the same time, attracts fresh entry into the neighbourhood.

The following proposition summarizes the discussion in this section.

Proposition 1. *When the rich and poor incomes are such that condition* (8) *holds, then*

(a) *the rich has complete market access while the poor has partial access*; *only those poor residing closer to the facilities have access to them*, *others get excluded*;

(b) *equilibrium price and number of firms are given by equations* (6) *and* (7) *respectively*;

(c) *equilibrium price and number of firms are independent of the rich income*; *while price is independent of the poor income also*, *number of firms increases with the poor income*;

(d) *both equilibrium price and number of firms increases as the proportion of poor decreases.*

4 Income Inequality, Market Access and Welfare of Poor

Now we use the generic case (2) to analyze the impact of income inequality on the market access and welfare of the poor.

Consider the rich consumers first. Since all the rich consumers are served, the market access of the poor can be thought of as in proportion to that of the rich. To calculate the aggregate consumer surplus of the rich community as a whole we proceed as follows. Surplus to a rich consumer located at a distance x from the firm from which it is buying is $Y_R\theta - p - tx - Y_R$.[12] Since there are n firms each with a market coverage of $\frac{1}{2n}$ on either side of its location, the aggregate consumer surplus of the rich community is

$$CS_R = 2n \int_0^{\frac{1}{2n}} \left[Y_R (\theta - 1) - p - tx\right] dx = Y_R (\theta - 1) - p - \frac{t}{4n}.$$

As expected, consumer surplus increases with income (Y_R) and number of firms (n), and decreases with travel cost (t) and price (p). Since price and number of firms are endogenous,

since, under case (2), $\frac{3+f}{2}\sqrt{\frac{tF}{1+f}} > Y_P(\theta-1) - c$ implies
$(3f^2+6f+7)\sqrt{\frac{tF}{1+f}} - 4(1+f)\left[Y_P(\theta-1)-c\right] > (1-f)^2\sqrt{\frac{tF}{1+f}} \geq 0.$

[12]Recall that the reservation utility of the rich is Y_R.

substituting their equilibrium values from equations (6) and (7) we derive the expression for aggregate consumer surplus of the rich community solely in terms of the parameters of the model:

$$CS_R = Y_R(\theta-1) + \frac{f}{2(1-f)} Y_P(\theta-1) - c\left[1 - \frac{f}{2(1-f)}\right] - \sqrt{\frac{tF}{1+f}}\left[1 + \frac{1+3f}{4(1-f)}\right]. \quad (9)$$

It is interesting to note that consumer surplus of the rich increases even when the income of the poor increases. As noted in the last section, as poor income increases price remains the same but number of firms increases. Increased number of firms implies greater accessibility of the product or service (leading to less travel cost) for the rich and hence their consumer surplus increases.

Coming to the poor consumers, consider their market access first. Not all the poor can afford to buy the product: only the poor up to the distance $\frac{Y_P(\theta-1)-p}{t}$ from any firm are buying the product; those in between the distance $\frac{Y_P(\theta-1)-p}{t}$ and $\frac{1}{2n}$ cannot afford it. Hence the aggregate market access of the poor community is

$$A_P = 2n \int_0^{\frac{Y_P(\theta-1)-p}{t}} dx = \frac{2n}{t}\left[Y_P(\theta-1) - p\right].$$

The tension between price and number of firms is clear: an increase in number of firms increases market access while a price increase reduces it. Substituting the equilibrium values of price and number of firms we get

$$A_P = \frac{2(1-f)}{(1+3f)\sqrt{\frac{tF}{1+f}} - 2f\left[Y_P(\theta-1) - c\right]}\left[Y_P(\theta-1) - c - \sqrt{\frac{tF}{1+f}}\right]. \quad (10)$$

Note that the aggregate market access of poor increases as poor income increases. In fact it is easy to check that $A_P \to 0$ as $Y_P \to \frac{c+\sqrt{\frac{tF}{1+f}}}{(\theta-1)}$, whereas $A_P \to 1$ as $Y_P \to \frac{c+\frac{3+f}{2}\sqrt{\frac{tF}{1+f}}}{(\theta-1)}$; in between these two bounds A_P increases steadily as Y_P increases. There are two effects at work. First is the direct effect: as income increases market access of the consumers increases. Second effect is the indirect effect working through the increase in number of firms as poor income increases. Both the effects work in the same direction reinforcing each other. Since price is independent of income, there is no counteracting force at work.

Finally consider the aggregate consumer surplus of the poor. Since the poor in between the distance $\frac{Y_P(\theta-1)-p}{t}$ and $\frac{1}{2n}$ from any firm does not buy the product, their consumer surplus is zero. Hence the aggregate consumer surplus of the poor is

$$CS_P = 2n\left[\int_0^{\frac{Y_P(\theta-1)-p}{t}} \left[Y_P(\theta-1) - p - tx\right] dx\right] = \frac{n}{t}\left[Y_P(\theta-1) - p\right]^2.$$

Similar to aggregate market access, an increase in number of firms increases aggregate consumer surplus of the poor while a price increase reduces it. Substituting the equilibrium values we derive

$$CS_P = \frac{(1-f)}{(1+3f)\sqrt{\frac{tF}{1+f}} - 2f\left[Y_P(\theta-1)-c\right]}\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f}}\right]^2. \tag{11}$$

Similar to market access, aggregate consumer surplus of the poor also increases steadily as poor income increases in between the lower and upper bounds.

Now to see the effect of income inequality on the market access and welfare of the poor we first conduct the following comparative static analysis: we vary f keeping Y_P and Y_R fixed. That is, we follow the poor with the same income level and compare the aggregate market access and consumer surplus of the poor community as a whole when they live in relatively richer societies (as f decreases from 1 to 0).

For this comparative static exercise let us rewrite the expression for aggregate market access of poor as

$$A_P = 2\left(\frac{1-f}{f}\right)\left(\frac{\chi\sqrt{1+f}-1}{3+\frac{1}{f}-2\chi\sqrt{1+f}}\right)$$

where $\chi \equiv \frac{Y_P(\theta-1)-c}{\sqrt{tF}}$ captures, in a nutshell, all the parameters of the model other than f. This expression becomes quite handy in depicting the market access of poor as a function of f treating χ as the parameter. Note from condition (8) that $\frac{1}{\sqrt{1+f}} < \chi < \frac{3+f}{2\sqrt{1+f}}$. Figure 1 depicts the aggregate market access of poor as f varies from 0 to 1 for some illustrative values of parameter χ within this range. In Figure 1 higher values of χ shifts the A_P curve upwards illustrating the point mentioned above that market access of the poor increases as the poor income increases.

Similarly we can express the aggregate consumer surplus of poor as

$$\frac{CS_P}{\sqrt{tF}} = \left(\frac{1-f}{f}\right)\left[\frac{\left(\chi\sqrt{1+f}-1\right)^2}{3+\frac{1}{f}-2\chi\sqrt{1+f}}\right]\frac{1}{\sqrt{1+f}}.$$

Figure 2 illustrates this relationship as a function of f for the same parameter values of χ as in Figure 1. Again higher values of χ shifts the CS_P curve upwards illustrating that consumer surplus of poor increases as the poor income increases.[13]

[13]Note that Figure 2 shows the aggregate consumer surplus of poor in proportion to $\sqrt{tF}$. To read the consumer surplus from the figure, we have to multiply the height of each point on the figure by $\sqrt{tF}$. This will have no impact on the inverted-U shape of the curve.

