Innovation and the Dynamics of Global Warming
Preliminary Draft: Please do not cite

Ralph A. Winter
Sauder School of Business, UBC
27th January 2011

Abstract
Global warming and greenhouse gases are a dynamic system with positive feedback effects. Fossil fuels are an exhaustible resource. These two facts mean that in the simplest of models, innovation in clean energy technology makes global warming worse, not better. This paper explores the impact of innovation in the simplest model linking the theory of exhaustible resources with the dynamics of global warming.

1 Introduction
Innovation and development of clean energy sources, such as wind and solar energy, are emerging as the key strategy in the battle against global warming. The strategy rests on a seemingly obvious proposition: innovation that lowers the costs of alternative energy sources must lead to substitution away from fossil fuels, reducing carbon emissions and mitigating the problem of global warming.

The proposition, unfortunately, is false. Even dramatic innovation in clean energy, whether the result of private investment or subsidies, can set global temperatures on a permanently higher path. Under some circumstances, banning innovation in alternative energy sources could improve the environment.

At constant prices for fossil fuels, reductions in the cost of clean energy would of course encourage the use of these alternative fuels, mitigating global warming. But the price of fossil fuels is endogenous. Development of energy alternatives leads to a reduction in the price of fossil fuels, encouraging the consumption of these carbon fuels, with an effect on carbon emissions that is opposite to the expected effect. Under a wide range of conditions, this price effect is large enough
to overwhelm the direct effect of the reduction clean energy costs, with a net effect of permanently higher temperatures.

The conventional wisdom on the benefits of clean energy innovation is based on a mistaken intuition that comes from static, