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Abstract

In the presence of temptation and self-control preferences as in Gul and Pesendorfer, the optimal policy is to subsidize savings when consumers are tempted by "excessive" impatience (Krusell, Kuruşçu and Smith, 2010). However, in the homogeneous agents model, taxation loses an important property in that it fails to reduce the inequality through redistribution. Thus the phenomenon that welfare improves on subsidizing savings may vanish when the agents differ in their abilities to earn income. They may well choose a positive tax if they are from low ability group where the redistribution effect of tax dominates the temptation effect. In a political economy, a situation may easily arise where a negative tax will never be implemented. When agents are homogeneous, as temptation grows, optimal subsidy on saving increases. The corresponding result in the heterogeneous agents case is that as temptation grows, the political support for the subsidy increases.

Keywords : Temptation, self-control, optimal tax, voting

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1 Introduction

Laibson (1997), based on the earlier work by Strotz (1956) and Phelps and Pollak (1968), developed a model where the agents have time inconsistent preferences as time passes. It can explain the phenomenon that as time progresses, consumers exhibit preference reversals. The literature on time inconsistency has been further enriched by the introduction of a new class of utility function (Gul and Pesendorfer (2001, 2004, 2005)). This function presents an alternative approach to modeling preference reversals. The utility function under Gul and Pesendorfer preference consists of two parts to formalize the concepts of temptation and self-control: a commitment utility measuring the preference on actual consumption choice and a temptation utility measuring the preference on consumption would have been chosen had she succumbed to temptation. As the agent makes the actual choice, she always incurs the cost of deviating from the temptation, i.e. cost of self-control, and thus the actual choice is a compromise between commitment utility and the cost of self control.

Using Gul and Pesendorfer type of utility function in a standard macroeconomic setting, Krusell, Kuruşçu and Smith (2010) have studied optimal taxation in a closed economy. They have shown that the optimal policy in this framework is to subsidize savings when consumers are tempted by impatience.[1] When the period utility is logarithmic, they have also shown that as the horizon grows large, the optimal policy recommendation is a constant subsidy which is in contrast to the well known Chamley (1986) and Jude (1985) result. Thus, subsidy on savings can be used as an instrument to improve welfare because it makes surrender to temptations less attractive. Since the

[1] Subsidizing savings is generally desirable in a setup where the agents have limited rationality. For example, Amador, Werning and Angeletos (2006) study the optimal trade-off between commitment and flexibility, and show that imposing a minimum level of savings is always a feature of the solution. Also, in a multiple-selves consumption-savings model, Laibson (1996) argued that optimal policy is to subsidize savings.

agents are homogeneous, an investment subsidy increases the welfare for all. Naturally in this setting, the tax instrument plays a very limited role. More specifically, taxation loses one of its most important properties in that it fails to reduce the inequality in the economy through redistribution of the tax revenue collected.

However, in an economy with heterogeneous agents, for a certain proportion of people, the temptation effect for which a subsidy is desired, may well be dominated by the redistribution effect of taxation. As a result, we can no longer claim that a subsidy on investment is welfare improving for all. This in turn answers the concerns that Krusell, Kuruşçu and Smith (2010) raised that "in reality these taxes are positive......and it is highly likely that these outcomes have political economy underpinnings".[2] To model heterogeneity, we assume that the agents differ in terms of their ability and given a wage rate, as ability increases, the earnings of the individual also increases. The effect of redistribution is monotonically decreasing with the level of income. Thus people from comparatively lower (degree depends on the distribution) ability group prefer a positive tax rate so that the gain from redistribution can improve their welfare by dominating the effect of a negative tax that was required to improve welfare in the presence of temptations in order to make it less attractive. That means, for these people in the lower side of the income distribution ladder, since the redistribution effect is so high that it dominates the temptation effect, if they are given an opportunity to choose their tax rates, they prefer a positive tax instead of a subsidy. For those people who have relatively higher ability, the redistribution effect cannot dominate the temptation effect and since a subsidy on savings make temptations less attractive, they prefer a subsidy so that it can compensate not only the effect of temptation but also the loss due to taxation for having higher earnings.

[2] Acemoglu, Golosov, and Tsyvinski (2008) presented a political-economic analysis of optimal taxation where they focus on the best equilibrium that satisfies the incentive compatibility constraints of politicians.

Thus, in this economy when agents are heterogenous it is certain that not all individuals will choose a negative tax rate. Naturally in this setup, under some conditions, it may very well be the case that a negative tax rate is never implemented. Also a further question may arise here. Since heterogeneity automatically brings two distinct preferences by the two groups of people with regards to the sign of the tax rate, it is worthwhile to investigate the role that temptation and self control play here. We show that as the strength of temptation increases, the support of the positive tax unambiguously falls, that is, as temptation increases, lesser population choose a positive tax and thus more people prefer a subsidy on savings.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the analytical results of partial equilibrium and general equilibrium for the case of logarithmic utility function. A more general utility function with a constant inter-temporal elasticity of substitution preference is studied in Section 4 by means of a numerical example to explore the effect of interest rate on the consumers and how the equilibrium responds when there is a change in the elasticity of intertemporal substitution. Finally section 5 concludes. The appendix contains all the proofs.

