Bailouts and Financial Fragility

Todd Keister
Research and Statistics Group
Federal Reserve Bank of New York
Todd.Keister@ny.frb.org

April 29, 2011

Abstract

How does the belief that policymakers will bail out investors in the event of a crisis affect the allocation of resources and the stability of the financial system? I study this question in a model of financial intermediation with limited commitment. When a crisis occurs, the efficient policy response is to use public resources to augment the private consumption of those investors facing losses. The anticipation of such a “bailout” distorts ex ante incentives, leading intermediaries to choose arrangements with excessive illiquidity and thereby increasing financial fragility. Prohibiting bailouts is not necessarily desirable, however: it induces intermediaries to become too liquid from a social point of view and may, in addition, leave the economy more susceptible to a crisis. A policy of taxing short-term liabilities, in contrast, can correct the incentive problem while improving financial stability.

Initial draft: September 2009. I am grateful to participants at numerous conference and seminar presentations and especially to Giovanni Calice, Amil Dasgupta, Huberto Ennis, Alexander Monge-Naranjo, and Jaume Ventura for helpful comments. I also thank Vijay Narasiman for excellent research assistance. Part of this work was completed while I was a Fernand Braudel Fellow at the European University Institute and then a visiting scholar at the Stern School of Business, New York University; the hospitality and support of both institutions are gratefully acknowledged. The views expressed herein are my own and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.
1 Introduction

The recent financial crisis has generated a heated debate about the economic effects of public-sector bailouts of private financial institutions. Most observers agree that the anticipation of such bailouts in the event of a crisis distorts the incentives faced by financial institutions and their investors. By insulating these agents from the full consequences of a negative outcome, an anticipated bailout results in a misallocation of resources and encourages risky behavior that may leave the economy more susceptible to a future crisis. Opinions differ widely, however, on the best way for policy makers to deal with this problem. Some observers argue that policy makers should focus on making credible commitments to not bail out financial institutions in the event of a future crisis. Such a commitment would encourage investors to provision for bad outcomes and, it is claimed, these actions would collectively make the financial system more stable. Others argue that the focus should instead be on improving the regulation and supervision of financial institutions. Proponents of this second view typically believe that it is either undesirable or infeasible to limit future policy makers’ actions and argue that the resulting distortions in incentives should be corrected through regulation.

Would it be desirable for policy makers to commit to never bail out financial institutions? Would doing so be an effective way to promote financial stability? I address these questions in a model of financial intermediation and fragility based on the classic paper of Diamond and Dybvig (1983). In particular, I study an environment with idiosyncratic liquidity risk and limited commitment, as in Ennis and Keister (2009a). Individuals deposit resources with financial intermediaries, and these resources are invested in a nonstochastic production technology. Intermediaries perform maturity transformation and thereby insure investors against their individual liquidity risk. This maturity transformation makes intermediaries illiquid and may leave them susceptible to a self-fulfilling run by investors. I introduce fiscal policy into this framework by adding a public good that is financed by taxing households’ endowments. In the event of a crisis, some of this tax revenue may be diverted from production of the public good and instead given as private consumption to investors facing losses in the financial system. These “bailout” payments aim to improve the allocation of the remaining resources in the economy, but may have undesirable effects on incentives.

I begin the analysis by characterizing a benchmark allocation that represents the efficient distri-
bution of resources in this environment conditional on investors running on the financial system in some state of the world. I show that this allocation always involves a transfer of public resources to private investors in that state. In other words, a bailout is part of an efficient social insurance arrangement when a crisis is possible. The logic behind this result is straightforward and fairly general. In normal times, the policy maker chooses the tax rate and the level of public good provision to equate the marginal social values of public and private consumption. A crisis results in a misallocation of resources, which raises the marginal value of private consumption for some investors. The optimal response to this situation is to decrease public consumption and transfer resources to these investors – a “bailout.”

In a decentralized setting, the anticipation of this type of bailout distorts the incentives of investors and their intermediaries. As a result, intermediaries choose to perform more maturity transformation, and hence become more illiquid, than in the benchmark allocation. This excessive illiquidity, in turn, implies that the financial system is more fragile in the sense that a self-fulfilling run can occur in equilibrium for a strictly larger set of parameter values. The incentive problem created by the anticipation of a bailout thus has two negative effects in this environment: it both distorts the allocation of resources in normal times and increases the financial system’s susceptibility to a crisis.

A strict no-bailouts policy is not necessarily desirable, however. Such a policy requires intermediaries to completely self-insure against the possibility of a crisis, which would lead them to become more liquid (by performing less maturity transformation) than in the benchmark efficient allocation. Despite this increase in liquidity, the economy would remain more fragile than in the benchmark allocation. A no-bailouts policy would also leave the level of public good provision inefficiently high if a crisis does occur. When the probability of a crisis is sufficiently small, a no-bailouts commitment is strictly inferior to a discretionary policy regime – it lowers equilibrium welfare without improving financial stability. For higher probabilities of a crisis, a no-bailouts policy may or may not be preferable, depending on parameter values, but it will never achieve the efficient allocation of resources. Interestingly, for some economies that are not fragile in a discretionary regime, a no-bailouts policy would introduce the possibility of a self-fulfilling run.

The idea that a credible no-bailout commitment can increase the fragility of the financial system may seem surprising at first, but the mechanism behind this result is easy to understand. A bailout policy provides insurance – it lessens the potential loss an investor faces if she does not withdraw
her funds and a crisis occurs. Removing this insurance increases each individual’s incentive to withdraw early if she expects others to do so, which makes the financial system more susceptible to a crisis. This argument is familiar in the context of retail banking: government-sponsored deposit insurance programs can be thought of as a type of “bailout” policy that is explicitly designed to play a stabilizing role. Despite this similarity, discussion of the insurance role of bailouts has been largely absent in the current policy debate.

An optimal policy arrangement in the environment studied here requires permitting bailouts to occur, so that investors benefit from the efficient level of insurance, while offsetting the negative effects on ex ante incentives. One way this can be accomplished is by placing a Pigouvian tax on intermediaries’ short-term liabilities, which can also be interpreted as a tax on the activity of maturity transformation. I show how the appropriate choice of tax rate implements the benchmark efficient allocation. Note that, in addition to improving the allocation of resources, this policy has a *macroprudential* component: it decreases the scope for financial fragility relative to either the discretionary or the no-bailouts regime.

There is a growing literature on the incentive effects of financial-sector bailouts and optimal regulatory policy in the presence of limited commitment. In most of the settings that have been studied, bailouts serve no useful purpose from an ex ante point of view. Chari and Kehoe (2010), for example, study an environment in which committing to a no-bailout policy would generate the constrained-efficient allocation of resources. When such commitment is infeasible, they show how renegotiation of contracts (i.e., “a bailout”) tends to undermine ex ante incentives and how regulation of private contracts can be welfare improving. In a similar vein, Farhi and Tirole (2009) study a setting where the policy maker would like to commit to not lower interest rates in the event of a crisis. In the absence of commitment, the anticipation of this type of bailout distorts banks’ incentives and introduces a role for regulation.¹

In the environment studied here, in contrast, committing to a no-bailout policy is not optimal because bailout payments provide socially-valuable insurance.² As a result, the paper presents a richer view of the issue in which the ex ante incentives generated by bailouts are not entirely neg-

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¹ Other related work includes Gale and Vives (2002), who study dollarization as a device for limiting a central bank’s ability to engage in bailouts, Cooper and Kempf (2009), who study the redistributive effects of deposit insurance when agents are ex ante heterogeneous, Niepmann and Schmidt-Eisenlohr (2010), who examine the strategic interaction between governments when bailouts have international spillover effects, and Ranciere and Tornell (2011), who show how the anticipation of a bailout can lead to welfare-reducing financial innovation.

² This aspect of the model is similar in some respects to Green (2010), who also shows how policies resembling a bailout can be part of a desirable insurance arrangement.
ative. In particular, the insurance provided by a bailout encourages intermediaries to undertake socially-valuable maturity transformation and makes investors more willing to stay invested. The problem, of course, is that intermediaries have an incentive to go too far and become too illiquid, which in turn makes investors more anxious to withdraw in a crisis. The analysis shows how policy makers must seek to balance these concerns, reigning in the incentive for excessive illiquidity without discouraging desirable activity or losing the benefits of socially-efficient insurance arrangements.

The next section presents the basic model, while Section 3 studies efficient allocations with financial crises. Sections 4 and 5 study equilibrium allocations under discretionary policy and under a no-bailouts regime, respectively, and Section 6 shows how a tax on short-term liabilities can improve the outcome relative to either of these approaches. Section 7 offers some concluding remarks.

2 The Model

I begin with a fairly standard version of the Diamond and Dybvig (1983) model and augment this basic framework by introducing a public good. This section describes the physical environment and the model of the decentralized economy.

2.1 The environment

There are three time periods, $t = 0, 1, 2$, and a continuum of investors, indexed by $i \in [0, 1]$. Each investor has preferences given by

$$U(c_1, c_2, g; \theta_i) = u(c_1 + \theta_i c_2) + v(g),$$

where $c_t$ is consumption of the private good in period $t$ and $g$ is the level of public good, which is provided in period 1. The functions $u$ and $v$ are assumed to be strictly increasing, strictly concave, and to satisfy the usual Inada conditions. In addition, the coefficient of relative risk aversion for the function $u$ is assumed to be constant and greater than one. The parameter $\theta_i$ is a binomial random variable with support $\Theta = \{0, 1\}$. If the realized value of $\theta_i$ is zero, investor $i$ is impatient and only cares about early consumption. An investor’s type $\theta_i$ is revealed to her in period 1 and remains private information. Let $\omega$ denote a profile of preference types for each investor and let $\Omega$ denote the set of all such profiles. Let $\pi$ denote the probability with which each individual investor will
be impatient. By a law of large numbers, \( \pi \) is also the fraction of investors in the population who will be impatient.