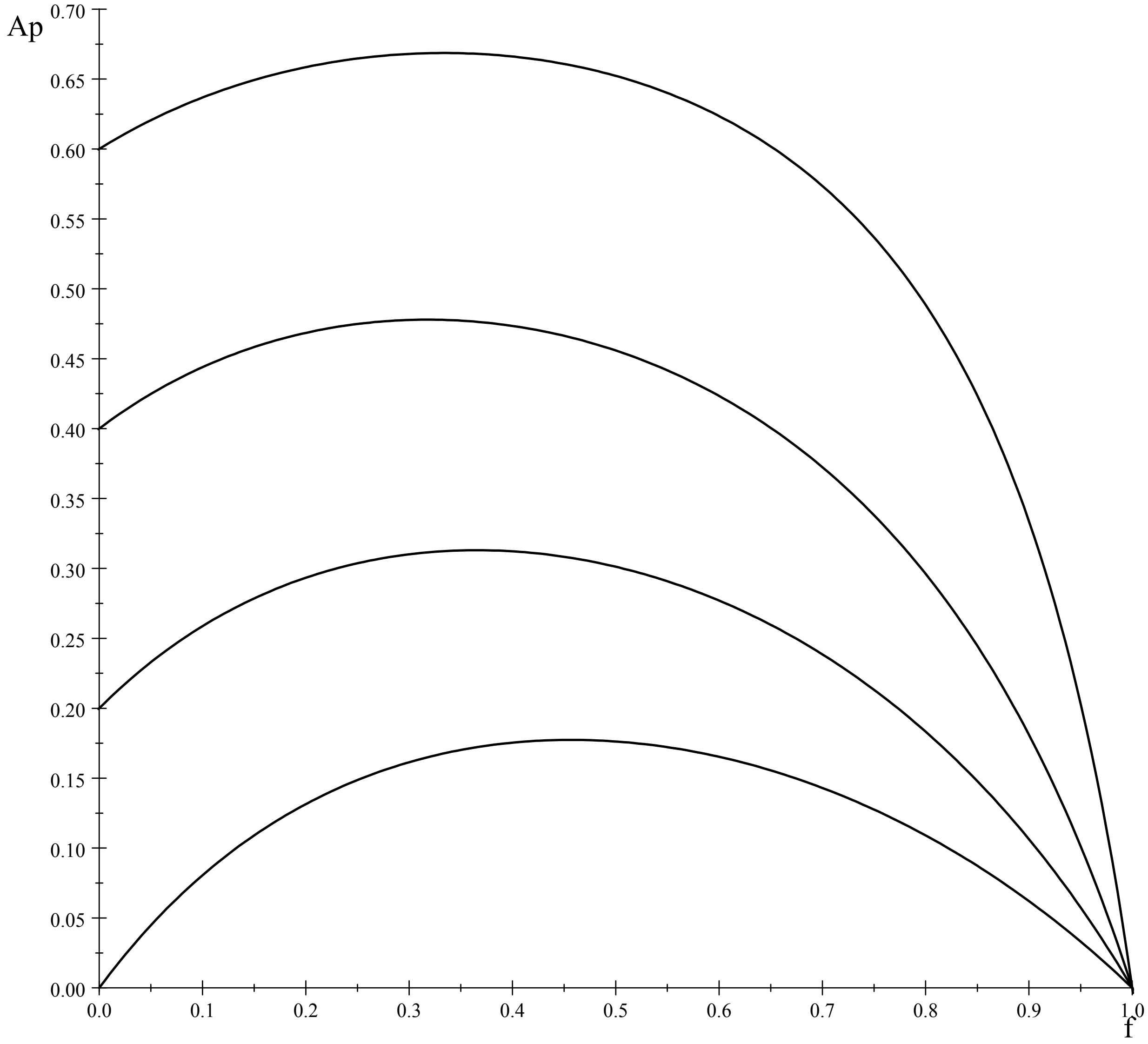

Figure 1: Aggregate Market Access of the Poor

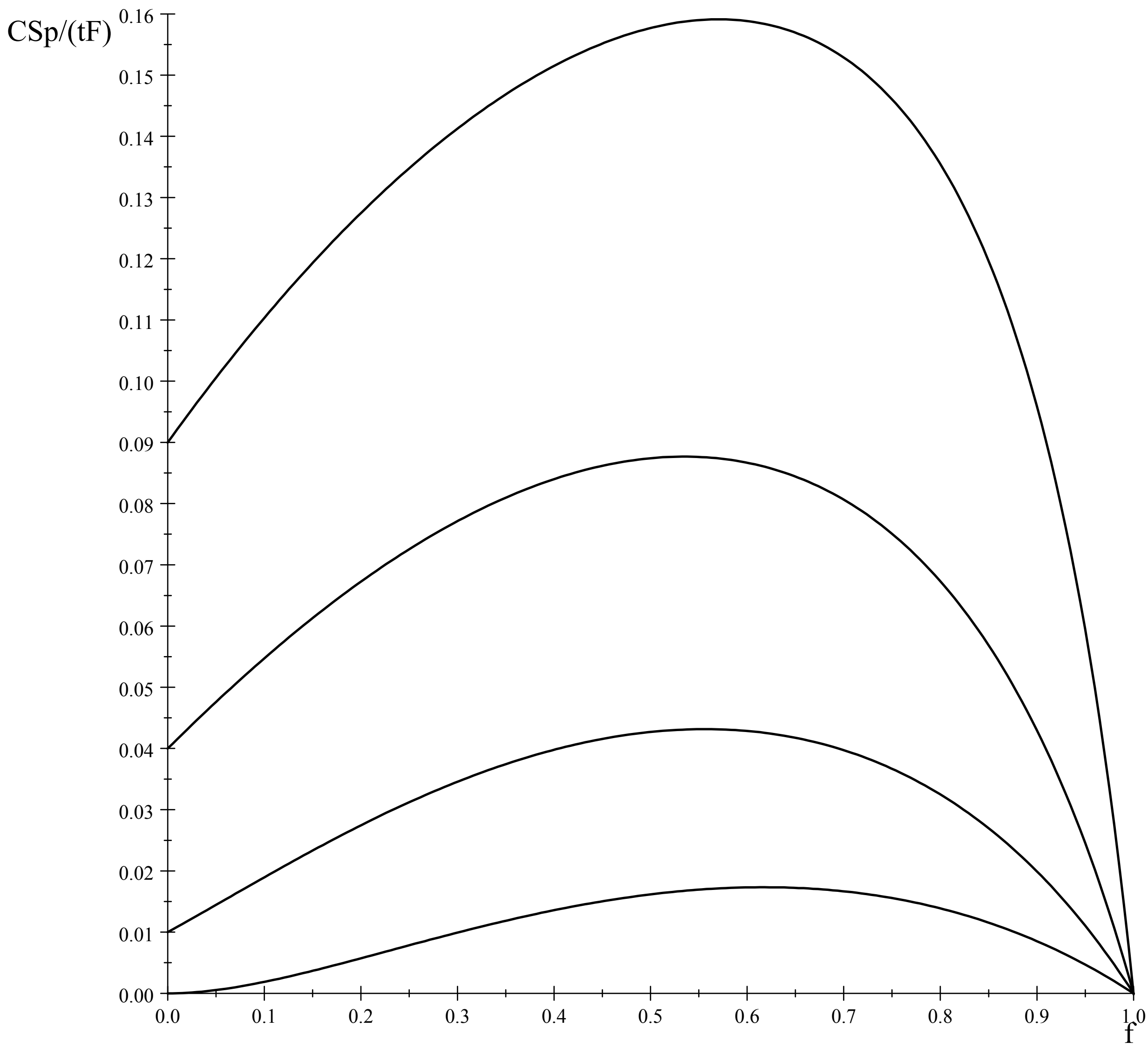

Figure 2: Aggregate Consumer Surplus of the Poor

It is very interesting to observe the "inverted-U" shape relationships between proportion of poor people in the neighbourhood (f) and their aggregate market access (A_P) and consumer surplus (CS_P).[14] That is, if we compare a cross-section of neighbourhoods, the poor community as a whole is initially better-off living in relatively richer neighbourhoods (as f decreases from 1). But, beyond a point, both the aggregate market access of the poor and their consumer surplus start declining as the neighbourhood becomes richer. Instead of comparing a cross-section of societies if we consider the same society then this result can be interpreted as follows. Since Y_P and Y_R remain the same, as the proportion of poor (f) decreases the society becomes richer. If we restrict our attention to those who still remain poor, then their market access and consumer surplus demonstrate inverted-U shape relationships as the society becomes richer.

The reason for these inverted-U shape relationships can be traced to the behaviour of equilibrium price and number of firms. As established in the last section (see Proposition 1(d)), both price and number of firms increases steadily as f decreases from 1 to 0. Consumers benefit from the increase in number of firms but lose from the increase in price. For the poor community as a whole the number of firms effect dominates initially: the poor located closer to the firms get to consume the product and the number of poor served increases as the number of firms increases. But, beyond a point, the adverse price effect takes over.