2 The Model

We consider a simple two-period economy consisting of a continuum of agents assumed to be of measure one. Each agent is young and endowed with one unit of labor in the first period. She becomes old and retires in the second period. The agent i has Gul and Pesendorfer preferences with two components of utility $u(c_{1,i}, c_{2,i})$ and $v(c_{1,i}, c_{2,i})$ representing commitment utility and temptation utility respectively. Her decision problem is

$$\max_{c_{1,i},c_{2,i}} \{u(c_{1,i},c_{2,i}) + v(c_{1,i},c_{2,i})\} - \max_{\widetilde{c}_{1,i},\widetilde{c}_{2,i}} v(\widetilde{c}_{1,i},\widetilde{c}_{2,i}). \tag{1}$$

Agent's actual choice maximizes the sum $u(c_{1,i},c_{2,i}) + v(c_{1,i},c_{2,i})$ of the commitment and temptation utilities, and for any choice bundle $(c_{1,i},c_{2,i})$, the cost of self-control is given by max $v(\widetilde{c}_{1,i},\widetilde{c}_{2,i}) - v(c_{1,i},c_{2,i})$.

The crucial assumption of this paper is that agents are not identical - they differ in their abilities to earn income. Let θ_i represent the ability to earn income by the agent i with $\theta_i \in [0,1]$ and let it follow the distribution $F(\theta_i)$. Given a wage rate w, the agent's income is $\theta_i w$. Thus θ_i measures the effective labor supply of agent i. Further, we assume that the mean of θ_i is greater than the median of θ_i, that is $E(\theta_i) = \overline{\theta} > \theta^{med}$.

There is a single final good produced using a constant returns to scale production function $F(K,L)$ where K denotes the capital input and L denotes the labor input. We assume that F takes the Cobb-Douglas form, i,e.,

$$F(K,L) = AK^{\alpha}L^{1-\alpha},\ A > 0,\ \alpha \in (0,1). \tag{2}$$

The final good can either be consumed or it can be saved to provide capital.

Each young agent supplies labor inelastically in competitive labor markets, earning a wage of $w\theta_i$. Since $\overline{\theta}$ measures the total effective labor supply in this economy, the wage rate is

$$w \equiv (1-\alpha) AK^{\alpha}L^{-\alpha} = (1-\alpha) AK^{\alpha}\overline{\theta}^{-\alpha} \tag{3}$$

Capital is traded in competitive capital markets, and earns a gross real return of r between two periods where

$$r \equiv \alpha AK^{1-\alpha}\overline{\theta}^{1-\alpha}, \tag{4}$$

with $r'(K) < 0$.

Suppose the government collects capital tax from the young and returns the total tax revenue as a lump-sum to the young. Following the proportional tax system in Krusell, Kuruşçu and Smith (2010), we let τ and s be the proportional tax imposed on investment and the lump-sum return respectively. We impose the balanced budget condition of the government in the following way:

$$s = \tau K \tag{5}$$

where

$$K \equiv \int_{\theta_i=0}^{1} k_i(\theta_i)\, dF(\theta_i)\,. \tag{6}$$

Then as a consumer, agent i maximizes (1) subject to the following budget constraints:

$$c_{1,i} = w\theta_i + s - (1+\tau)\, k_i\ ; \tag{7}$$

$$c_{2,i} = rk_i; \tag{8}$$

$$\widetilde{c}_{1,i} = w\theta_i + s - (1+\tau)\, \widetilde{k}_i\ ; \tag{9}$$

$$\widetilde{c}_{2,i} = r\widetilde{k}_i. \tag{10}$$

where $\widetilde{c}_{1,i}$, $\widetilde{c}_{2,i}$ and $\widetilde{k}_i$ denote the choice that would have been made had the consumer succumbed to temptation. Equations (7) and (8) are the budget constraints for the problem of maximizing the sum of commitment utility and temptation utility. Equations (9) and (10) correspond the problem of maximizing the temptation utility.

Finally, we assume that in this economy the capital tax rate τ is decided by voting. Thus each agent is not only a consumer but also a political voter. We suppose that the young generation votes for the capital tax rate τ. As a political voter, agent i chooses the level of τ_i that maximizes her value function. For simplicity, for rest of the analysis, we assume that the period-wise utility functions are additively separable.

3 With Logarithmic utility

To obtain analytical results, we consider the instantaneous utility function that takes a logarithmic form. More specifically we assume $u(c_{1,i}, c_{2,i}) = \ln c_{1,i} + \delta \ln c_{2,i}$ and $v(c_{1,i}, c_{2,i}) = \gamma \{\ln c_{1,i} + \delta\beta \ln c_{2,i}\}$, where the parameters in this representation have the same interpretations as in Krusell, Kuruşçu and Smith (2010). While δ represents the long-run discount rate, $\delta\beta$ is the discount rate in the short run, where $\beta < 1$ measures the temptation impatience relative to commitment impatience. The parameter γ represents the strength of the temptation; in particular, the consumer completely surrenders to temptation when $\gamma \to \infty$ while $\gamma \to 0$ represents the full commitment case. The cost of self control is expressed by the term $\gamma \{\ln \widetilde{c}_{1,i} + \delta\beta \ln \widetilde{c}_{2,i} - (\ln c_{1,i} + \delta\beta \ln c_{2,i})\}$. We present both the partial and general equilibrium results under this logarithmic utility framework, starting with the partial equilibrium as presented below.

3.1 Partial equilibrium

In order to compare the results with Krusell, Kuruşçu and Smith (2010), we first consider a partial equilibrium framework where the factor prices of labor and capital are exogenously given at w and r respectively. In the following we proceed step by step.

First, we need to solve the problem of the agent as a consumer. We solve for the commitment utility first and then for the temptation utility. The problem involving commitment utility is to maximize

$$\max_{c_{1,i},c_{2,i}} \ (1+\gamma)\ln c_{1,i} + \delta(1+\beta\gamma)\ln c_{2,i} \tag{11}$$

subject to (7) and (8). By substituting the budget constraints into the objective function (commitment utility), we have

$$\max_{k_i}(1+\gamma)\ln[w\theta_i + s - (1+\tau)k_i] + \delta(1+\beta\gamma)\ln(rk_i).$$

The first order condition yields

$$\frac{(1+\gamma)(1+\tau)}{w\theta_i + s - (1+\tau)k_i} = \frac{\delta(1+\beta\gamma)}{k_i}, \tag{12}$$

and the agent's capital investment k_i is then

$$k_i = \frac{\delta(1+\beta\gamma)(w\theta_i + s)}{(1+\tau)[1+\gamma+\delta(1+\beta\gamma)]}. \tag{13}$$

The temptation utility maximization problem is given by

$$\max_{\widetilde{c}_{1,i},\widetilde{c}_{2,i}} \ \ln\widetilde{c}_{1,i} + \delta\beta\ln\widetilde{c}_{2,i}$$

subject to (9) and (10). By solving the problem, we have

$$\widetilde{k}_i = \frac{\delta\beta(w\theta_i + s)}{(1+\tau_i)(1+\delta\beta)}. \tag{14}$$

Lemma 1 *For any individual i, $k_i > \widetilde{k}_i$.*

By substituting (13) into (6) and using the condition (5), we can obtain the total capital investment in this economy as

$$K = \frac{\delta\left(1+\beta\gamma\right)w\overline{\theta}}{(1+\gamma)(1+\tau)+\delta(1+\beta\gamma)}. \tag{15}$$

Now we can solve the voting problem for each agent. Note that as a voter, each agent takes account of the equilibrium conditions (5) and (15) when voting for the capital tax. Alternatively speaking, each voter has an optimal tax rate τ_i that maximizes her own value function $V(\tau_i)$

$$\begin{aligned} V(\tau_i) &= (1+\gamma)\ln\left[w\theta_i+\tau_i K-(1+\tau_i)\,k_i\right]+\delta\left(1+\beta\gamma\right)\ln\left(rk_i\right) \\ &\quad -\gamma\ln\left[w\theta_i+\tau_i k-(1+\tau_i)\,\widetilde{k}_i\right]-\gamma\delta\beta\ln\left(r\widetilde{k}_i\right) \end{aligned} \tag{16}$$

where k_i, $\widetilde{k}_i$ and K are given by (13), (14) and (15) respectively. Since we are interested in the sign of the capital tax rate in an economy with Gul and Pesendorfer preferences, instead of concentrating on median voter's choice, we look for the political support for positive tax rate, i.e. the proportion of the population that prefers positive tax rate to negative tax rate.[3] We find that the voter's preferred tax rate is decreasing in θ_i and thus as stated in the following Proposition, there exists a threshold value of θ_i such that for all agents with smaller (larger) θ_i would support a tax (subsidy) on capital investment.

Proposition 1 *Let* $\theta^*_{PE} \equiv \dfrac{(1+\delta)\left(1+\beta\gamma\right)}{1+\gamma+\delta\left(1+\beta\gamma\right)}\overline{\theta}$. *Then* $\theta^*_{PE} < \overline{\theta}$ *and the optimal*

[3] As shown in the proof of Proposition 1, the single peakness assumption for some θ_i, higher than but close to $\overline{\theta}$, is violated and median voter theorem can not be applied to our model. This occurs because of the fact that τ could be negative. If the choice set of τ is positive, all agents would have single peakness preference.

feasible investment tax τ_i^* chosen by the individual i has the following property:

$$\tau_i^* = \begin{cases} \geqslant 0, & \text{if } \theta_i \leq \theta_{PE}^*, \\ < 0, & \text{if } \theta_i > \theta_{PE}^* \end{cases}. \tag{17}$$

In the above proposition, we express how the relative position matters in their choice of tax when agents face a temptation. When the agents are homogeneous, a negative tax is optimal since it makes succumbing to temptations less attractive for any individual and since homogeneity ensures that this policy it is optimal for all. However when we make the more appealing assumption that the agents are heterogenous, the optimal tax chosen by the individual depends on her relative position in the ability distribution. Since the effect of redistribution monotonically decreases with the level of income, people from comparatively lower ability group prefer a positive tax rate so that the gain from redistribution can improve their welfare by dominating the effect of a negative tax which was required to improve welfare in the presence of temptations in order to make it less attractive. Therefore, for such agents, since the redistribution effect is so high that it dominates the temptation effect, if they are given an opportunity to choose their tax rates, they prefer a positive tax instead of a subsidy on investment. For those people who have a relatively higher ability, the redistribution effect cannot dominate the temptation effect and since a negative tax on savings make temptations less attractive, they prefer a negative tax so that it can compensate not only the effect of temptation but also the loss due to taxation for having higher income. The above result guarantees that in a political equilibrium, there is a possibility that a negative tax on investment may never be the outcome of the voting process. More specifically, if the median voter's type θ^{med} is such that $\theta^{med} < \theta_{PE}^*$, more than half of the population would support a positive tax rate and a negative tax on investment will never be implemented. To conclude how the choice of tax differs with respect to the crucial parameters of the economy, we present the following corollary which directly follows from the above proposition.

Corollary 1 *Incorporating temptations* ($\gamma > 0$) *necessarily reduces the support of a positive tax. As temptation grows, the support of a positive tax rate decreases. However, an increase in the mean of the ability distribution* $\overline{\theta}$ *increases the support of the positive tax.*

The implication of the above result is very clear. First, incorporating temptations necessarily reduces the political support for the positive tax. This implies that under full commitment, the threshold θ_i is higher compared to when temptation is present. When self-control and temptation is introduced in the model, the marginal agents who chose a positive tax in the absence of temptation, will optimally prefer a negative tax because the temptation effect now dominates the redistribution effect from the tax program. As the strength of the temptation γ increases, the temptation effect now becomes larger and the support of the positive tax thus decreases. A similar reasoning is also true for a decrease in the value of β so that the temptation impatience relative to commitment impatience, that is, the gap between the short-run and the long-run discount rate increases and hence the temptation grows larger. We notice a link between the above corollary and the result shown in Krusell, Kuruşçu and Smith (2010). In their result, optimal subsidy on saving increases as temptation grows larger. In our heterogeneous agents case, the support of the negative tax rate increases as temptation grows larger. Thus our result may be thought of as a counterpart of the result in a homogeneous agent version. Also, it is easy to verify that as the mean of the ability distribution $\overline{\theta}$ increases, the support of the positive tax increases as well. $\overline{\theta}$ measures the total resource or income in this economy and as it increases, government's tax revenue increases such that agents can gain more from the redistribution program. Therefore the fraction of the people who choose positive tax increases since the redistribution effect becomes stronger as $\overline{\theta}$ increases.

3.2 General equilibrium

We continue our analysis with the same logarithmic utility function but in a general equilibrium framework. By substituting the wage rate (3) into (13), we obtain the general equilibrium level of capital K. Using that expression of K, we find the equilibrium wage and interest rate. It can be verified that the equilibrium values of K, r and w are as follows:

$$K = \overline{\theta}\left[\frac{\delta\left(1+\beta\gamma\right)\left(1-\alpha\right)A}{(1+\gamma)(1+\tau)+\delta(1+\beta\gamma)}\right]^{\frac{1}{1-\alpha}} \tag{18}$$

$$r = \alpha\frac{(1+\gamma)(1+\tau)+\delta(1+\beta\gamma)}{\delta\left(1+\beta\gamma\right)\left(1-\alpha\right)} \text{ and} \tag{19}$$

$$w = \left(1-\alpha\right)A\left[\frac{\delta\left(1+\beta\gamma\right)\left(1-\alpha\right)A}{(1+\gamma)(1+\tau)+\delta(1+\beta\gamma)}\right]^{\frac{\alpha}{1-\alpha}}. \tag{20}$$

Then as in the case of partial equilibrium, we can derive the political support for the positive capital tax rate.

Proposition 2 *Let*

$$\theta^*_{GE} \equiv \frac{\left(1+\delta\right)\left(1+\beta\gamma\right)}{\delta\left(1+\beta\gamma\right)+\left(1+\gamma\right)\dfrac{\alpha\left(1+\delta\right)}{\left(1-\alpha\right)\delta}}\overline{\theta}.$$

Then if $\theta^*_{GE} < \overline{\theta}$, the optimal feasible investment tax τ^*_i chosen by the individual i has *the following property:*

$$\tau_i^* \begin{cases} \geq 0, & \text{if } \theta_i \leq \max\{1, \theta_{GE}^*\} \\ < 0, & \text{if } \theta_i > \theta_{GE}^* \end{cases} . \tag{21}$$

Note θ_{GE}^* contains an additional term at the denominator, $\alpha(1+\delta)/[(1-\alpha)\delta]$, when we compare the expression with the partial case. Also note that for reasonable values of α and δ (like $\alpha = 0.4$ and $\delta = 0.5$), the additional item in θ_{GE}^* is positive. The comparative static results that hold in corollary 1 are also true under the general equilibrium. Further, the results show that the support for positive tax rate decreases as the output elasticity of capital α increases or the elasticity of labor $(1-\alpha)$ decreases. An increase in α guarantees an increase in the rate of interest and a decrease in the wage rate. Since in a log utility setup interest rate has no effect on savings, a decrease in wage induces the marginal agents to opt for a negative tax expecting that the effect of loss in wage may be compensated by the effect of receiving subsidy through redistribution of tax income. As in the case of partial equilibrium, if the median voter's type θ^{med} is such that $\theta^{med} < \theta_{GE}^* < \overline{\theta}$, a negative tax on investment will not be implemented in the political equilibrium.

4 With more general utility function

In this section we present our result with a more general utility function. Particularly, we assume

$$u(c_{1,i}, c_{2,i}) = \frac{c_{1,i}^{1-\sigma}}{1-\sigma} + \delta \frac{c_{2,i}^{1-\sigma}}{1-\sigma} \tag{22}$$

and

$$v(c_{1,i}, c_{2,i}) = \gamma \left(\frac{c_{1,i}^{1-\sigma}}{1-\sigma} + \delta\beta \frac{c_{2,i}^{1-\sigma}}{1-\sigma} \right). \tag{23}$$

Since this form of the utility function is very common in applied macroeconomic literature, we find it important to discuss our result in this general framework. Since this form gives us the flexibility to explain the sensitivity of the results with respect to the intertemporal elasticity of substitution, we can examine whether the voting results in this framework would deviate from the above results when agent's decision depends on the interest rate. Again we focus on the threshold value θ^* but due to the complexity of the model, we rely on a simulation example to study the case. We proceed as follows: in the first step, we calibrate the parameters for a benchmark economy and then in the second step we initiate the comparative static for θ^* by varying the values for different parameters of this economy. Before the simulation exercise, we present an analytical result which is a variant of Lemma 1.

Lemma 2 *Independent of the elasticity of intertemporal substitution, for any individual* i*,* $k_i > \widetilde{k}_i$*.*

All the calculations are based on general equilibrium model and it can be verified that the partial equilibrium model also has the same pattern. In this numerical example, each period lasts for 25 years, i.e., the remaining life expectancy of an agent entering into the work force is 50 years. Thus by choosing the standard discount rate that is used in the literature, $\delta = 0.99^{25*4} = 0.366$. The values of constant elasticity of intertemporal substitution (σ^{-1}) and the share of capital income are also standard, i.e. $\sigma = 2$ and $\alpha = 1/3$. The productivity level A is a scale parameter and is set to 10. According to U.S. Census Bureau (2010), the U.S. mean household income of quintiles in 2009 is \$11552, \$29257, \$49534, \$78694 and \$170844. Under this income distribution, the mean income for the whole population is \$67976. Then by normalizing the highest income level, setting \$170844 to one, we have $\overline{\theta} = 0.4$ and $\theta^{med} = 0.29$. Following Laibson, Repetto and Tobacman (2004) and subsequently

Krusell, Kuruşçu and Smith (2009), we let $\beta = 0.7$. Finally we set $\gamma = 1$ such that $\theta^* = \theta^{med} = 0.29$ in our benchmark case. In the following diagrams, we demonstrate the impacts on θ^* by varying the values of γ, β, $\overline{\theta}$ and σ. We notice that the first three figures confirm the result we discussed under Section 3.1 where we dealt with the problem with a logarithmic utility function. With the help of the general form of utility function, we are able to do the comparative static analysis with respect to σ too. It has been verified that when we reduce the value of σ (say from 2 to 0.5) in the benchmark case, all these relationships remain unchanged.