Each investor is endowed with one unit of the private good in period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into private consumption in the later periods. A unit of the good invested in period 0 yields \( R > 1 \) units in period 2, but only one unit in period 1. This investment technology is operated in a central location, where investors can pool resources in an intermediation technology to insure against individual liquidity risk. Investors are isolated from each other in periods 1 and 2 and no trade can occur among them. Upon learning her preference type, each investor chooses either to contact the intermediation technology in period 1 to withdraw funds or to wait and withdraw in period 2. There is also a technology for transforming units of the private good one-for-one into units of the public good. This technology is operated in period 1, using goods that were placed into the investment technology in period 0.

An (ex post) allocation in this environment is a pair \((c, g)\), where \( c : [0, 1] \to \mathbb{R}_+^2 \) is an assignment of a private consumption level to each investor in each period and \( g \in \mathbb{R}_+ \) is a level of public good provision. An allocation is feasible if it can be produced from the period-0 endowments using the technologies described above, that is, if

\[
\int_0^1 c_1(i) \, di + \frac{1}{R} \int_0^1 c_2(i) \, di \leq 1 - g.
\]

Let \( \mathbb{A} \) denote the set of feasible allocations. A state-contingent allocation is a mapping \( c : \Omega \to \mathbb{A} \) from the set of realized preference types to the set of feasible allocations.

Investors who choose to withdraw in period 1 arrive one at a time in a randomly-determined order. As in Wallace (1988, 1990), these investors must consume immediately upon arrival. This sequential-service constraint implies that the payment made to such an investor can only depend on the information received by the intermediation technology up to that point. In particular, this payment can be contingent on the number of early withdrawals that have taken place so far, but not on the total number of early withdrawals that will occur because this latter number will not be known until the end of the period.

Since investors are ex ante identical, it is natural to measure ex ante welfare in this economy as the period-0 expected utility of each investor. For ex post measures of welfare, after preference types (and potentially some consumption levels) have been realized, I use an equal-weighted sum.
of individual utilities to measure welfare. The expression

\[ W = \int_0^1 E \left[ U (c_1 (i), c_2 (i), g; \theta_i) \right] di \]

captures both of these notions and is, therefore, used to measure welfare throughout the analysis.

### 2.2 The decentralized economy

In the decentralized economy, the intermediation technology is operated by a large number of competitive intermediaries, each of which aims to maximize the expected utility of its investors. Each intermediary serves a large number of investors and, hence, knows that a fraction \( \pi \) will be impatient. Because investors’ types are private information, the payment an investor receives from her intermediary cannot depend directly on her realized type. Instead, the intermediary allows each investor to choose the period in which she will withdraw. I follow Green and Lin (2003), Peck and Shell (2003), Ennis and Keister (2009b) and other recent work in allowing intermediaries to offer any payment schedule that is consistent with the information flow generated by the sequential service constraint. In particular, intermediaries and the policy-maker are able to react if they observe an unusually high number of early withdrawals, rather than following a simple rule such as paying investors at face value until all funds are depleted.

Intermediaries act to maximize the expected utility of their investors at all times. In reality, there are important agency problems that cause the incentives of financial intermediaries to differ from those of their investors and creditors. I abstract from these agency problems here in order to focus more directly on the distortions in investors’ incentives that are created by the anticipation of a bailout. As in Ennis and Keister (2009a, 2010), intermediaries cannot commit to future actions. This inability to commit implies that they are unable to use the type of suspension of convertibility plans discussed in Diamond and Dybvig (1983) or the type of run-proof contracts studied in Cooper and Ross (1998). Instead, the payment given to each investor will be a best response to the current situation when she withdraws.

The public good is provided by a benevolent policy maker who has the ability to tax endowments in period 0. The revenue from this tax is placed into the investment technology and transformed into period 1 private goods. In period 1, the policy maker can use these private goods to produce units of the public good or, if a crisis is underway, can transfer some of these private goods as “bailout” payments to the financial intermediaries.\(^3\)

\(^3\) Notice that this type of bailout policy is entirely consistent with the sequential service constraint, since all taxes are
2.3 Financial crises

In order to allow a run on the financial system to occur with nontrivial probability, I introduce an extrinsic signal on which investors can potentially condition their actions. Let \( S = \{s_1, s_2\} \) be the set of possible states. Investor \( i \) chooses a strategy that assigns a decision to withdraw in either period 1 or period 2 to each possible realization of her preference type \( \theta_i \) and of the state

\[
y_i : \Theta \times S \rightarrow \{1, 2\}.
\]

I will say the financial system is \textit{fragile} if there exists an equilibrium in which investors run on their intermediaries by all attempting to withdraw early, regardless of their preference types, in one of the states.

Other approaches to modeling financial fragility would lead to similar results. Suppose, for example, that the fraction of impatient investors varied across states, as in Allen and Gale (2000) and others. An economy could then be called \textit{fragile} if there exists an equilibrium in which investors run on the financial system when this fraction is high. In this modified situation, a run would have two distinct components: some of the additional withdrawals would come from investors who are truly impatient, but this “real” shock would be amplified in equilibrium as patient investors attempt to withdraw early as well. The model studied here is the limiting case in which the proportion of additional impatient investors in the high state is zero. In other words, the model here abstracts from the initial shock – treating it as a “sunspot” – and focuses entirely on the amplification of this shock through the decisions of patient investors. Many observers claim that such amplification effects were large during the recent crises compared to the magnitude of the underlying shocks to the financial system.\(^4\)

Neither the intermediaries nor the policy maker observe the realized state; they must try to infer this information from the flow of withdrawals. This approach is standard\(^5\) and, combined with the sequential service constraint, implies that some payments must be made to withdrawing investors...
before the intermediaries or policy maker know whether or not a run is underway. Suppose that all investors attempt to withdraw early in state $s_2$. Intermediaries and the policy maker know that at least $\pi$ investors will withdraw in both states and, therefore, as the first $\pi$ withdrawals take place they are unable to infer anything about the realized state. If the fraction of early withdrawals goes past $\pi$, however, they can immediately infer that the state is $s_2$ and that a run is underway.

The model must specify how intermediaries and the policy maker respond at this point and how the remaining investors react to this response. In general, this interaction may be quite complex and different patterns of behavior are possible (see Ennis and Keister, 2010). To simplify matters, I assume here that once it has discovered a run is underway, an intermediary is able to implement the efficient allocation of its remaining resources among the remaining investors. As part of this allocation, only those remaining investors who are impatient withdraw early; the remaining patient investors wait until period 2 to withdraw.\footnote{There are several different ways in which this allocation could come about. It could, for example, be the result of a screening technology that can be used in the event of a run (as in Ennis and Keister, 2009a) or of equilibrium behavior in a game played by the intermediary and the remaining investors (as in Ennis and Keister, 2010). To simplify the presentation, I assume here that the intermediary can directly impose this allocation.} This assumption is not crucial for any of the results below; it is simply a parsimonious way of specifying the post-run allocation of resources. The important point is that this allocation is decided in a way that reflects ex post considerations and does not aim to influence investors’ ex ante incentive to run, since at this point a run has already occurred. Any allocation rule with this property will potentially open the door to financial fragility and, hence, make the bailout policy an important determinant of the equilibrium outcome.

\section{Efficient Allocations and Bailouts}

In this section, I study equilibrium in a setting where a benevolent planner controls both the financial system and the public sector. The planner cannot dictate the withdrawal decisions of investors and, hence, financial crises may occur. I assume the planner faces the same informational constraints that intermediaries and the policy maker face in the decentralized economy. In particular, the planner correctly anticipates investors’ withdrawal strategies, but is unable to observe the realized state. Instead, it must infer the state from the observed withdrawal behavior of investors. By definition, financial arrangements will be efficient in this setting, and the welfare-maximizing allocation of resources will obtain, conditional on the pattern of withdrawal behavior. For this reason, the results here provide a useful benchmark for the decentralized economies studied below.
3.1 The q-efficient allocation

Suppose that only impatient investors withdraw early in state $s_1$, but all investors attempt to withdraw early in state $s_2$. What is the planner’s best response to this profile of withdrawal strategies? In this scenario, the fraction of investors who attempt to withdraw early will be $\pi$ in state $s_1$ and 1 in state $s_2$. As the first $\pi$ withdrawals are taking place, therefore, no information about the state is revealed to the planner. The best policy must give the same consumption level to all of these investors; any feasible allocation in which these investors consume different amounts is strictly dominated by another feasible allocation in which their consumption levels are equalized. Let $c_E$ denote the payment given to these investors, who withdraw “early.” If withdrawals cease after a fraction $\pi$ of investors has withdrawn, the planner can infer that the remaining investors are all patient and will withdraw in period 2. The planner will then divide the remaining resources between a common payment $c_L$ for those investors who withdraw “late” and an amount $g$ of the public good.

If, on the other hand, the fraction of investors withdrawing in period 1 goes past $\pi$, the planner is immediately able to infer that state $s_2$ has occurred. At this point, the planner is able to implement the efficient continuation allocation among the remaining investors. This allocation gives a common amount $\hat{c}_E$ of consumption to each remaining impatient investor in period 1 and a common amount $\hat{c}_L$ to each remaining patient investor in period 2. Let $\hat{g}$ denote the amount of public good provided in this case. Notice the importance of the sequential service constraint here: a fraction $\pi$ of investors must be served, and will consume, before the planner is able to infer the state and thus determine the appropriate consumption levels.

The problem of finding the planner’s best response to this strategy profile can, therefore, be reduced to choosing the consumption levels $(c_E, c_L, \hat{c}_E, \hat{c}_L)$ and the levels of public good provision $(g, \hat{g})$ to solve

$$\max (1 - q) \left[ \pi u(c_E) + (1 - \pi) u(c_L) + v(g) \right] +$$

$$q \left[ \pi u(c_E) + (1 - \pi) \left( \pi u(\hat{c}_E) + (1 - \pi) u(\hat{c}_L) \right) + v(\hat{g}) \right]$$

subject to

$$\pi c_E + (1 - \pi) \frac{c_L}{R} + g \leq 1, \quad (1)$$

$$(1 - \pi) \left( \pi \hat{c}_E + (1 - \pi) \frac{\hat{c}_L}{R} \right) + \hat{g} \leq 1 - \pi c_E, \quad (2)$$

and

$$c_L \geq c_E, \quad \hat{c}_L \geq \hat{c}_E.$$
where \( q \geq 0 \) is the probability of the crisis state \( s_2 \). Expression (1) is the resource constraint that applies in state \( s_1 \), while (2) applies in state \( s_2 \). The final two constraints are incentive compatibility conditions that, in a decentralized economy, ensure withdrawing early is not a dominant strategy. One can show that these latter constraints never bind at the solution. I refer to the solution to this problem as the \( q \)-efficient allocation; it represents the best allocation of resources in this environment conditional on a crisis occurring with probability \( q \).

Letting \((1 - q) \mu\) and \( q \mu \) denote the multipliers on constraints (1) and (2), respectively, the solution is characterized by the conditions

\[
\begin{align*}
\mu \left( c^*_E \right) &= (1 - q) \mu + q \mu \\
Ru' \left( c^*_L \right) &= v' (g) = \mu, \quad \text{and} \\
u' \left( \hat{c}^*_E \right) &= Ru' \left( \hat{c}^*_L \right) = v' (\hat{g}) = \hat{\mu}.
\end{align*}
\]

The first condition says that the marginal value assigned to resources paid out before the planner knows whether a run is underway should be equal to the expected future marginal value of resources. The other equations can be interpreted as the standard Samuelson condition for the efficient provision of a public good in each of the two states.

Let \( c^* = (c^*_E, c^*_L, g^*, \hat{c}^*_E, \hat{c}^*_L, \hat{g}^*) \) denote the solution and let \((\mu^*, \hat{\mu}^*)\) denote the corresponding values of the (normalized) multipliers. It is straightforward to show that each element of \( c^* \) varies continuously with the probability of a crisis \( q \) and that evaluating \( c^* \) in the limit as \( q \to 0 \) yields the first-best allocation of resources in this environment.

### 3.2 Financial Fragility

The allocation \( c^* \) represents the planner’s best response to the strategy profile in which patient investors wait until period 2 to withdraw in state \( s_1 \), but run in state \( s_2 \). When is this profile consistent with equilibrium? Since \( c^*_E < c^*_L \) holds, a patient investor will always strictly prefer to wait in state \( s_1 \), as specified. The question, therefore, is under what conditions these investors have an incentive to join the run in state \( s_2 \). An investor who runs will receive \( c^*_E \) if she arrives before the planner discovers that a run is underway and \( \hat{c}^*_L \) otherwise. If, instead, she deviated from this strategy profile and waited until period 2, she would receive \( \hat{c}^*_L \) for sure. This strategy profile is consistent with equilibrium, therefore, if and only if the allocation derived above satisfies

\[
c^*_E \geq \hat{c}^*_L.
\]
Using the first-order conditions (3) – (5), this condition can be written as

\[
\frac{\mu^*}{\bar{\mu}^*} \leq \frac{R^{-1} - q}{1 - q}.
\]

(7)

It is straightforward to show that there exist parameter values such that this condition is satisfied. In other words, financial fragility can arise in this model even when the financial system is operated by a benevolent planner and there are no distortions from the bailout policy. Let \( \Phi^* \) denote the set of economies that are fragile under the \( q \)-efficient allocation of resources. An economy is characterized by a set of parameter values; let \( e \equiv (R, \pi, u, v, q) \) denote a typical economy. Then we have \( e \in \Phi^* \) if and only if (6) holds.

### 3.3 Illiquidity

For any given allocation, define the degree of illiquidity in the financial system to be

\[
\rho \equiv \frac{c_E}{1 - g}.
\]

Since each investor has the option of withdrawing early, \( c_E \) represents the face value of the short term liabilities of the financial system in per-capita terms. The short-run value of intermediaries’ assets per capita is equal to the fraction of endowments that are invested to provide private consumption, \( 1 - g \). Hence \( \rho \) represents the ratio of the short-term liabilities of the financial system to the short-run value of its assets. I will say that the financial system is illiquid whenever \( \rho > 1 \) holds.

The following proposition shows that the financial system is illiquid under the \( q \)-efficient allocation for any value of \( q \). As is standard in Diamond-Dybvig models, this illiquidity is what potentially opens the door to self-fulfilling financial crises. In addition, the proposition shows that the efficient response to an increase in the probability of a crisis is to decrease the degree of illiquidity. Proofs of all propositions are contained in the appendix.

**Proposition 1** \( \rho^* > 1 \) holds for all \( q \geq 0 \) and \( \rho^* \) is strictly decreasing in \( q \).

### 3.4 Bailouts

The next proposition establishes a key feature of the \( q \)-efficient allocation: less public good is provided in the event of a crisis than in normal times.
Proposition 2  \( \hat{g}^* < g^* \) holds for all \( q \geq 0 \).

Recall that \( g^* \) is the quantity of resources initially set aside to provide the public good. If a crisis occurs, some of these resources are instead used to provide private consumption to those investors who have not yet been able to withdraw. The property \( \hat{g}^* < g^* \) can, therefore, be interpreted as a bailout of the financial system: all investors pay a cost in terms of a lower level of the public good (an “austerity program”) in order to augment the private consumption of those agents facing losses on their financial investments.\(^7\)

Proposition 2 shows that this bailout is part of the efficient allocation of resources when a crisis is possible. The logic behind the result is fairly general and seems likely to appear in a wide range of settings. The planner’s fiscal plan is designed so that the marginal social value of public consumption will equal the marginal value of the private consumption in normal times. When a crisis occurs, it leads to a misallocation of resources that lowers private consumption for some investors, which raises their marginal value of consumption. The efficient response must, therefore, be to shift some resources away from public consumption and into the private consumption of these investors. Notice that this “bailout” is efficient even from an ex ante point of view, as it provides investors with insurance against the losses they may suffer in the event of a crisis.

4 Equilibrium under Discretion

I now turn to the allocation of resources that emerges in an equilibrium of the decentralized economy, again focusing on scenarios in which a crisis occurs in state \( s_2 \). The equilibrium allocation is constructed by working backward, beginning with the division of resources among the remaining investors in the event of a run.

4.1 The post-run allocation and bailout policy

Suppose the realized state is \( s_2 \) and a run occurs. Once it discovers that a run has taken place, each intermediary \( j \) efficiently divides whatever resources it has available among its remaining investors. Let \( \tau \) denote the fraction of investors’ endowments collected in taxes in the initial period, so that \( 1 - \tau \) is the size of the deposit made by each investor. Let \( c_{E,j} \) denote the amount received by each of the first \( \pi \) investors to withdraw from intermediary \( j \) and let \( b_j \geq 0 \) denote the size of the bailout.

\(^7\) Note that total government spending is unaffected by a financial crisis in this model, since all tax revenue is collected in the initial period and the government budget is always balanced. What changes during a crisis is the composition of government spending between public services and transfer payments.
payment per investor received by the intermediary. Then the resources available to intermediary $j$, per remaining investor, are given by

$$\psi_j = \frac{1 - \tau - \pi c_{E,j} + b_j}{1 - \pi}. \quad (8)$$

The intermediary sets the consumption levels $(\widehat{c}_{E,j}, \widehat{c}_{L,j})$ to solve

$$\hat{V} (\psi_j) \equiv \max \pi u (\widehat{c}_{E,j}) + (1 - \pi) u (\widehat{c}_{L,j}) \quad (9)$$

subject to

$$\pi \widehat{c}_{E,j} + (1 - \pi) \frac{\widehat{c}_{L,j}}{R} \leq \psi_j \quad \text{and} \quad \widehat{c}_{L,j} \geq \widehat{c}_{E,j}. \quad (10)$$

The solution to this problem is characterized by the first-order conditions

$$u' (\widehat{c}_{E,j}) = R u' (\widehat{c}_{L,j}) = \hat{\mu}_j, \quad (11)$$

where $\hat{\mu}_j$ is the multiplier on the resource constraint (10).

The policy maker divides its revenue $\tau$ between a level of the public good $\widehat{g}$ and bailout payments $b_j$. Let $\sigma_j$ denote the fraction of investors in the economy who have deposited with intermediary $j$. The problem of choosing the optimal bailout policy can be written as

$$\max_{\{b_j, \widehat{g}\}} \sum_j \sigma_j (1 - \pi) \hat{V} (\psi_j) + v (\widehat{g})$$

subject to the relationship (8) and the budget constraint

$$\widehat{g} + \sum_j \sigma_j b_j = \tau.$$

The solution to this problem is characterized by first-order conditions

$$\hat{V}' (\psi_j) = u' (\widehat{g}) \quad \text{for all } j,$$

which immediately imply

$$\psi_j = \psi_{j'}, \quad \text{for all } j \text{ and } j'.$$

(12)

In other words, the ex post efficient bailout payments equalize the resources available for private
consumption across intermediaries. The incentive problems that will be caused by this bailout policy are clear: an intermediary with fewer remaining resources (because it chose a higher value of $c_{E,j}$) will receive a larger bailout.\footnote{Note that, in principle, a similar incentive problem could arise in state $s_1$ if the policymaker made bailout payments to intermediaries that chose an unusually high level of $c_{E,j}$, in that state as well. I assume that bailout payments are only made in the event of a financial crisis. This assumption could be justified by reputation concerns, which will be significant for decisions made in normal times but much less important for a policymaker facing a rare event like a financial crisis.} The total size of the bailout payments is then given by

$$b \equiv \sum_j \sigma_j b_j = \tau - \hat{g} \quad (13)$$

4.2 The ex ante allocation

The remaining elements to be determined are the payments given by intermediaries to the first $\pi$ investors who withdraw and the tax rate. Since all intermediaries face the same decision problem, I omit the $j$ subscript and use $c_E$ to denote the payment offered by a representative intermediary. The equilibrium value of $c_E$ must solve

$$\max_{\{c_E, c_L\}} (1 - q) (\pi u(c_E) + (1 - \pi) u(c_L)) + q \left( \pi u(c_E) + (1 - \pi) \hat{V} \right) \quad (14)$$

subject to

$$\pi c_E + (1 - \pi) \frac{c_L}{R} = 1 - \tau, \quad \text{and}$$

$$c_L \geq c_E \quad (15)$$

Intermediaries and their investors anticipate the fact that, in the event of a crisis, the consumption of each remaining investor will depend only on the aggregate amount of resources in the economy and not on the condition of the investor’s own intermediary. For this reason, an intermediary takes the value $\hat{V}$ as given when choosing the payment $c_E$.

The first-order conditions that characterize the solution to this problem when the incentive-compatibility constraint (16) does not bind are

$$u'(c_E) = (1 - q) \mu = (1 - q) R u'(c_L), \quad (17)$$

where $(1 - q) \mu$ is the multiplier on the resource constraint (15). Comparing the first inequality with (3) illustrates the distortion of incentives: the equilibrium payment $c_E$ balances the marginal
value of resources in the early period against the marginal value of resources in the late period in the no-run state, ignoring the value of resources in the event of a run. The larger the probability of a run $q$ is, the more distorted the allocation of resources becomes. We can also see from this expression that the incentive compatibility constraint will be satisfied at the interior solution as long as

$$q \leq \frac{R - 1}{R},$$

but will otherwise be violated. When the constraint does bind, the equilibrium values are determined by the condition $c_L = c_E$ together with the resource constraint (15).

Define the value function

$$V^D(\tau) = \pi u(c_E) + (1 - q)(1 - \pi)u(c_L) + v(\tau) + q(1 - \pi)\hat{V}\left(\frac{1 - \tau - \pi c_E}{1 - \tau}\right) + v(\tau - b)$$

where $c_E$ and $c_L$ are the solution to problem (14) and $b$ is given by (13). The policy maker will choose the tax rate $\tau$ in the initial period to maximize the function $V^D$. Notice that (18) differs from the objective in (14) because the policy maker recognizes that the value $\hat{V}$ depends on the total quantity of resources remaining after the first $\pi$ withdrawals have taken place, whereas individual intermediaries and investors taken this value as given.

The first-order condition characterizing the policy maker’s choice of tax rate can be written as

$$v'(\tau) = \mu + \frac{q}{1 - q}\frac{d}{d\tau}\frac{d}{d\tau}$$

If the probability of a crisis $q$ were zero, the tax rate would be set to equate the marginal utility of the public good with the marginal value of goods used for private consumption, $\mu$. When $q$ is positive, however, the policy maker must also take into account the fact that changes in $\tau$ will lead to changes in the equilibrium level of $c_E$, which in turn affects the total quantity of resources available in the event of a run. This effect is captured by the second term on the right-hand side of (19).

Let $c^D$ denote the complete allocation derived above. It is straightforward to show that $c^D$ varies continuously with the probability of a crisis $q$ and converges to the efficient allocation as $q$ goes to zero. This allocation is indeed an equilibrium of the decentralized economy if and only if $c^D_E \geq \hat{c}^D_L$ holds, that is, if and only if patient investors find it optimal to withdraw early in state $s_2$. 

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Let $\Phi^D$ denote the set of economies $e$ for which this condition holds. Welfare in this equilibrium is given by

$$W^D \equiv \max_{\{\tau\}} V^D(\tau).$$

### 4.3 Illiquidity and fragility

The distortion created by the bailout policy gives each intermediary an incentive to become more illiquid by offering a larger return to its investors who withdraw early. The next proposition shows that, in the aggregate, this effect increases illiquidity in the financial sector as a whole.

**Proposition 3** $\rho^D > \rho^*$ holds for all $q > 0$. In addition, $\rho^D$ is strictly increasing in $q$ for $q < (R - 1)/R$ and constant for larger values of $q$.

Recall that under the $q$-efficient allocation of resources, an increase in the probability of a crisis leads to a more liquid financial system (see Proposition 1). Proposition 3 shows that the opposite occurs in the competitive equilibrium. When a financial crisis – and the associated bailout – is more likely, investors prefer a higher short-run return and intermediaries become less liquid. Together, the propositions show that the gap between the efficient level of illiquidity and the level that emerges in equilibrium becomes wider as the probability of a crisis increases.

This higher degree of illiquidity increases the scope for financial fragility in the model, as shown by the following strict inclusion relationship.

**Proposition 4** $\Phi^D \supset \Phi^*$.

This result gives a precise sense in which the incentive problem caused by bailouts makes the financial system more fragile. Consider an economy that is not in the set $\Phi^*$. For these parameter values, the $q$-efficient allocation of resources is such that a patient investor has no incentive to withdraw early, even if he believes everyone else will try to do so. As a result, the financial system is stable in the sense that a crisis cannot occur in equilibrium. In the competitive equilibrium, however, intermediaries become more illiquid than in the $q$-efficient allocation and investors would find themselves in a worse position in the event of a run. This fact increases the incentive for a patient investor to withdraw early if he believes other investors will run. In some cases, this increase is large enough to make joining the run an optimal response, meaning that the distortions
created by the bailout policy introduce an equilibrium in which a self-fulfilling financial crisis occurs.

This set-theoretic approach to measuring fragility has a natural interpretation in terms of changes in the probability of a financial crisis. Suppose that at the beginning of period zero, the parameter values $e$ are drawn at random from some probability distribution $f$. If the realized $e$ is such that the economy is not fragile, investors do not run on the financial system in either state. If the economy is fragile, however, investors run on the financial system in state $s_2$. The ex ante probability assigned to a crisis by this process will be strictly higher in the decentralized economy than under the $q$-efficient allocation for any probability distribution $f$ that has full support. In this sense, the likelihood of a financial crisis is inefficiently high in the decentralized economy.9

In the next two sections, I analyze two policy measures designed to mitigate the incentive problem and potentially improve welfare compared to this discretionary policy regime: a strict no-bailouts policy and a tax on short-term liabilities.

5 Committing to No Bailouts

I now examine a policy regime that has received considerable attention in the financial press and elsewhere: a commitment to not providing any bailout payments, that is, to setting $b = 0$ in all states of nature. Note that a very limited form of commitment is being introduced here, in the sense that the policy maker can commit to follow this simple rule but not a more intricate plan. Whether or not it is feasible to commit to such a rule in reality is debatable. The question I ask here is whether such a policy – if feasible – would be desirable.10

5.1 Equilibrium

In the event of a run, each intermediary responds by implementing the efficient allocation of its

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9 An alternative approach would be to attempt to resolve the multiplicity of equilibrium by introducing private information as in the literature on global games pioneered by Carlsson and van Damme [5]. However, this approach places rather strict requirements on the information structure of the model. Papers that have used the global games methodology in Diamond-Dybvig type models have done so by placing arbitrary restrictions on contracts between intermediaries and their investors (see, for example, Rochet and Vives [24] and Goldstein and Pauzner [16]). These restrictions themselves are potential sources of financial fragility, quite separate from the issues related to bailouts under consideration here. The approach taken here captures the effects of changes in the incentives faced by investors in a reasonably clear and transparent way, and does not place any additional restrictions on agents other than those imposed by the physical environment.

10 Note that committing to a pre-specified bailout size $b > 0$ would not correct the incentive problem that arises in the discretionary regime. The distortion in the model comes not from the size of the bailout payment per se, but from the distribution of the bailout payment across intermediaries according to (13).
remaining resources among its investors, as in problem (9). These resources will be allocated according to the first-order condition (11), and their value is measured by the function $\hat{V}$. The equilibrium values of $c_E$ and $c_L$ will solve

$$
\max_{\{c_E, c_L\}} \pi u (c_E) + (1 - \pi) \left( (1 - q) u (c_L) + q \hat{V} \left( \frac{1 - \tau - \pi c_E}{1 - \pi} \right) \right)
$$

subject to

$$
\pi c_E + (1 - \pi) \frac{c_L}{R} \leq 1 - \tau, \quad \text{and} \quad c_L \geq c_E.
$$

Note that the function $\hat{V}$ is evaluated at the level of resources (per investor) that the intermediary will have after $\pi$ withdrawals, a quantity that depends on the intermediary’s choice of $c_E$. Intermediaries now recognize that, in the event of a run, the only resources that will be available for the private consumption of the remaining investors will be those funds held by the intermediary.

The solution to this problem is characterized by the first-order conditions

$$
u' (c_E) = (1 - q) \mu + q \hat{\mu}
$$

and

$$Ru' (c_L) = \mu,
$$

where $(1 - q)\mu$ is the multiplier on resource constraint and the first equation uses the envelope condition $\hat{V}' = \hat{\mu}$. Comparing (21) with (17) shows the effect of the no-bailout policy and how it mitigates the incentive problem. Under this policy, an intermediary must balance the value of the early payment $c_E$ not only against the value of late consumption in the no-run state $\mu$, but also against the value of resources in the run state $\hat{\mu}$.

Define the value function

$$V^{NB} (\tau) = \pi u (c_E) + (1 - \pi) \left( (1 - q) u (c_L) + q \hat{V} \left( \frac{1 - \tau - \pi c_E}{1 - \pi} \right) \right) + v (\tau),$$

where $c_E$ and $c_L$ are the solution to (20). As indicated in this expression, the level of the public good is equal to tax revenue $\tau$ in both states. The policy maker will choose the tax rate to maximize
The first-order condition for this problem can be written as

\[ v' (\tau) = (1 - q) \mu + q \tilde{\mu}. \] (23)

Let \( c^{NB} \) denote the equilibrium allocation under a no-bailout policy. Let \( \Phi^{NB} \) denote the set of economies for which \( c^{NB}_E \geq \hat{c}^{NB}_L \) holds and, hence, there is an equilibrium in which all investors attempt to withdraw early in state \( s_2 \). Equilibrium welfare under this policy regime is given by

\[ W^{NB} \equiv \max_{\{\tau\}} V^{NB} (\tau). \]

### 5.2 Illiquidity and fragility

One can show that the degree of illiquidity under the no-bailout regime is strictly decreasing in \( q \).

Recall that this result is the opposite of that obtained in the previous section. When intermediaries and investors anticipate a bailout in the event of a run, an increase in the probability of a run leads them to adopt a more illiquid position. Here, in contrast, an increase in the probability of a run leads intermediaries to adopt a more liquid position. Comparing \( \rho^{NB} \) to the degree of illiquidity in the \( q \)-efficient allocation, however, shows that the no-bailout policy actually leads intermediaries to become too liquid. These results are summarized in the following proposition.

**Proposition 5** \( \rho^{NB} < \rho^* \) holds for all \( q > 0 \) and \( \rho^{NB} \) is strictly decreasing in \( q \).

This proposition shows that the no-bailout policy introduces a new distortion in ex ante incentives. Instead of performing too much maturity transformation, and taking on too much illiquidity, intermediaries perform too little under this policy. The reason is that intermediaries must now completely self-insure against the possibility of a run. In the \( q \)-efficient allocation, in contrast, the bailout policy provides intermediaries with some insurance against this event. As a result, the shadow value of resources in the financial system in state \( s_2, \hat{\mu} \), is higher under a no-bailouts policy than in the \( q \)-efficient allocation, which leads intermediaries to choose too much liquidity from a social point of view.

Despite encouraging financial intermediaries to become more liquid, the no-bailout policy still generates greater scope for financial fragility than the \( q \)-efficient allocation.
Proposition 6 \( \Phi^{NB} \supset \Phi^* \). Moreover, there exist economies in \( \Phi^{NB} \) that are not in \( \Phi^D \).

The intuition behind this result can be seen by considering the limiting case as \( q \) goes to zero. The components of the allocation that apply to the no-run state \( (c_E^{NB}, c_L^{NB}, g^{NB}) \) converge to the corresponding components of the \( q \)-efficient allocation, but the post-run components of the allocation \( (\widehat{c}_E^{NB}, \widehat{c}_L^{NB}, g^{NB}) \) do not. Because no bailout payments are made, the level of the public good is higher than in the \( q \)-efficient allocation and the private consumption levels \( \widehat{c}_E^{NB} \) and \( \widehat{c}_L^{NB} \) are lower. It follows that the fragility condition \( c_E \geq \widehat{c}_L \) will hold for a strictly larger set of parameter values.

The second part of Proposition 6 demonstrates that some economies that are not fragile under the discretionary policy regime become fragile when a no-bailout policy is implemented. This result is somewhat surprising in light of the arguments made by many commentators during the recent financial crisis and the subsequent debate over financial regulatory reform. The intuition behind this result is clear: by increasing \( \widehat{c}_L \), a bailout reduces the cost to an investor of leaving her funds deposited in the event of a run. In other words, the anticipation of a bailout also has a positive effect on ex ante incentives by encouraging investors to keep their funds deposited in the financial system. The no-bailout policy removes this positive effect and, as a result, can increase financial fragility.

5.3 Welfare

In cases where the economy is fragile under both the policy regimes, the desirability of a no-bailout policy will depend on how it affects equilibrium welfare. In general, equilibrium welfare may be either higher or lower than in the discretionary regime, depending on parameter values. As the next proposition shows, however, a sharp comparison is possible when the value of \( q \) is small, that is, when a financial crisis is sufficiently unlikely. In such situations, committing to a no-bailout policy (i) never enhances financial stability and (ii) necessarily lowers welfare.

Proposition 7 For any \( (R, \pi, u, v) \), there exists \( \overline{q} > 0 \) such that \( q < \overline{q} \) and \( e \in \Phi^D \) implies both \( e \in \Phi^{NB} \) and \( W^D > W^{NB} \).

5.4 An example

A numerical example can be used to illustrate the results presented above. The utility functions for
this example are
\[ u(c) = \frac{(c)^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v(g) = \frac{\delta(g)^{1-\gamma}}{1-\gamma}, \]
and the fundamental parameter values are given by \((R, \pi, \gamma, \delta) = (1.1, 0.5, 6, 0.01)\). When \(q\) is small, the financial system is fragile under the \(q\)-efficient allocation of resources for these values and, hence, is fragile under both the discretionary and the no-bailout policy regimes. Panel (a) in Figure 1 shows the degree of illiquidity \(\rho\) in each regime as a function of the probability of a crisis \(q\). When \(q = 0\), the first-best value of \(\rho\) obtains in all three scenarios. As a crisis becomes more likely, the degree of illiquidity in the efficient allocation declines, in accordance with Proposition 1. Under the no-bailout policy, illiquidity declines even faster as intermediaries adopt more conservative positions, in line with Proposition 5. Under the discretionary policy, in contrast, illiquidity rises as \(q\) increases. The kink in this curve corresponds to point where the incentive compatibility constraint begins to bind in problem (14). Beyond this point the degree of illiquidity stays constant, in line with Proposition 3.

Panel (b) of the figure compares equilibrium welfare under the discretionary and no-bailout regimes. The curve plotted in the figure represents the benefit of the discretionary regime over the no-bailouts regime, \(W^D - W^{NB}\). Two competing forces are at work in determining the shape of this curve. The ex ante distortion – as depicted in panel (a) – is larger in the discretionary case; this fact tends to make the no-bailout policy attractive. However, the no-bailout regime also leads to an ex post inefficient allocation of resources in the event of a run. For small enough values of \(q\), these ex post concerns dominate and the discretionary policy yields higher welfare, in line with
Proposition 7. As \( q \) increases further and the ex ante distortions become larger, however, the former effect eventually dominates. For values of \( q \) above approximately 0.08, the curve becomes negative and welfare is higher under the no-bailouts policy. Once \( q \) passes the threshold level \((R - 1) / R\), however, the incentive compatibility constraint binds in the discretionary equilibrium. As a result, the ex ante distortion in the discretionary case remains constant as \( q \) increases further. For the no-bailout policy, however, the welfare loss from having an inefficient allocation of resources in the event of a run continues to grow as the probability of this event increases. For values of \( q \) above 0.12, the curve becomes positive and the discretionary policy again yields higher welfare.

Figure 2 illustrates how financial fragility differs across policy regimes by presenting a projection of the sets \( \Phi^*, \Phi^D, \) and \( \Phi^{NB} \) onto a two-dimensional diagram. The horizontal axis of the figure corresponds to the probability of a crisis, \( q \), while the vertical axis measures one of the fundamental parameters, \( \pi \). Different shades are used to represent economies that are fragile under the different policy regimes. The darkest area in the figure represents the economies belong to all three sets. For these combinations of parameter values, the financial system is fragile even under the q-efficient allocation of resources. As the probability of a crisis \( q \) rises, illiquidity falls in this allocation (Proposition 3) and, as a result, the set of values of \( \pi \) leading to fragility becomes smaller, as shown in the figure.

The set \( \Phi^D \) is represented by the lightest colored (and lower most) area, together with the two darkest areas where it overlaps with the other sets. Notice that economies with low values of \( \pi \) tend to be fragile under the discretionary policy regime. This pattern reflects the fact that intermediaries tend to take on more illiquidity when there are relatively few impatient investors, which implies that the magnitude of the distortion under the discretionary regime is largest when \( \pi \) is small. The set \( \Phi^{NB} \) is represented by the next-lightest colored (and upper most) area, together with the two darkest areas. Under this regime, economies with low values of \( \pi \) tend to be stable, but those with high values of \( \pi \) tend to be fragile. If \( \pi \) is large, there are relatively few remaining investors when a bailout payment is made, which implies that even a moderate-sized bailout payment will have a large effect on investors’ incentives. Hence, the destabilizing effect of removing this insurance is largest when \( \pi \) is close to one.

Figure 3 presents this same diagram for a variety of different parameter values, showing how changes in the parameters \( \gamma \) and \( \delta \) affect the size and shape of the sets \( \Phi^*, \Phi^D, \) and \( \Phi^{NB} \).
Figure 2: The sets $\Phi^*$, $\Phi^D$ and $\Phi^{NB}$. Darker areas indicate the intersection of sets.

Figure 3: The sets $\Phi^*$, $\Phi^D$, and $\Phi^{NB}$ for different parameter values
6 Taxing Short-term Liabilities

Another policy option is to place no restrictions on the bailout policy, but to offset the distortion through regulation or some other ex ante intervention. To illustrate the effects of such an intervention, I now allow the policy maker to impose a tax on intermediaries’ short-term liabilities; this policy can also be thought of as a tax on the activity of maturity transformation. This particular tax is one of several possible policies that would have equivalent effects in the simple model studied here, including directly imposing an appropriately-chosen cap on short-term liabilities. The goal is to investigate the effectiveness of a policy regime that aims to influence intermediaries’ choices through ex ante intervention rather than through restrictions on the ex post bailout payments. A Pigouvian tax on short-term liabilities is one way to illustrate the results of such an approach.

Suppose each intermediary must pay a fee that is proportional to the total value of its short-term liabilities,

\[ \text{fee}_j = \eta \pi \sigma_j c_E, \]

where, as above, \( \sigma_j \) denotes the fraction of investors who deposit with intermediary \( j \). The tax rate is this policy is \( \eta \pi \), where \( \eta \) is chosen by the policy maker. For simplicity, I make the policy revenue neutral by giving each intermediary a lump-sum transfer \( N \sigma_j (1 - \tau) \), where \( N \) is equal to the average fee collected per unit of deposits. This assumption is only to facilitate comparison with the earlier cases.

6.1 Equilibrium

Under this policy, the equilibrium payment \( c_E \) will maximize the objective in (14), but subject to the modified resource constraint

\[ \pi c_E + (1 - \pi) \frac{c_L}{R} \leq 1 - \tau - \eta \pi c_E + N (1 - \tau). \]  

(24)

The first-order conditions of this modified problem are

\[ u'(c_E) = (1 + \eta) (1 - q) \mu = (1 + \eta) (1 - q) R u'(c_L), \]

where \( (1 - q) \mu \) is again the multiplier on the resource constraint. We know that the post-run allocation of resources will be efficient, and hence will satisfy the usual first-order conditions (5).
Revenue neutrality implies

\[ N (1 - \tau) = \eta \pi E. \]

Substituting this condition into (24) yields the standard resource constraint for the no-run state.

### 6.2 The optimal Pigouvain tax rate

Can the tax rate \( \eta \) can be set so that the equilibrium allocation with ex ante intervention matches the \( q \)-efficient allocation? In the \( q \)-efficient allocation, we have

\[ u' (c^*_E) = (1 - q) Ru' (c^*_L) + q Ru' (\hat{c}^*_L) \]

In order for the equilibrium allocation to be efficient, therefore we need

\[ \eta (1 - q) Ru' (c^*_L) = q Ru' (\hat{c}^*_L) \]

or

\[ \eta = \frac{q \hat{\mu}^*}{(1 - q) \mu^*} \equiv \eta^*, \quad (25) \]

where \( (1 - q) \mu^* \) and \( q \hat{\mu}^* \) are the multipliers on the resource constraints (1) and (2), respectively, evaluated at the \( q \)-efficient allocation. When \( \eta \) is set equal to \( \eta^* \), the competitive equilibrium allocation will satisfy all of the conditions characterizing the \( q \)-efficient allocation. Since these conditions uniquely determine the efficient allocation, we have the following result.

**Proposition 8** When the tax rate \( \eta \) is set according to (25), the equilibrium allocation with a tax on short-term liabilities is equal to the \( q \)-efficient allocation.

The ratio of the two multipliers in (25) can be interpreted as the price of period-2 consumption in state \( s_2 \) relative to state \( s_1 \). The optimal tax rate \( \eta^* \) induces each intermediary to place an additional value on period-2 resources equal to the marginal social value of resources in the event of a run, which exactly offsets the distortion created by the bailout policy. This result shows ex ante intervention to be a powerful policy tool in the environment studied here. An appropriately chosen tax rate allows the policy maker to provide investors with the optimal level of insurance against the losses associated with a financial crisis without leading intermediaries to choose excessively high levels of illiquidity. Not only does this policy improve the allocation of resources in normal times, it also conveys a macroprudential benefit, decreasing financial fragility relative to either the discretionary or the no-bailouts regime.

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7 Concluding Remarks

There is widespread agreement that the anticipation of receiving a public-sector bailout in the event of a crisis distorts the incentives of financial institutions and other investors. By partially insulating these agents from the effects of a negative outcome, bailouts diminish their incentive to provision for such outcomes and encourage excessively risky behavior. Such concerns have featured prominently in the recent debate on financial regulatory reform and have lead some commentators to argue that governments and central banks should aim to make credible commitments that limit future bailouts.

The model presented here shows that there is another side to this issue, however, and that the anticipation of a bailout can have positive ex ante effects as well. These positive effects appear in two distinct forms. First, bailouts are part of an efficient insurance arrangement. A financial crisis leads to a misallocation of resources that raises the marginal social value of private consumption. The optimal response for a policy maker is to decrease public consumption, using these resources to augment the private consumption of agents facing losses. This policy raises ex ante welfare by providing risk-averse agents with insurance against the losses associated with a crisis.

In addition, the insurance provided by a bailout policy can have a stabilizing effect on the financial system. Financial crises are commonly thought to have an important self-fulfilling component, with individual investors each withdrawing funds in part because they fear the withdrawals of others will deepen the crisis and create further losses. The anticipation of a bailout lessens the potential loss an investor faces if she does not withdraw her funds. As such, it decreases the incentive for investors to withdraw, which, in turn, makes the financial system less susceptible to a crisis. Committing to a no-bailouts policy removes this insurance and, in some cases, can actually create fragility in the financial system.

It should be emphasized that the bailout policies studied here are efficient; they do not lead to rent-seeking behavior, nor are they motivated by outside political considerations. In reality, these types of distortions are important concerns. The message of the paper is not that any type of bailout policy is acceptable as long as the ex ante effects are offset through taxation. Limits on the ability of policy makers to undertake inefficient redistribution during a crisis may well be desirable. Rather, the message is that restrictions on bailouts alone cannot ensure that investors face the correct ex ante incentives. In a reasonably standard economic environment, the efficient allocation of resources requires that investors receive some insurance in the form of a bailout.
Providing this insurance distorts incentives, and some form of regulation or other ex ante policy intervention is needed to offset this distortion.

Extending the analysis to richer environments may generate insight into the relative merits of different types of ex ante intervention. In the model presented here, taxing short-term liabilities and imposing a cap on such liabilities are equally effective policies. In a setting where intermediaries make additional decisions and, perhaps, take unobserved actions (such as portfolio allocations, effort in monitoring investments, etc.), this equivalence may no longer hold. Studying such environments using the approach developed here seems a promising avenue for future research.
Appendix A. Proofs of Propositions

**Proposition 1:** $\rho^* > 1$ holds for all $q \geq 0$ and $\rho^*$ is strictly decreasing in $q$.

*Proof:* The proof is divided into three steps.

**Step 1:** Show that $\rho^* > 1$ holds. The assumption that the coefficient of relative risk aversion for $u$ is greater than 1 implies\(^{11}\)

$$u' (1 - g) > Ru' (R (1 - g)).$$

If the $q$-efficient allocation satisfies $u' (c^*_E) \leq Ru' (c^*_L)$, then the resource constraint (1) and the concavity of $u$ would immediately imply $c^*_E > 1 - g$ and the result would be established. Suppose instead, then, that

$$u' (c^*_E) > Ru' (c^*_L) \quad (26)$$

holds. In this case, the first-order conditions (3) – (5) imply $\hat{\mu}^* > \mu^*$ and the vector inequality

$$(c^*_E, c^*_L, g^*) \succ (c^*_E, \hat{c}^*_L, \hat{g}^*). \quad (27)$$

In other words, if (26) holds, each component of the post-run allocation must be strictly smaller than the corresponding component of the no-run allocation. Note that the resource constraints (1) and (2) will both hold with equality at the $q$-efficient allocation and can be written as

$$\pi c^*_E + (1 - \pi) \frac{c^*_L}{R} = 1 - g^* \quad \text{and}$$

$$\pi \hat{c}^*_E + (1 - \pi) \frac{\hat{c}^*_L}{R} = \frac{1 - \pi c^*_E - \hat{g}^*}{1 - \pi}.$$ 

The vector inequality (27) then implies

$$\frac{1 - \pi c^*_E - \hat{g}^*}{1 - \pi} < 1 - g^*. \quad (28)$$

Straightforward algebra shows that this inequality is equivalent to $c^*_E > 1 - g^*$, that is, $\rho^* > 1$.

**Step 2:** Show that $\mu^* < \hat{\mu}^*$ holds. The reasoning follows that in Step 1, but in reverse order. Given that $\rho^* > 1$ and (28) hold, the first-order conditions (3) – (5) together with the resource constraints (1) and (2) imply that the vector inequality (27) must hold, as must $\mu^* < \hat{\mu}^*$.

**Step 3:** Show that $\rho^*$ is strictly decreasing in $q$. Note that the resource constraint (1) can be written

\(^{11}\) This is a well-know property; see Diamond and Dybvig (1983, footnote 3) for a proof.
\[ \rho^{-1} = \pi + (1 - \pi) \frac{1}{R} \left( \frac{c_E}{c_L} \right)^{-1}. \]  

(29)

Thus \( \rho^* \) is strictly decreasing in \( q \) if and only if the ratio \( \frac{c_E^*}{c_L^*} \) is strictly decreasing in \( q \). The first-order conditions (3) – (5) together with the resource constraints (1) and (2) implicitly define the \( q \)-efficient allocation as a function of \( q \) in a neighborhood of the solution \( c^* \). Differentiating (1) and (4) with respect to \( q \) and combining the resulting equations yields

\[ \frac{dc_L^*}{dq} = -\alpha_1 \frac{dc_E^*}{dq}, \quad \text{where} \quad \alpha_1 \equiv \frac{\pi}{(1 - \pi) \frac{1}{R} + \frac{R u''(c_L^*)}{u'(g^*)}} > 0. \]

(30)

To show that the ratio \( c_E^*/c_L^* \) is strictly decreasing in \( q \), therefore, it suffices to show that \( c_E^* \) is strictly decreasing in \( q \). Differentiating (2) and (5) with respect to \( q \) yields

\[ \frac{dc_L^*}{dq} = -\alpha_2 \frac{dc_E^*}{dq}, \]

where

\[ \alpha_2 \equiv \frac{\pi}{(1 - \pi) \left( \frac{R u''(c_E^*)}{u'(c_E^*)} + (1 - \pi) \frac{1}{R} \right) + \frac{R u''(c_L^*)}{u'(g^*)}} > 0. \]

(31)

Differentiating (3) with respect to \( q \) yields

\[ u''(c_E^*) \frac{dc_E^*}{dq} - (1 - q) R u''(c_L^*) \frac{dc_L^*}{dq} - q R u''(c_L^*) \frac{dc_L^*}{dq} = R \left( u'(c_L^*) - u'(c_E^*) \right). \]

Define

\[ \alpha_3 = R \left( u'(c_L^*) - u'(c_E^*) \right) > 0. \]

The fact that this expression is strictly positive follows from \( \hat{\mu}^* > \mu^* \) and the first-order conditions (4) and (5). Combining the previous equation with (30) and (31) yields

\[ \frac{dc_E^*}{dq} = \frac{\alpha_3}{u''(c_E^*) + (1 - q) R u''(c_L^*) \alpha_1 + q R u''(c_L^*) \alpha_2} < 0, \]

as desired.

\[ \text{Proposition 2: } \hat{g}^* < g^* \text{ holds for all } q \geq 0. \]

\[ \text{Proof: } \text{The proof of Proposition 1 establishes that } \mu^* < \hat{\mu}^* \text{ holds for all } q \text{ (see Step 2 of the proof).} \]

The first-order conditions (4) and (5) then immediately imply \( \hat{g}^* < g^* \).  

\[ \blacksquare \]
Proposition 3: $\rho^D > \rho^*$ holds for all $q > 0$. In addition, $\rho^D$ is strictly increasing in $q$ for $q < (R - 1) / R$ and constant for larger values of $q$.

Proof: First, since the multipliers $\mu^*$ and $\hat{\mu}^*$ are always strictly positive, we clearly have

\[(1 - q) + q \frac{\hat{\mu}^*}{\mu^*} > (1 - q).\]

This inequality implies

\[\frac{(1 - q) \mu^* + q \hat{\mu}^*}{R^{-1} \mu^*} > \frac{(1 - q) \mu^D}{R^{-1} \mu^D}\]

or

\[\frac{u'(c^*_E)}{u'(c^*_L)} > \frac{u'(c^D_E)}{u'(c^D_L)}.\]

Because the function $u$ is of the constant-relative-risk-aversion form, expected utility preferences over pairs $(c_E, c_L)$ are homothetic and, therefore, the above inequality implies

\[\frac{c^*_E}{c^*_L} < \frac{c^D_E}{c^D_L},\]

which, using (29), immediately implies $\rho^* < \rho^D$, as desired.

Next, from the first-order conditions (17) we have

\[\frac{u'(c^D_E)}{u'(c^D_L)} = \frac{1 - q}{R} \quad \text{for} \quad q < \frac{R - 1}{R}.\]

Using the homotheticity of preferences, this equation implies that the ratio $c^*_E / c^*_L$ is strictly increasing in $q$. Equation (29) then shows that $\rho^D$ is also strictly increasing in $q$ over this range. For larger values of $q$, the incentive compatibility constraint $c_E \leq c_L$ binds in the equilibrium allocation. In this case, (29) implies that $\rho^D$ is independent of $q$. 

Proposition 4: $\Phi^D \supset \Phi^*$.

Proof: The proof is divided into two steps.

Step 1: Show that $c^D_E > c^*_E$ holds for all $q > 0$. First, use the resource constraint (15) to replace $c_L$ in the first-order condition (17) and then differentiate the latter with respect to $\tau$, which yields

\[\frac{dc_E}{d\tau} = -\frac{1}{\pi + (1 - \pi) \frac{u''(c_E)}{(1 - q)R^2(u'(c_L))}}.\]
This expression can be used to show

\[-1 < \pi \frac{dc_E}{d\tau} < 0.\]

Combined with (19), this inequality implies for any \( q > 0 \), we have

\[v'(g^D) > \mu^D - \frac{q}{1-q} \hat{\mu}^D.\] (32)

Next, suppose \( c_E^D \leq c_E^* \) held for some \( q > 0 \). Then (3) and (17) would imply

\[(1-q) \mu^D \geq (1-q) \mu^* + q \hat{\mu}^*.\] (33)

In addition, \( 1 - \pi c_E^D \geq 1 - \pi c_E^* \) would hold; combined with (5) and (11) this would imply

\[\hat{\mu}^D \leq \hat{\mu}^*.\] (34)

Combining (34) and (33) and rearranging terms yields

\[\mu^D - \frac{q}{1-q} \hat{\mu}^D \geq \mu^*.\] (35)

Using (32) and (4), this inequality would imply \( g^D < g^* \). Also note that (35) would imply \( \mu^D > \mu^* \), which through (4) and (17) would give \( c_E^D < c_E^* \). However, combining these inequalities with the assumption \( c_E^D \leq c_E^* \), the resource constraint (15), and the equilibrium condition \( g = \tau \) shows that \( c^* \) would violate the resource constraint (1), a contradiction. Hence, \( c_E^D > c_E^* \) must hold.

**Step 2:** Show \( \Phi^D \supset \Phi^* \). Consider any economy in \( \Phi^* \), that is, for which \( c_E^* \geq c_L^* \) holds. Using the result from Step 1 that \( c_E^D > c_E^* \) holds, the first-order conditions (3) and (17) imply

\[(1-q) \mu^D < \frac{1}{R} \hat{\mu}^*.\]

Furthermore, \( 1 - \pi c_E^D < 1 - \pi c_E^* \) implies \( \hat{\mu}^D > \hat{\mu}^* \). We thus have

\[(1-q) \mu^D < \frac{1}{R} \hat{\mu}^D,\] (36)

which implies \( c_E^D > c_L^D \) and, hence, the economy is also in \( \Phi^D \). Moreover, the fact that the inequality in (36) is strict implies that the inclusion relationship is also strict: there exist economies for which \( c_E^* \) is slightly smaller than \( c_L^* \), but (36) still holds. Alternatively, it is easy to find examples of economies that belong to \( \Phi^D \) but not to \( \Phi^* \); see Figure 2.
Proposition 5: $\rho^{NB} < \rho^*$ holds for all $q > 0$ and $\rho^{NB}$ is strictly decreasing in $q$.

Proof: The proof, which is similar to that of Proposition 1, is divided into three steps.