It is important to highlight the role of the spatial structure, in particular to point out that we are getting the inverted-U shape in both market access and consumer surplus because the number of firms are also changing endogenously. When we conduct the same analysis with number of firms fixed, both market access and consumer surplus of the poor decreases steadily as f decreases; that is, we do not see any inverted-U shape in the relationships. The reason is that price increases steadily without any compensating increase in the number of firms.

The following proposition summarizes the relationships between the proportion of poor and the aggregate market access and welfare of the poor community as a whole.

Proposition 2. *When the rich and poor incomes are such that the rich has complete market access while the poor has only partial access, then there exists an "inverted-U" shape relationship between the proportion of poor people in the neighbourhood* (f) *and their aggregate market access* (A_P) *and consumer surplus* (CS_P)*: as f decreases from* 1 *to* 0, *both A_P and CS_P initially increase, reach a maximum and then fall.*

[14] These "inverted-U" shape relationships are established in details in Appendix A.1.

In the comparative static exercise conducted above note that since Y_P and Y_R remain the same as the proportion of poor (f) decreases the society becomes richer. In order to capture the role of inequality in its purest form let us next examine the effect of mean-preserving spread: we increase Y_R together with an increase in f keeping Y_P and the average income of the society $(fY_P + (1-f)\,Y_R)$ fixed.

The result of this comparative static exercise follows in a straight-forward way from the last two propositions. Note from equations (10) and (11) that neither market access nor consumer surplus of poor depends on the rich income. Hence the effect of mean-preserving spread works only through the increase in the proportion of poor. Because of the inverted-U relationship encountered in Proposition 2 it follows that the effect of the mean-preserving spread depends on which part of the inverted-U we start from. If the initial proportion of poor is to the left of the peak of the inverted-U, then both market access and consumer surplus of poor will increase for a substantial range of increase in the spread before reverting back to the downward trend. On the other hand, if the initial proportion of poor is to the right of the peak of the inverted-U, then market access and consumer surplus of poor decrease with the increase in the spread.

The intuition for this result can again be traced to the behaviour of equilibrium price and number of firms. Observe that mean-preserving spread works again only through the increase in the proportion of poor since neither price nor number of firms depends on the rich income. It follows that an increase in the mean-preserving spread decreases both price and number of firms. We have already noted in the context of Proposition 2 that to the left of the peak of the inverted-U price effect dominates while the number of firms effect dominates to the right of the peak.

The following proposition summarizes the relationships between mean-preserving spread and market access and welfare of the poor.

Proposition 3. *When the rich and poor incomes are such that the rich has complete market access while the poor has only partial access, then the effect of mean-preserving spread depends on the initial proportion of poor.*

(a) *If the initial proportion of poor is to the left of the peaks of the inverted-U relationships, then both market access and consumer surplus of poor increase in the beginning, reach a maximum and then fall as the spread increases.*

(b) *If the initial proportion of poor is to the right of the peaks of the inverted-U relationships, then both market access and consumer surplus of poor decrease steadily as the spread increases.*

5 Characterizing the Equilibrium

In the last two sections we have analyzed in detail the generic case (2) where all the rich consumers are served but only some of the poor consumers are served, others are left out of the market. Analyses of the other cases are similar and, for the sake of brevity, we do not repeat the detailed analyses in the text and relegate it to Appendix A.2. Instead, in this section we summarize the income ranges of rich and poor under which different cases arise and discuss the implications of income inequality in characterizing the nature of equilibrium.

5.1 Summary of Different Equilibrium Possibilities

5.1.1 Complete Market Access for both Rich and Poor

Complete market access for the poor occurs when their income is high enough; this happens under case (1): $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$, and case (4): $Y_R > \overline{Y}$ and $Y_P = \overline{Y}$. Analysis of these two cases leads to the following proposition.

Proposition 4.

(a) *When the rich and poor incomes are such that*

$$Y_R > Y_P > \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)},$$

then firms compete for both consumer types – rich and poor, and all consumers of each type are served.

(b) *If, instead, the rich and poor incomes are such that*

$$\frac{c + \dfrac{3+f}{2}\sqrt{\dfrac{tF}{1+f}}}{(\theta - 1)} < Y_P < \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)} < Y_R,$$

then firms compete for the rich, but the marginal poor who is indifferent between two adjacent firms is also indifferent between buying and not buying; all consumers of each type are served though.

5.1.2 Complete Market Access for Rich and No Access for Poor

On the other extreme complete exclusion of the poor occurs when their income is low enough. When the poor income is low, and, at the same time, firms charge a high enough price, it becomes impossible for a poor consumer to afford the product even when they are located at the same location as the firm. Firms completely ignore the presence of the poor and choose the price considering as if there are only rich individuals residing in the neighbourhood. This happens under case (3): $Y_R > \overline{Y}$ and $Y_P < \underline{Y}$. Analysis of this case can be summarized in the following proposition.

Proposition 5. *When the rich and poor incomes are such that*

$$Y_P < \frac{c + \sqrt{\frac{tF}{1-f}}}{(\theta - 1)} \quad \text{and} \quad Y_R > \frac{c + \frac{3}{2}\sqrt{\frac{tF}{1-f}}}{(\theta - 1)},$$

then firms compete only for the rich and all the rich consumers are served; but all the poor consumers are left out.

5.1.3 Partial Market Access for Poor

In section 3 we have discussed one situation of partial market access for poor when rich income is high enough that firms compete for all the rich consumers. Another case of poor's partial market access arises when the rich income is not that high; it is reasonably high in the sense that the marginal rich consumer is also indifferent between buying and not buying. This happens under case (5): $Y_R = \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$. The following proposition summarizes the analysis of this case.

Proposition 6. *When the rich and poor incomes are such that*

$$\frac{c + \sqrt{\frac{tF}{2}}}{(\theta - 1)} < Y_P < \frac{c + \sqrt{\frac{tF}{1+f}}}{(\theta - 1)}$$

$$\text{and}$$

$$\frac{c + \sqrt{2tF}}{(\theta - 1)} < (1-f)\,Y_R + fY_P < \frac{c + \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}}{(\theta - 1)},$$

then firms not only exert monopoly power over the poor, but even the marginal rich is also indifferent between buying and not buying. All the rich consumers are served though. The poor has a partial access – only those residing closer to the facilities are served, others get excluded.

Figure 3 summarizes all these equilibrium possibilities by plotting the lower and upper bounds of incomes for different values of f, the proportion of poor people.

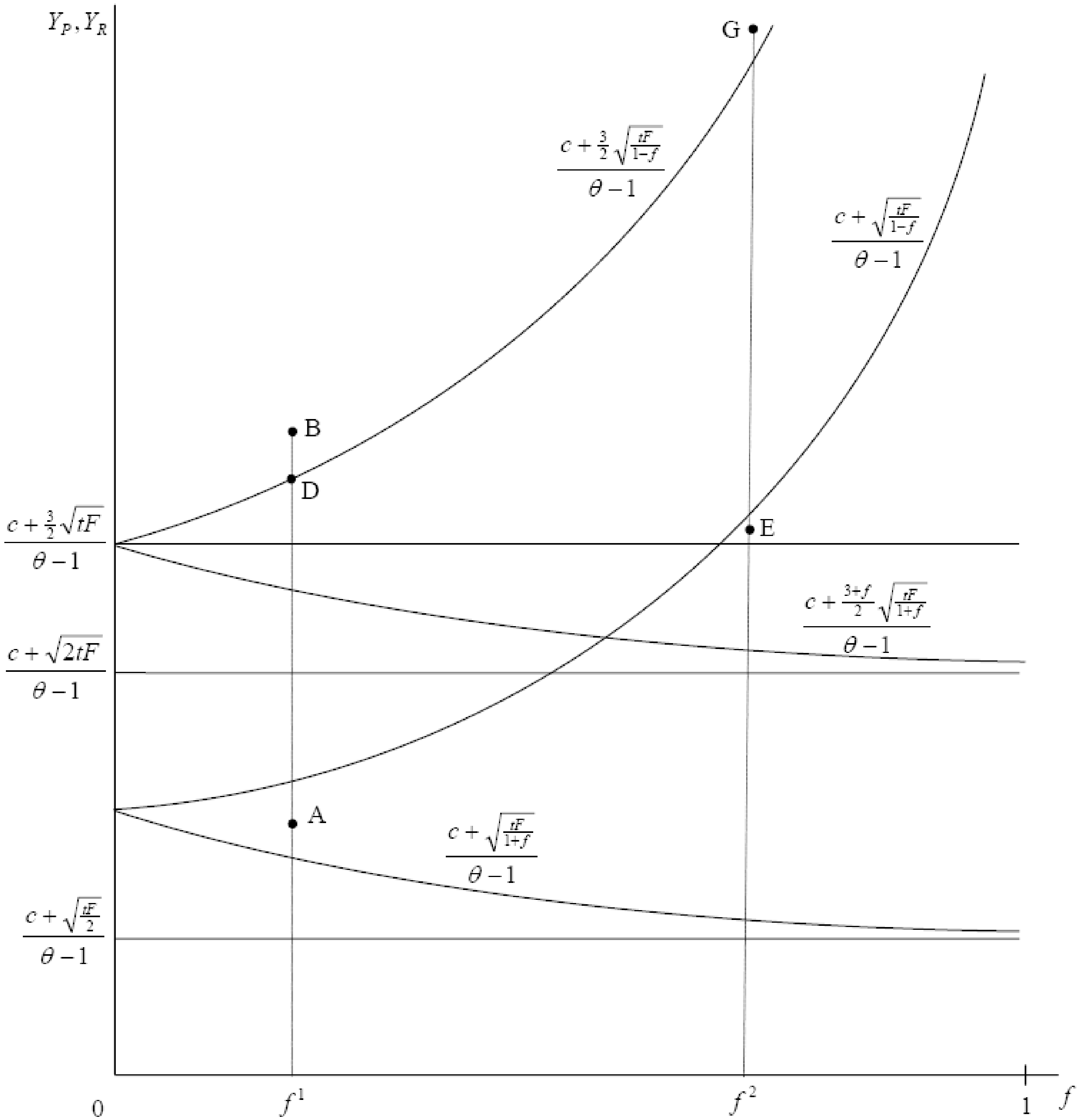

Figure 3: Different Equilibrium Possibilities

5.2 Implications of Income Inequality

Our analysis of the different equilibrium possibilities summarized in the last section has a number of implications of income inequality.

5.2.1 Upper threshold for $\mathbf{Y}_P$

From propositions 1 and 4 it is clear that there exists an upper income threshold for Y_P, call it $\overline{Y}_P$, defined by

$$\overline{Y}_P \equiv \frac{c + \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}}{(\theta - 1)}$$

such that all poor consumers are served only if $Y_P \geq \overline{Y}_P$.

Proposition 4(a) shows the existence of another income threshold, $\frac{c+\frac{3}{2}\sqrt{tF}}{(\theta-1)} > \overline{Y}_P$, such that if the income of the poor is above this threshold, then not only all poor consumers are served but, in addition, each firm has to compete with its adjacent firms for *both* poor and rich customers. Equilibrium price and number of firms reflect this competition (see Appendix A.2).

5.2.2 Lower thresholds for $\mathbf{Y}_P$

There are two lower income thresholds for the poor, $\underline{Y}_P$, such that no poor consumer is served if $Y_P < \underline{Y}_P$. Interestingly which threshold is relevant depends on the income of the rich.

When the rich income is high enough so that firms are competing for the rich, then it follows from proposition 1 that the lower income threshold for the poor is given by

$$\underline{Y}_P^1 \equiv \frac{c + \sqrt{\frac{tF}{1+f}}}{(\theta - 1)}.$$

But when the rich income is reasonably low in the sense that the marginal rich is indifferent between buying and not buying (proposition 6), then this lower income threshold becomes

$$\underline{Y}_P^2 \equiv \frac{c + \sqrt{\frac{tF}{2}}}{(\theta - 1)}.$$

Implication of Income Gap between Rich and Poor:

Notice that $\underline{Y}_P^2 < \underline{Y}_P^1$, that is, the lower income threshold of the poor is lower when the rich income is reasonably low. Thus the poor are better off when the income gap between the rich and poor is lower.

When the poor income is in between the upper and lower income thresholds, there are pockets of the city where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

5.2.3 Possibility of Multiple Equilibria

Interestingly this model identifies the possibility of multiple equilibria. Consider, for example, the income distribution depicted by points A and B in Figure 3: there are f^1 proportion of poor with income given by the height of A and $(1 - f^1)$ proportion of rich with income B. The income distribution is such that parameter configurations for both cases (2) and (3) are satisfied, generating the multiple equilibria. The equilibrium under case (2) is a good equilibrium where at least some poor (if not all of them) get served, those who are located closer to the firms. The equilibrium outcome under case (3) is a bad outcome: all the poor are excluded; the firms completely ignore their presence and choose the price as if there were only rich individuals residing in the neighbourhood. It is worthwhile to point out that the other adaptations of the Salop (1979) framework – for example, Bhaskar and To (1999, 2003) and Brekke et al. (2008) – could not identify this multiple equilibria possibility as they have concentrated only on the generic case (2).

Implication of Income Gap:

Note once again the implication of higher income gap between rich and poor. If the rich income were below the height of D, then this complete exclusion possibility of the poor would not have arisen. It is the higher income gap that exposes the poor to this vulnerable situation.

The implication of income gap could be even more damaging for a multiple equilibria situation like the one depicted by the other income distribution shown in Figure 3: there are f^2 proportion of poor with income given by the height of E and $(1 - f^2)$ proportion of rich with income G. Here the multiplicity occurs with cases (1) and (3). Recall from proposition 4(a) that case (1) is the best possible outcome that can happen to the poor – income of the poor is high enough so that all the poor are served, and, at the same time, the firms are forced to compete for them. But even then a higher income gap exposes them to the possibility of complete exclusion.

The Case of Minority Poor:

Poor are more likely to be completely excluded when they are a minority, that is, when f is low: firms may completely ignore the poor even when the rich are not ultra rich just because the rich are more in number. For instance, in Figure 3, with the same income levels A for poor and B for rich, the complete exclusion possibility does not arise when the proportion of poor is f^2; but this possibility does arise when the proportion of poor is f^1.

5.3 Comparison with a Single-Income Neighbourhood

In section 4 we have identified scenarios where the poor could be better-off living in relatively richer societies. To see how the possibility arises in the simplest possible way it is interesting to compare our model economy with two income groups with a single-income neighbourhood. A single-income neighbourhood refers to a city inhabited by a single income group; that is, there is a measure 1 of consumers with the same income Y distributed uniformly along the city circumference. The single-income neighbourhood model is analyzed in Appendix A.3 and the relevant comparison is highlighted below.

The Feasibility Income Threshold in a Single-Income Neighbourhood:

In a single-income neighbourhood it is not feasible for any firm to operate unless the common income is at least $\frac{c+\sqrt{2tF}}{(\theta-1)}$. If the income is below this feasibility threshold, the willingness to pay is so low that it is not possible for the firms to recover the fixed cost of production. The implication for a single-income *poor* neighbourhood with common income Y_P is that nobody gets to enjoy the product or service when $Y_P < \frac{c+\sqrt{2tF}}{(\theta-1)}$.[15]

Comparing Single-Income with Mixed-Income Neighbourhoods:

With reference to a single-income neighbourhood, a mixed-income neighbourhood is the one that we are considering so far where there are f proportion of poor with income Y_P and $(1-f)$ proportion of rich with income Y_R distributed uniformly along the circumference of the city. Since both the lower income thresholds of the poor, $\underline{Y}_P^1$ and $\underline{Y}_P^2$, are strictly less

[15]Note that $\frac{c+\sqrt{2tF}}{(\theta-1)} = \lim_{f\to 1} \frac{c+\frac{3+f}{2}\sqrt{\frac{tF}{1+f}}}{(\theta-1)} = \lim_{f\to 1} \overline{Y}_P$.

than the feasibility threshold, $\dfrac{c+\sqrt{2tF}}{(\theta-1)}$, it is clear that poor are better-off staying in the mixed-income neighbourhood as long as the poor income is below this feasibility threshold. At least some poor get to enjoy the product or service in the mixed-income neighbourhood as the firms recover their fixed costs due to the higher willingness to pay of the rich. This is not possible in a single-income *poor* neighbourhood.