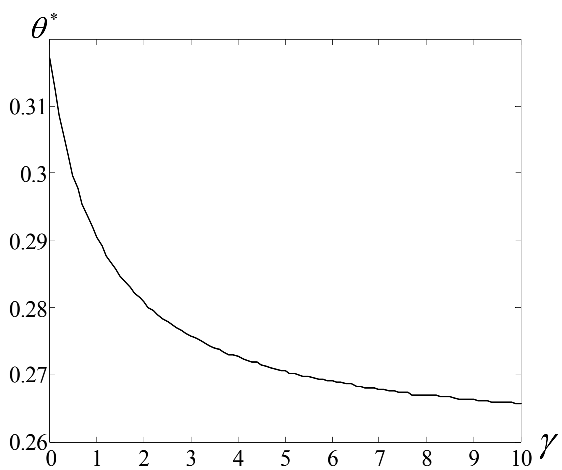

Figure 1: Impacts of γ

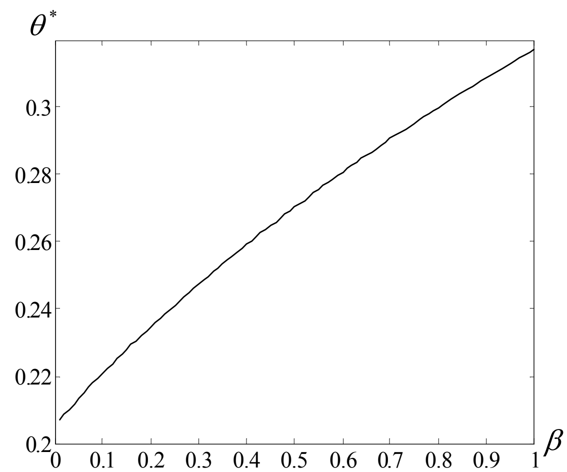

Figure 2: Impacts of β

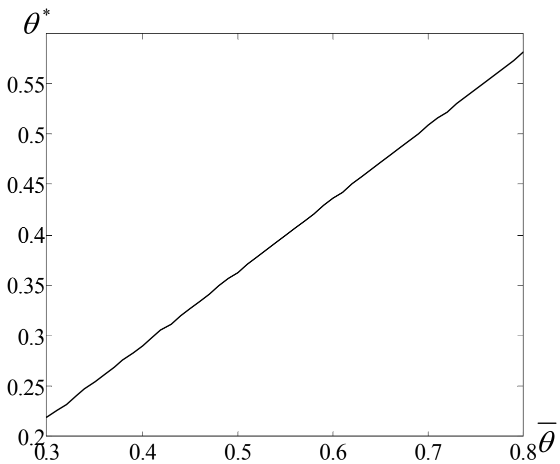

Figure 3: Impacts of $\overline{\theta}$

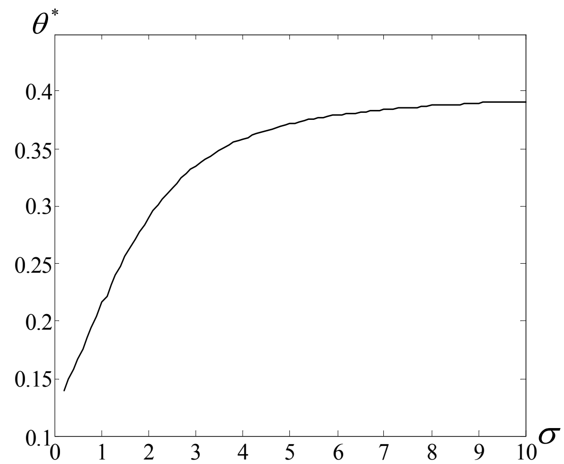

Figure 4: Impacts of σ

5 Conclusion

Gul and Pesendorfer (2001, 2004, 2005) have identified a class of preferences that can model problems of temptation and self-control. When it is applied to a standard macroeconomic setting, it has been shown by Krusell, Kuruşçu and Smith (2010) that the optimal intervention policy is to subsidize savings for all the consumers, when they are tempted by excessive impatience, since a subsidy improves welfare by making succumbing to temptation less attractive. However, this kind of observation is restricted only to an economy with homogeneous agents. When agents differ in their types and are thus assumed to differ in their income levels, those agents in the lower side of the ability distribution, when allowed to choose optimally, will not prefer subsidy on savings. We thus conclude that when there is heterogeneity among agents there will be an interplay between the redistributive effect of taxation and the temptation effect. Depending on this, agents choose their optimal tax rates, which can well turn out to be positive. Further we show that as temptations grow larger, the support of the positive tax decreases, that is, the proportion of agents for whom a subsidy proves to be welfare improving, increases. The above study also reveals that there may be a situation when a subsidy on investment may never be implemented in a political equilibrium.

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Appendix Proof

Proof of Lemma 1. Consider the following function

$$f_i(x) = \frac{\delta x (w\theta_i + s)}{(1+\tau)(1+\delta x)} \text{ where } x = \left\{ \frac{1+\beta\gamma}{1+\gamma}, \beta \right\}.$$

Note that $f_i(x = (1+\beta\gamma)/(1+\gamma)) = k_i$ and $f_i(x = \beta) = \widetilde{k}_i$. It can easily be checked that $\partial f_i/\partial x > 0$. Since $(1+\beta\gamma)/(1+\gamma) > \beta$, $\partial f_i/\partial x > 0$ implies that $k_i > \widetilde{k}_i$. Hence proved. ■

Proof of Proposition 1. By substituting the values of k_i, k, and $\widetilde{k}_i$ into the value function and removing the constant part, it can be shown that the effective value function $V_e(\tau_i)$ which contains τ_i can be written as

$$V_e(\tau_i) = (1+\delta)\ln\left(\frac{\theta_i}{\overline{\theta}} + \frac{\delta(1+\beta\gamma)\tau_i}{(1+\gamma)(1+\tau_i) + \delta(1+\beta\gamma)}\right) - \delta\ln(1+\tau_i). \tag{24}$$

For $V_e(\tau_i)$ to be well defined, both the two terms $\left(\frac{\theta_i}{\overline{\theta}} + \frac{\delta(1+\beta\gamma)\tau_i}{(1+\gamma)(1+\tau_i) + \delta(1+\beta\gamma)}\right)$ and $(1+\tau_i)$ should be strictly positive, which implies that $\tau_i > -1$ and

$$\tau_i > -\frac{\frac{\theta_i}{\overline{\theta}}(\delta + \beta\delta\gamma) + \frac{\theta_i}{\overline{\theta}}(1+\gamma)}{\delta + \beta\delta\gamma + \frac{\theta_i}{\overline{\theta}}(1+\gamma)}. \tag{25}$$

Note that when $\theta_i < \overline{\theta}$, the right hand side of (25) is larger than -1 and the choice set for voter i is defined by (25). On the other hand, if $\theta_i > \overline{\theta}$, the right hand side of (25) is less than -1 and $\tau_i > -1$ becomes the choice set for voter i.

We prove our proposition in three parts. First we consider the case when $\theta_i < \overline{\theta}$.