Step 1: Show that $\rho^{NB} < \rho^*$ holds. First, recall that Proposition 2 establishes $\hat{g}^* < g^*$. Using the resource constraints (1) and (2), this implies

$$\frac{c^*_L}{R} < \pi \hat{c}^*_E + (1 - \pi) \frac{c^*_L}{R}$$

or

$$1 < R \pi \frac{c^*_E}{c^*_L} + (1 - \pi) \frac{c^*_L}{c^*_E}. $$

Under a no-bailout policy, $\hat{g} = g$ holds by definition and the resource constraints imply

$$1 = R \pi \frac{c^*_{NB}}{c^*_L} + (1 - \pi) \frac{c^*_{NB}}{c^*_L}. \quad (37) $$

It must be the case, therefore, that at least one of the following two inequalities holds:

$$\frac{\hat{c}^*_E}{c^*_L} > \frac{c^*_{NB}}{c^*_L} \quad \text{or} \quad \frac{\hat{c}^*_L}{c^*_L} > \frac{c^*_{NB}}{c^*_L}. \quad (38)$$

If the first of these inequalities holds, then by the homotheticity of preferences we have

$$\frac{u'(\hat{c}^*_E)}{Ru'(c^*_L)} < \frac{u'(c^*_{NB})}{Ru'(c^*_{NB})},$$

which is equivalent to

$$\frac{\hat{\mu}^*}{\mu^*} < \frac{\mu^{NB}}{\mu^{NB}}. \quad (39)$$

The second inequality in (38) would lead to the same conclusion. Working from (39), we have

$$(1 - q) + q \frac{\hat{\mu}^*}{\mu^*} < (1 - q) + q \frac{\mu^{NB}}{\mu^{NB}},$$

which can be rewritten as

$$\frac{(1 - q) \mu^* + q \hat{\mu}^*}{R^{-1} \mu^*} \leq \frac{(1 - q) \mu^{NB} + q \hat{\mu}^{NB}}{R^{-1} \mu^{NB}},$$

or

$$\frac{u'(c^*_E)}{u'(c^*_L)} < \frac{u'(c^*_{NB})}{u'(c^*_{NB})}. $$

Again using the homotheticity of preferences, this last inequality implies

$$\frac{c^*_E}{c^*_L} > \frac{c^*_{NB}}{c^*_L}. $$
Using (29), this inequality immediately implies $\rho^* > \rho^{NB}$, as desired.

**Step 2:** Show that $\mu^{NB} < \mu^{NB}$ holds. The assumption that the coefficient of relative risk aversion in $u$ is greater than one implies $R^2E > c^NB$. Equation (37) then implies that the ratio $\overline{c}^NB/c^NB$ must be smaller than one, which through the first-order conditions (11) and (22) implies $\tilde{\mu}^{NB} > \mu^{NB}$, as desired.

**Step 3:** Show that $\rho^{NB}$ is strictly decreasing in $q$. From (29), we know that $\rho^{NB}$ is strictly decreasing in $q$ if and only if the ratio $c^NB/c^NB$ is strictly decreasing in $q$. Differentiating the resource constraint (1) along with the first-order conditions (21) and (23) with respect to $q$ and combining the resulting equations yields

$$\frac{dc^NB}{dq} = -\beta_1 \frac{dc^NB}{dq},$$

where $\beta_1 \equiv \pi + \frac{u''(c^NB)}{v_0'(g^NB)} > 0$. (40)

To show that the ratio $c^NB/c^NB$ is strictly decreasing in $q$, therefore, it suffices to show that $c^NB$ is strictly decreasing in $q$. Doing the same with the post-run resource constraint (2) (including the no-bailout restriction $\widehat{g} = g$) and conditions (11) and (23) yields

$$\frac{dc^NB}{dq} = -\beta_2 \frac{dc^NB}{dq},$$

where

$$\beta_2 \equiv \frac{\pi + \frac{u''(c^NB)}{v_0'(g^NB)}}{(1 - \pi) / R + \pi \frac{Ru''(c^NB)}{u''(g^NB)}} > 0.$$ (41)

Finally, differentiating (21) with respect to $q$ yields

$$u''(c^NB) \frac{dc^{NB}}{dq} - (1 - q) Ru''(c^NB) \frac{dc^{NB}}{dq} - qRu''(c^{NB}) \frac{dc^{NB}}{dq} = R \left( u'(c^{NB}) - u'(c^NB) \right).$$

Define

$$\beta_3 \equiv R \left( u'(c^{NB}) - u'(c^NB) \right) > 0.$$ (42)

The fact that this expression is strictly positive follows from $\tilde{\mu}^{NB} > \mu^{NB}$ and the first-order conditions (11) and (22). Combining the previous equation with (40) and (41) yields

$$\frac{dc^NB}{dq} = \frac{\beta_3}{u''(c^NB) + (1 - q) Ru''(c^NB) \beta_1 + qRu''(c^NB) \beta_2} < 0,$$

as desired.

$\blacksquare$
Proposition 6: $\Phi^{NB} \supset \Phi^*$. Moreover, there exists economies in $\Phi^{NB}$ that are not in $\Phi^D$.

Proof: For any economy in $\Phi^*$, we know that condition (7) holds. Together with inequality (39) from the proof of Proposition 5, this implies

$$\frac{\mu^{NB}}{\bar{\mu}^{NB}} < \frac{R^{-1} - q}{1 - q}. \quad (42)$$

Straightforward algebra then shows $c_E^{NB} > \tilde{c}_L^{NB}$, so that the economy is also in $\Phi^{NB}$. Moreover, the fact that the inequality in (42) is strict implies that the inclusion relationship is also strict: there exist economies for which (7) is violated by a small amount, but (42) still holds. Alternatively, it is easy to find examples of economies that belong to $\Phi^{NB}$ but not to $\Phi^*$; see Figure 2.

Figure 2 also presents examples of economies that are in $\Phi^{NB}$ but not in $\Phi^D$.

Proposition 7: For any $(R, \pi, u, v)$, there exists $\bar{q} > 0$ such that $q < \bar{q}$ and $e \in \Phi^D$ implies both $e \in \Phi^{NB}$ and $W^D > W^{NB}$.

Proof: For any $(R, \pi, u, v)$, in the limit as $q$ goes to zero, the ex ante distortion disappears and the value of $c_E$ is the same under each of the policy regimes,

$$\lim_{q \to 0} c_E^{NB} (q) = \lim_{q \to 0} c_E^{D} (q) = \lim_{q \to 0} c_E^* (q).$$

However, it follows from Proposition 2 and the resource constraint (2) that $\tilde{c}_L$ will be lower in the no-bailouts regime,

$$\lim_{q \to 0} \tilde{c}_L^{NB} (q) < \lim_{q \to 0} \tilde{c}_L^{D} (q) = \lim_{q \to 0} \tilde{c}_L^* (q).$$

Therefore, there exists some $\bar{q} > 0$ such that

$$\frac{c_E^{NB} (q)}{c_L^{NB} (q)} > \frac{c_E^{D} (q)}{c_L^{D} (q)} \quad \text{for all } q < \bar{q}.$$

If $e \in \Phi^D$ for any such value of $q$, then $c_E^{D} (q) \geq \tilde{c}_L^{D} (q)$ holds by definition. The inequality above then implies $c_E^{NB} (q) > \tilde{c}_L^{NB} (q)$ and, hence, $e \in \Phi^{NB}$ also holds, establishing the first part of the proposition.

For the second part of the proposition, note that when $e \in \Phi^D$ the two policy regimes yield the same equilibrium welfare in the limit as the probability of a crisis goes to zero,

$$\lim_{q \to 0} W^{NB} (q) = \lim_{q \to 0} W^D (q).$$
When \( q \) is close to zero, there is almost no distortion of ex ante incentives and both policy regimes deliver the first-best allocation of resources. The proposition will, therefore, be established if we can show that welfare initially falls faster under the no-bailouts regime as \( q \) rises, that is, if we can show

\[
\lim_{q \to 0} \frac{dW^{NB}(q)}{dq} < \lim_{q \to 0} \frac{dW^{D}(q)}{dq}.
\] (43)

In the limiting case as \( q \) goes to zero, this derivative under the discretionary regime can be written as

\[
\lim_{q \to 0} \frac{dW^{D}(q)}{dq} = -(1 - \pi) u(c^*_L) - v(g^*) + (1 - \pi) (\pi u(\hat{c}^*_E) + (1 - \pi) u(\hat{c}^*_L)) + v(\hat{g}^*).
\]

This expression uses the fact that the equilibrium allocation \( c^D \) converges to the efficient allocation \( c^* \) as \( q \) goes to zero, so that \( c^D_E \) can be replaced by \( c^*_L \), \( g^D \) by \( g^* \), etc. To evaluate the derivative under the no-bailouts regime, note the no-run components of the allocation \( (c_E, c_L, g) \) converge to those in \( c^* \) as \( q \) goes to zero, but the run components \( (\hat{c}_E, \hat{c}_L, \hat{g}) \) do not; this happens precisely because no bailout takes place. In this case, the limit of the derivative can be written as

\[
\lim_{q \to 0} \frac{dW^{NB}(q)}{dq} = -(1 - \pi) u(c^*_L) - v(g^*) + (1 - \pi) (\pi u(\hat{c}^{NB}_E) + (1 - \pi) u(\hat{c}^{NB}_L)) + v(\hat{g}^{NB}).
\]

Notice that the first two terms in these derivatives are the same, but the last two terms differ. Moreover, note that, by definition, \( (\hat{c}^*_E, \hat{c}^*_L, \hat{g}^*) \) maximizes continuation utility

\[
(1 - \pi) (\pi u(\hat{c}_E) + (1 - \pi) u(\hat{c}_L)) + v(\hat{g})
\]

subject to the resource constraint (2), while \( (\hat{c}^{NB}_E, \hat{c}^{NB}_L, g^{NB}) \) does not. It follows that (43) holds, which establishes the result. ■
References