6 Conclusion

The chief contribution of this paper is to model the interaction between neighbourhood effects and income inequality in a simple and tractable way by integrating consumers' income distribution with the spatial distribution of their location. While the basic analytical structure is adapted from the industrial organization literature (Salop, 1979; Bhaskar and To, 1999, 2003; Brekke et al., 2008), this literature does not explore the implications of income inequality. On the other hand, the literature on income inequality has not typically investigated the implications of inequality operating through industrial structure. This paper complements this literature by exploring the impact of income inequality working through price and number of firms.

We find inverted-U shape relationships between income inequality and market access and welfare of the poor. If we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies by having access to a wider varieties of products and services. But, beyond a point, the aggregate consumer surplus of the poor starts declining as the society becomes richer: the welfare gain from increase in access to wider varieties of products and services is not enough to offset the corresponding rise in price. Our results square well with the inverted-U relationship between health status and neighbourhood inequality found by Feng and Yu (2007) and Li and Zhu (2006) using the China Health and Nutrition Survey (CHNS) data.

As an added bonus, we identify the possibility of multiple equilibria so far overlooked by the industrial organization literature: a bad equilibrium where all the poor are excluded can exist simultaneously with a good equilibrium where at least some poor (if not all of them) get served by the market. We have isolated the higher income gap between rich and poor as the key factor that exposes the poor to this complete exclusion possibility. Finally we compare a mixed-income neighbourhood where rich and poor live side by side with a single-income homogeneous neighbourhood and find that the poor are better-off living in the mixed neighbourhood as long as the poor income is below a certain feasibility threshold.

7 Appendix

A.1 Inverted-U Relationships[16]

- **Relationship between Proportion of Poor (f) and their Aggregate Market Access (A_P)**

Recall that the expression for aggregate market access of poor is

$$A_P = 2\left(\frac{1-f}{f}\right)\left(\frac{\chi\sqrt{1+f}-1}{3+\frac{1}{f}-2\chi\sqrt{1+f}}\right)$$

where $\chi \equiv \dfrac{Y_P(\theta-1)-c}{\sqrt{tF}}$ and $\dfrac{1}{\sqrt{1+f}} < \chi < \dfrac{3+f}{2\sqrt{1+f}}$. We establish the inverted-U relationship between A_P and f in three steps: Step I: $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=0} > 0$; Step II: $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=1} < 0$; Step III: A_P reaches a maximum between 0 and 1.

Step I: When $f = 0$, χ varies between 1 and 1.5. The following Maple plot of $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=0}$ when χ varies between 1 and 1.5 shows that $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=0} > 0$.

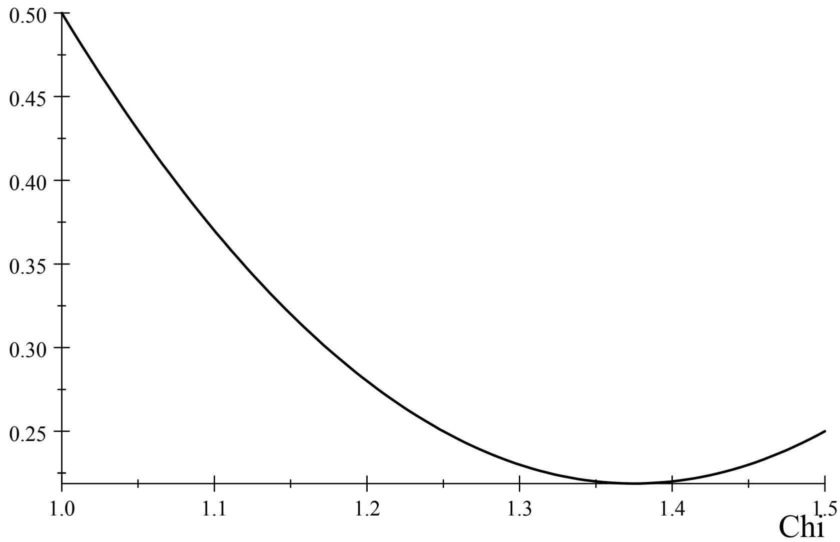

Figure A.1: Maple Plot of $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=0}$

[16] In establishing the inverted-U relationships between f and A_P and between f and CS_P we need to take first and second derivatives of A_P and CS_P with respect to f and evaluate the second derivatives at the points where the first derivative is zero. Since the algebraic expressions are quite cumbersome, we do not report them here. Instead, we report the Maple plots (supported by Scientific WorkPlace) of the relevant expressions.

Step II: When $f = 1$, χ varies between $\frac{1}{\sqrt{2}}$ and $\sqrt{2}$. The following Maple plot of $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=1}$ when χ varies between $\frac{1}{\sqrt{2}}$ and $\sqrt{2}$ shows that $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=1} < 0$.

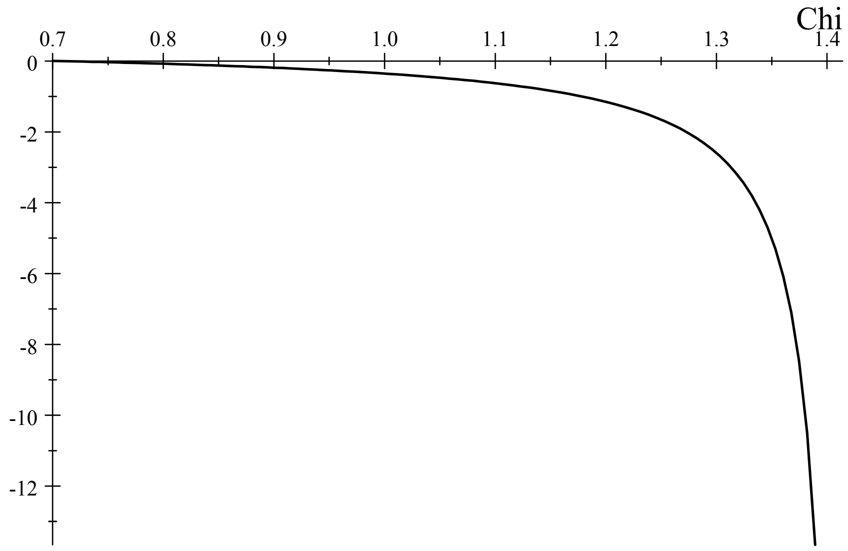

Figure A.2: Maple Plot of $\left.\dfrac{\partial A_P}{\partial f}\right|_{f=1}$

Step III: The following Maple plot shows the combinations of (f, χ) for which $\dfrac{\partial A_P}{\partial f} = 0$.

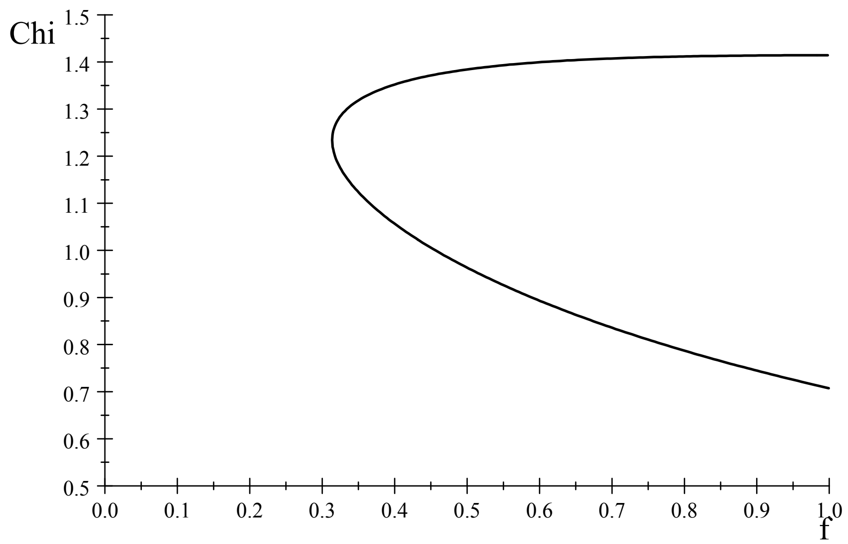

Figure A.3: Maple Plot of $\dfrac{\partial A_P}{\partial f} = 0$

For A_P to reach a maximum it is sufficient to show that $\dfrac{\partial^2 A_P}{\partial f^2}$ evaluated at these points is negative. The following Maple plot shows that this indeed is the case.