Taking the first order condition of $V_e(\tau_i)$ with respect to τ_i gives

$$\frac{\delta(1+\delta)\left[(1+\gamma)+\delta\left(1+\beta\gamma\right)\right](1+\beta\gamma)}{\frac{\theta_i}{\overline{\theta}}\left[(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)\right]^2+\tau_i^*\delta\left(1+\beta\gamma\right)\left[(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)\right]}=\frac{\delta}{1+\tau_i^*}$$

where τ_i^* denotes the optimal choice of τ_i by the individual i. Therefore the above equation can be written as:

$$\begin{aligned}
& (1+\delta)\left[(1+\gamma)+\delta\left(1+\beta\gamma\right)\right](1+\beta\gamma)\left(1+\tau_i^*\right) \\
= & \frac{\theta_i}{\overline{\theta}}\left[(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)\right]^2+\tau_i^*\delta\left(1+\beta\gamma\right)\left[(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)\right].
\end{aligned} \tag{26}$$

If we define the left and right hand side of the above equation by $B\left(\tau_i\right)$ and $G\left(\tau_i\right)$ respectively, that is,

$$\begin{aligned}
B\left(\tau_i^*\right) &= (1+\delta)\left(1+\gamma\right)\left(1+\beta\gamma\right)\left(1+\tau_i^*\right) \text{ and} \\
G\left(\tau_i^*\right) &= \frac{\theta_i}{\overline{\theta}}\left[(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)\right]^2+\tau_i^*\delta\left(1+\beta\gamma\right)\left[(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)\right],
\end{aligned}$$

τ_i^* is the solution to $B\left(\tau_i^*\right)=G\left(\tau_i^*\right)$. Let $\tau_i^{\min}$ be the minimum value of τ. Note that while $B\left(\tau_i^*\right)$ is linear and increasing in τ_i^* with $B\left(-1\right)=0$ and $B\left(\tau_i^{\min}\right)>0$, $G''\left(\tau_i^*\right)>0$ and $G\left(\tau_i^{\min}\right)=0$. Evidently $\tau_i^*\geqslant 0$ is the unique solution to (26) if and only if $B\left(0\right)\geqslant G\left(0\right)$, i.e.,

$$(1+\delta)\left[(1+\gamma)+\delta\left(1+\beta\gamma\right)\right](1+\beta\gamma)\geqslant\frac{\theta_i}{\overline{\theta}}\left[(1+\gamma)+\delta(1+\beta\gamma)\right]^2.$$

Therefore, if we denote $\dfrac{(1+\delta)\left(1+\beta\gamma\right)}{(1+\gamma)+\delta(1+\beta\gamma)}\overline{\theta}$ by θ_{PE}^*, we obtain (17).

Since the above argument is vaild for $\theta_{PE}^*<\overline{\theta}$, if we want the above result to hold for all θ_i, we need to prove that for any $\theta_i\geqslant\overline{\theta}$, the agent prefers a negative tax rate. Note that when $\theta_i=\overline{\theta}$, we have $\tau_i^{\min}=-1$ and the above result is valid since

$\theta^*_{PE} < \overline{\theta}$, $\tau^*_i < 0$ for $\theta_i = \overline{\theta}$. Thus the remaining case is when $\theta_i > \overline{\theta}$. Under the situation $\theta_i > \overline{\theta}$, we can have two possible scenarios as follows: case (a) when θ_i is sufficiently large such that $G > B$ for all τ, i.e., $V'_e(\tau_i) < 0$ for all τ_i and case (b) when θ_i is sufficiently low such that G intersects B twice, i.e., there exists two solutions of τ^*_i where $V'_e(\tau^*_i) = 0$ and both are negative. Let $\tau^{*,1}_i$ and $\tau^{*,2}_i$ be those two solutions and say $\tau^{*,1}_i < \tau^{*,2}_i$. It is easy to check that for case (a), a tax rate $\tau^*_i = -(1-\varepsilon)$ with any small $\varepsilon > 0$ is optimal, however, for (b), if there exist two solutions, then the smaller $\tau^{*,1}_i$ is the local minimum and larger $\tau^{*,2}_i$ is the local maximum. To see this, we evaluate the second derivative of $V''(\tau)$ at $\tau_i = \tau^*_i$. It can be verified that $V''(\tau_i)\Big|_{\tau=\tau^{*,1}_i} > 0$ and $V''(\tau_i)\Big|_{\tau=\tau^{*,2}_i} < 0$. If this does not hold, then there will exist more than two solutions of τ^*_i for which the equation $V'(\tau^*_i) = 0$ holds. But this is impossible since for $V'(\tau^*_i) = 0$, we can have at most two solutions (since B is linear and G is convex). Therefore, $V(\tau_i)$ is first convex and then concave in τ_i and for $\tau_i > \tau^{*,2}_i$, it is monotonically decreasing in τ_i. As a consequence, $V(\tau_i)$ has two local maximum at $\tau_i = -(1-\varepsilon)$ and $\tau_i = \tau^{*,2}_i$, both of which are negative tax rate.[4] Hence the proof. ■

Proof of Corollary 1. It is easy to verify that θ^*_{PE} decreases when γ increases and β decreases. Also θ^*_{PE} increases monotonically with $\overline{\theta}$. Hence the proof. ■

Proof of Proposition 2. The proof is similar to the proof of Proposition 1. Once we remove the constant part of the value function, it becomes equivalent to

$$V = \ln(w\theta_i + \tau_i K) + \delta \ln\left(r\frac{w\theta_i + \tau_i K}{1+\tau_i}\right) = (1+\delta)\ln(w\theta_i + \tau_i K) + \delta\ln(r) - \delta\ln(1+\tau_i). \tag{27}$$

[4]Note that in this case, though agent i's optimal choice is still negative tax, the value function has two peaks. That is why median voter theorem can not be applied to this model and we instead look at the proportion of population that support positive or negative tax rate.