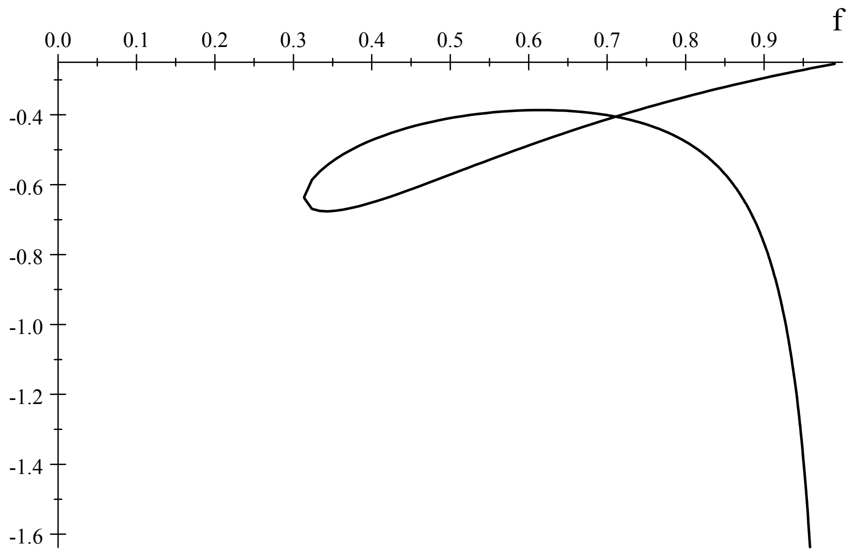

Figure A.4: Maple Plot of $\dfrac{\partial^2 A_P}{\partial f^2}$ **Evaluated at** (f, χ) **where** $\dfrac{\partial A_P}{\partial f} = 0$

- **Relationship between Proportion of Poor (f) and their Aggregate Consumer Surplus (CS_P)**

The expression for aggregate consumer surplus of poor is

$$\frac{CS_P}{\sqrt{tF}} = \left(\frac{1-f}{f}\right)\left[\frac{\left(\chi\sqrt{1+f}-1\right)^2}{3+\frac{1}{f}-2\chi\sqrt{1+f}}\right]\frac{1}{\sqrt{1+f}}.$$

We establish the inverted-U relationship between CS_P and f by following the same three steps as in case of aggregate market access of poor.

Step I: The following Maple plot of $\left.\frac{\partial\left(CS_P/\sqrt{tF}\right)}{\partial f}\right|_{f=0}$ when χ varies between 1 and 1.5 shows that $\left.\frac{\partial CS_P}{\partial f}\right|_{f=0} > 0$.

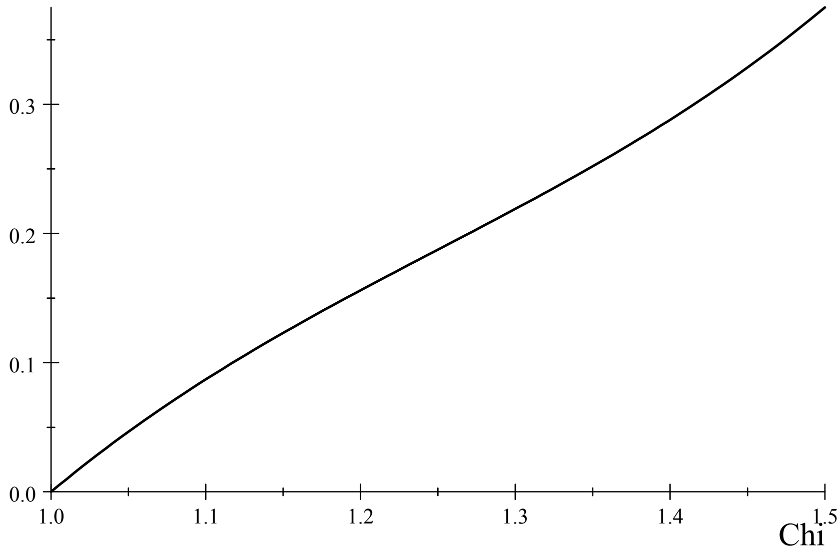

Figure A.5: Maple Plot of $\left.\frac{\partial\left(CS_P/\sqrt{tF}\right)}{\partial f}\right|_{f=0}$

Step II: The following Maple plot of $\left.\frac{\partial\left(CS_P/\sqrt{tF}\right)}{\partial f}\right|_{f=1}$ when χ varies between $\frac{1}{\sqrt{2}}$ and $\sqrt{2}$ shows that $\left.\frac{\partial CS_P}{\partial f}\right|_{f=1} < 0$.

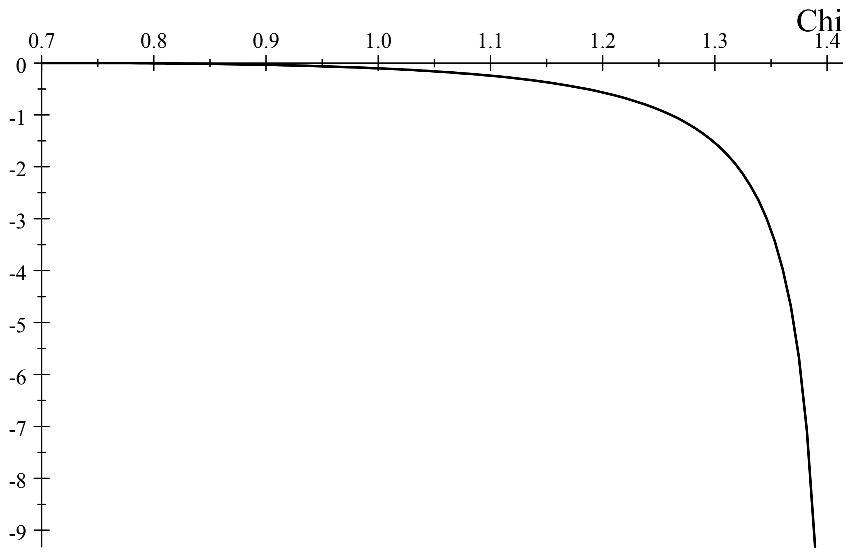

Figure A.6: Maple Plot of $\left.\frac{\partial\left(CS_P/\sqrt{tF}\right)}{\partial f}\right|_{f=1}$

Step III: The following Maple plot shows the combinations of (f, χ) for which $\dfrac{\partial CS_P}{\partial f} = 0$.

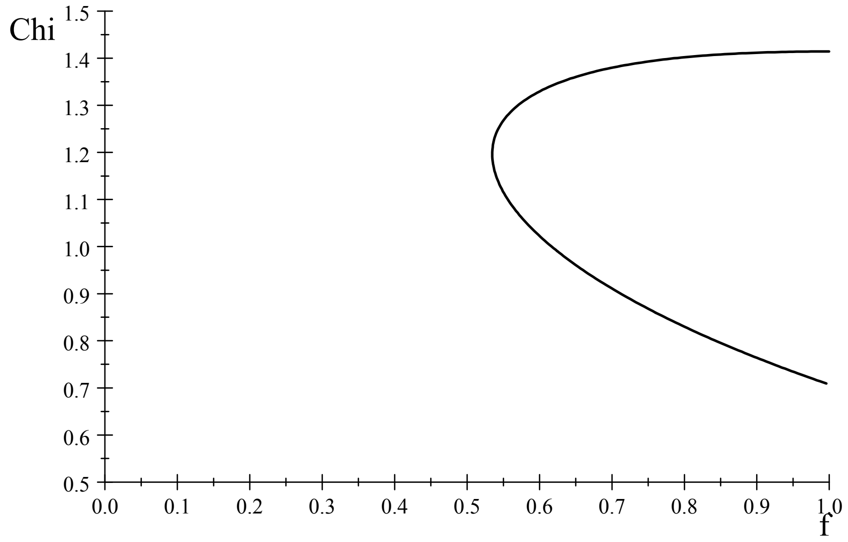

Figure A.7: Maple Plot of $\dfrac{\partial CS_P}{\partial f} = 0$

For CS_P to reach a maximum it is sufficient to show that $\dfrac{\partial^2 CS_P}{\partial f^2}$ evaluated at these points is negative. The following Maple plot shows that this indeed is the case.

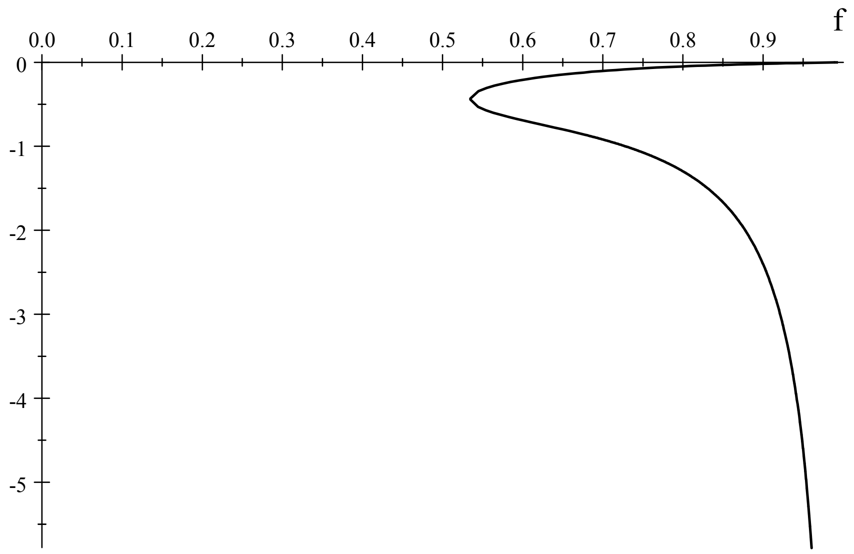

Figure A.8: Maple Plot of $\dfrac{\partial^2 CS_P}{\partial f^2}$ **Evaluated at** (f, χ) **where** $\dfrac{\partial CS_P}{\partial f} = 0$

A.2 Details of the Different Cases

Case (1): $\mathbf{Y}_R > \overline{Y}$ and $\mathbf{Y}_P > \overline{Y}$: Complete Market Access for both Rich and Poor

From the demand structure discussed in section 2.1 it follows that:

$$D_j = (1-f)\cdot[\delta_{j,j+1}+\delta_{j,j-1}] + f\cdot[\delta_{j,j+1}+\delta_{j,j-1}] = \frac{(p_{j-1}+p_{j+1}-2p_j)+\dfrac{2t}{n}}{2t}. \qquad \text{(A.1)}$$

This implies that $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$. In stage 2, given the entry decision in stage 1, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$2p_j - \frac{p_{j-1}}{2} - \frac{p_{j+1}}{2} = \frac{t}{n} + c,\ \ j = 1,2,...,n.$$

This linear system has the unique solution

$$p_1 = p_2 = ... = p_n = \frac{t}{n} + c \equiv p. \qquad \text{(A.2)}$$

In stage 1, firms' entry decision is determined by the zero-profit condition. Using (A.1) and (A.2) the common expression for profit becomes

$$\pi_j = (p_j - c)\, D_j - F = \frac{t}{n^2} - F,\ \ j = 1,2,...,n,$$

so that the zero-profit condition implies

$$\frac{t}{n^2} - F = 0. \qquad \text{(A.3)}$$

Using (A.2) and (A.3) we derive the equilibrium price and number of firms:

$$p = c + \sqrt{tF}, \text{ and } \frac{1}{n} = \sqrt{\frac{F}{t}}.$$

Now we identify the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$, where the upper income threshold $\overline{Y}$ is

$$\overline{Y} = \frac{p}{\theta-1} + \frac{t}{2n(\theta-1)}.$$

Substituting the equilibrium values of price and number of firms into the expressions for $\overline{Y}$ implies:

$$Y_P\,(\theta-1) - c > \frac{3}{2}\sqrt{tF}.$$

Thus we conclude that case (1) arises when

$$Y_R > Y_P > \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta-1)}.$$

Case (3): $\mathbf{Y}_R > \overline{Y}$ and $\mathbf{Y}_P < \underline{Y}$: Complete Market Access for Rich; No Access for Poor

In this case the demand for firm j is given by

$$D_j = (1-f) \cdot [\delta_{j,j+1} + \delta_{j,j-1}] = (1-f) \cdot \left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + \dfrac{2t}{n}}{2t} \right]. \tag{A.4}$$

This implies that $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1-f}{t}$. In stage 2, given the entry decision in stage 1, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$2p_j - \frac{p_{j-1}}{2} - \frac{p_{j+1}}{2} = \frac{t}{n} + c, \;\; j = 1, 2, ..., n.$$

This linear system has the unique solution

$$p_1 = p_2 = ... = p_n = \frac{t}{n} + c \equiv p. \tag{A.5}$$

In stage 1, firms' entry decision is determined by the zero-profit condition. Using (A.4) and (A.5) the common expression for profit becomes

$$\pi_j = (p_j - c)\, D_j - F = \frac{t\,(1-f)}{n^2} - F, \;\; j = 1, 2, ..., n,$$

so that the zero-profit condition implies

$$\frac{t\,(1-f)}{n^2} - F = 0. \tag{A.6}$$

Using (A.5) and (A.6) we derive the equilibrium price and number of firms:

$$p = c + \sqrt{\frac{tF}{1-f}}, \text{ and } \frac{1}{n} = \sqrt{\frac{F}{t\,(1-f)}}.$$

Now we identify the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $Y_P < \underline{Y}$, where the upper and lower income thresholds are given by $\overline{Y} = \dfrac{p}{\theta - 1} + \dfrac{t}{2n(\theta-1)}$ and $\underline{Y} = \dfrac{p}{\theta-1}$. Substituting the equilibrium values of price and number of firms we find that $Y_P < \underline{Y}$ implies $Y_P < \dfrac{c + \sqrt{\dfrac{tF}{1-f}}}{\theta - 1}$, whereas $Y_R > \overline{Y}$ implies $Y_R > \dfrac{c + \dfrac{3}{2}\sqrt{\dfrac{tF}{1-f}}}{\theta - 1}$.

Case (4): $\mathbf{Y}_R > \overline{Y}$ and $\mathbf{Y}_P = \overline{Y}$:

This is the case of a 'kinked equilibrium' as in Salop (1979). One extreme of the kink is case (1) described above where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$. The other extreme is case (2) discussed in section 3 where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\left(\frac{1+f}{t}\right)$.

Note that since $Y_P = \overline{Y} = \frac{p}{\theta - 1} + \frac{t}{2n(\theta-1)}$, we have

$$p = Y_P(\theta - 1) - \frac{t}{2n}.$$

For the first extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$, proceeding as in case (1) we can derive the equilibrium price and number of firms as

$$p = c + \sqrt{tF}, \text{ and } \frac{1}{n} = \sqrt{\frac{F}{t}}.$$

Since $p = c + \sqrt{tF}$ and, at the same time, $p = Y_P(\theta - 1) - \frac{t}{2n}$, this implies

$$\frac{1}{n} = \frac{2}{t}\left[Y_P\left(\theta - 1\right) - c\right] - 2\sqrt{\frac{F}{t}}.$$

But we have $\frac{1}{n} = \sqrt{\frac{F}{t}}$. It follows that this extreme case arises under the special circumstance when

$$Y_P\left(\theta - 1\right) - c = \frac{3}{2}\sqrt{tF}. \tag{A.7}$$

For the other extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\left(\frac{1+f}{t}\right)$, proceeding as in case (2) we have

$$p = c + \sqrt{\frac{tF}{1+f}}, \text{ and } \frac{1}{n} = \frac{1}{t\left(1-f\right)}\left[\left(1+3f\right)\sqrt{\frac{tF}{1+f}} - 2f\left(Y_P\left(\theta - 1\right) - c\right)\right].$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$Y_P\left(\theta - 1\right) - c = \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}. \tag{A.8}$$

Combining these two extremes it follows from (A.7) and (A.8) that case (4) arises when

$$\frac{c + \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}}{(\theta - 1)} < Y_P < \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)}.$$

Case (5): $\mathbf{Y}_R = \overline{Y}$ and $\underline{Y} < \mathbf{Y}_P < \overline{Y}$:

Since $Y_R = \overline{Y}$, this exemplifies another case of 'kinked equilibrium'. One extreme of the kink is case (2) discussed in section 3 where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\left(\frac{1+f}{t}\right)$. For the other extreme the demand from the rich is such that total demand is given by

$$\begin{aligned} D_j &= (1-f)\cdot\left[\eta_{j,j+1}(Y_R)+\eta_{j,j-1}(Y_R)\right]+f\cdot\left[\eta_{j,j+1}(Y_P)+\eta_{j,j-1}(Y_P)\right] \\ &= 2(1-f)\left[\frac{Y_R(\theta-1)-p_j}{t}\right]+2f\left[\frac{Y_P(\theta-1)-p_j}{t}\right], \end{aligned}$$

so that the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\left[\frac{2(1-f)+2f}{t}\right] = -\frac{2}{t}$.