Further by substituting the equilbrium values of K, w and r in the above equation and then eliminating again the constant parts, the effective value function V_e becomes

$$\begin{aligned} V_e &= (1+\delta)\ln\left(\frac{\theta_i}{\overline{\theta}}+\frac{\tau_i^*\delta(1+\beta\gamma)}{(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)}\right) \\ &\quad+\left[\delta-\frac{\alpha(1+\delta)}{1-\alpha}\right]\ln\left((1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)\right)-\delta\ln(1+\tau_i^*)\,. \end{aligned}$$

As before, when $\theta_i < \overline{\theta}$, the range of τ_i is defined by (25) and when $\theta_i \geq \overline{\theta}$, $\tau_i > -1$.

In the case of $\theta_i < \overline{\theta}$, the first order condition from the maximization of V_e with respect to τ_i results as follows:

$$\frac{\frac{\delta(1+\delta)(1+\beta\gamma)(1+\gamma)}{[(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)]^2}}{\frac{\theta_i}{\overline{\theta}}+\frac{\tau_i^*\delta(1+\beta\gamma)}{[(1+\gamma)(1+\tau_i)+\delta(1+\beta\gamma)]}}-\frac{\delta}{1+\tau_i^*}+\frac{(1+\gamma)\left[\delta-\frac{\alpha(1+\delta)}{1-\alpha}\right]}{(1+\gamma)(1+\tau_i^*)+\delta(1+\beta\gamma)}=0$$

where τ_i^* denotes the individual i's optimally chosen tax. Simplifying the above, we get the following equation

$$\begin{aligned} &\left\{\frac{\theta_i\,[1+\gamma)(1+\tau_i^*)]}{\overline{\theta}}+\left(\tau_i^*+\frac{\theta_i}{\overline{\theta}}\right)\delta(1+\beta\gamma)\right\}\left[\delta^2(1+\beta\gamma)+\frac{\alpha(1+\delta)(1+\tau_i^*)(1+\gamma)}{1-\alpha}\right] \\ &=\delta(1+\delta)(1+\beta\gamma)\left[(1+\gamma)+\delta(1+\beta\gamma)\right](1+\tau_i^*)\,. \end{aligned} \tag{28}$$

We denote the left hand side and right hand side by $J(\tau_i^*)$ and $H(\tau_i^*)$ respectively. While it can be checked that $H(\tau_i)$ is linear and increasing in τ_i^* with $H(-1)=0$ and $H\left(\tau_i^{\min}\right)>0$, it can be verified that $J''(\tau_i^*)>0$ and $J\left(\tau_i^{\min}\right)=0<H\left(\tau_i^{\min}\right)$. Thus, evidently, τ_i^* is the unique solution of the above equation and $\tau_i^*\geq 0$ if and only if $H(0)\geq J(0)$, i.e.,

$$\delta(1+\delta)(1+\beta\gamma)\geq\frac{\theta_i}{\overline{\theta}}\left[\delta^2(1+\beta\gamma)+(1+\gamma)\frac{\alpha(1+\delta)}{1-\alpha}\right],$$

which leads to (21).

In the case when $\theta_i < \overline{\theta}$, we follow the same procedure as we did in the proof of proposition 1. Hence the proof. ■

Proof of Lemma 2. It can be verified that from the first order condition of utility maximization,

$$\begin{aligned} \frac{c_{2,i}}{c_{1,i}} &= \left[\frac{\delta(1+\beta\gamma)r}{(1+\gamma)(1+\tau_i)}\right]^{\frac{1}{\sigma}} \equiv A, \ \frac{\widetilde{c}_{2,i}}{\widetilde{c}_{1,i}} = \left[\frac{\delta\beta r}{(1+\tau_i)}\right]^{\frac{1}{\sigma}} \equiv B \text{ and} \\ k_{2,i} &= \frac{A[w\theta_i+s]}{[r+A(1+\tau_i)]}, \ \widetilde{k}_{2,i} = \frac{B[w\theta_i+s]}{[r+B(1+\tau_i)]}. \end{aligned}$$

Just like the proof of Lemma 1, let $f_i(x) = \dfrac{x[w\theta_i+s]}{[r+x(1+\tau_i)]}$. Then

$$f_i(x) = \begin{cases} k_{2,i} \text{ when } x = A \\ \widetilde{k}_{2,i} \text{ when } x = B \end{cases}$$

and it can be easily verified that $f_i'(x) > 0$ as long as $r > 0$. Since $A > B$, we clearly have $k_{2,i} > \widetilde{k}_{2,i}$. Hence the proof. ■