Note that since $Y_R = \overline{Y} = \frac{p}{\theta-1} + \frac{t}{2n(\theta-1)}$, we have

$$p = Y_R(\theta-1) - \frac{t}{2n}.$$

For the first extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\left(\frac{1+f}{t}\right)$, proceeding as in case (2) we can derive the equilibrium price and number of firms as

$$p = c + \sqrt{\frac{tF}{1+f}}, \text{ and } \frac{1}{n} = \frac{1}{t(1-f)}\left[(1+3f)\sqrt{\frac{tF}{1+f}} - 2f\left(Y_P(\theta-1)-c\right)\right].$$

Since $p = c + \sqrt{\frac{tF}{1+f}}$ and, at the same time, $p = Y_R(\theta-1) - \frac{t}{2n}$, this implies

$$\frac{1}{n} = \frac{2}{t}\left[Y_R(\theta-1)-c\right] - \frac{2}{t}\sqrt{\frac{tF}{1+f}}.$$

But we have $\frac{1}{n} = \frac{1}{t(1-f)}\left[(1+3f)\sqrt{\frac{tF}{1+f}} - 2f\left(Y_P(\theta-1)-c\right)\right]$. It follows that this extreme case arises under the special circumstance when

$$\left[(1-f)Y_R + fY_P\right](\theta-1) - c = \frac{3+f}{2}\sqrt{\frac{tF}{1+f}}. \tag{A.9}$$

For the other extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{2}{t}$, using the first-order condition, demand structure and the zero-profit condition we derive

$$p = c + \sqrt{\frac{tF}{2}}, \text{ and } \frac{1}{n} = \sqrt{\frac{2F}{t}} + \frac{2f\left(Y_R - Y_P\right)(\theta-1)}{t}.$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$[(1-f)\,Y_R + fY_P]\,(\theta-1) - c = \sqrt{2tF}. \tag{A.10}$$

At the same time, $Y_P > \underline{Y}$ implies, for this extreme case,

$$\frac{c+\sqrt{\dfrac{tF}{2}}}{(\theta-1)} < Y_P. \tag{A.11}$$

Combining (A.9), (A.10) and (A.11) and the fact that the lower bound for Y_P is $\dfrac{c+\sqrt{\dfrac{tF}{1+f}}}{(\theta-1)}$ under case (2) which is just the other extreme for case (5), we conclude that case (5) arises when

$$\frac{c+\sqrt{\dfrac{tF}{2}}}{(\theta-1)} < Y_P < \frac{c+\sqrt{\dfrac{tF}{1+f}}}{(\theta-1)}$$

and

$$\frac{c+\sqrt{2tF}}{(\theta-1)} < (1-f)\,Y_R + fY_P < \frac{c+\dfrac{3+f}{2}\sqrt{\dfrac{tF}{1+f}}}{(\theta-1)}.$$

A.3 Single-Income Neighbourhood

Consider a circular city where there is a measure 1 of consumers with the same income Y distributed uniformly along the city circumference. Since there is only one income, we have the following three cases to consider:

(1) $Y > \overline{Y}$;

(2) $Y = \overline{Y}$;

(3) $\underline{Y} < Y < \overline{Y}$.

Case (1): $\boldsymbol{Y} > \overline{Y}$:

From the demand structure discussed in section 2.1 it follows that:

$$\begin{aligned} D_j &= \delta_{j,j+1} + \delta_{j,j-1} \\ &= \frac{(p_{j-1} + p_{j+1} - 2p_j) + \frac{2t}{n}}{2t}. \end{aligned}$$

This implies that $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$. Now, similar to the analysis of the two-income groups, using the first-order condition, demand structure and the zero-profit condition we derive

$$p = c + \sqrt{tF},$$

$$\frac{1}{n} = \sqrt{\frac{F}{t}}.$$

Substituting these equilibrium values of price and number of firms into the expressions for $\overline{Y}$ we conclude that case (1) arises when

$$Y > \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)}.$$

In this case firms compete for the consumers and all consumers are served.

Case (2): $\boldsymbol{Y = \overline{Y}}$:

This, once again, is a case of a 'kinked equilibrium'. One extreme of the kink is case (1) described above where the price response to demand is given by $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$. For the other extreme, demand is given by

$$\begin{aligned} D_j &= \eta_{j,j+1}(Y) + \eta_{j,j-1}(Y) \\ &= 2\left[\frac{Y(\theta-1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{2}{t}$.

Note that since $Y = \overline{Y} = \dfrac{p}{\theta - 1} + \dfrac{t}{2n(\theta-1)}$, we have

$$p = Y(\theta - 1) - \frac{t}{2n}.$$

For the first extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$, proceeding as in case (1) we can derive the equilibrium price and number of firms as

$$p = c + \sqrt{tF}, \text{ and } \frac{1}{n} = \sqrt{\frac{F}{t}}.$$

Since $p = c + \sqrt{tF}$ and, at the same time, $p = Y(\theta - 1) - \dfrac{t}{2n}$, this implies

$$\frac{1}{n} = \frac{2}{t}\left[Y(\theta - 1) - c\right] - 2\sqrt{\frac{F}{t}}.$$

But we have $\dfrac{1}{n} = \sqrt{\dfrac{F}{t}}$. It follows that this extreme case arises under the special circumstance when

$$Y(\theta - 1) - c = \frac{3}{2}\sqrt{tF}. \tag{A.12}$$

For the other extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{2}{t}$, using the first-order condition, demand structure and the zero-profit condition we derive

$$p = c + \sqrt{\frac{tF}{2}}, \text{ and } \frac{1}{n} = \sqrt{\frac{2F}{t}}.$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$Y(\theta - 1) - c = \sqrt{2tF}. \tag{A.13}$$

Combining (A.12) and (A.13) we conclude that case (2) arises when

$$\sqrt{2tF} < Y(\theta - 1) - c < \frac{3}{2}\sqrt{tF},$$

that is, when

$$\frac{c + \sqrt{2tF}}{(\theta - 1)} < Y < \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)}.$$

In this case also all the consumers are served, but the marginal consumer who is indifferent between two adjacent firms is also indifferent between buying and not buying.

Case (3): $\underline{Y} < \mathbf{Y} < \overline{Y}$ **:**

This is the second extreme of case (2) discussed above where demand is given by

$$\begin{aligned} D_j &= \eta_{j,j+1}(Y) + \eta_{j,j-1}(Y) \\ &= 2\left[\frac{Y(\theta - 1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{2}{t}$. As above, using the first-order condition, demand structure and the zero-profit condition, we derive

$$p = c + \sqrt{\frac{tF}{2}}, \text{ and } \frac{1}{n} = \sqrt{\frac{2F}{t}}.$$

In equilibrium $D_j = \frac{1}{n}$. Then $D_j = 2\left[\frac{Y(\theta - 1) - p_j}{t}\right]$ and $p = c + \sqrt{\frac{tF}{2}}$ give

$$\frac{1}{n} = \frac{2}{t}\left[Y(\theta - 1) - c - \sqrt{\frac{tF}{2}}\right].$$

Since $\frac{1}{n} = \sqrt{\frac{2F}{t}}$, and, at the same time, $\frac{1}{n} = \frac{2}{t}\left[Y(\theta - 1) - c - \sqrt{\frac{tF}{2}}\right]$, it follows that

$$Y(\theta - 1) - c = \sqrt{2tF}.$$

So we conclude that case (3) can occur under this limiting case where $Y(\theta - 1) - c = \sqrt{2tF}$. Following Salop (1979) we can ignore this limiting case.

- From the analysis of the three cases under the single-income neighbourhood it is clear that the minimum income required for any firm to operate is such that $Y(\theta - 1) - c = \sqrt{2tF}$. That is, the feasibility income threshold is $\frac{c + \sqrt{2tF}}{(\theta - 1)}$.

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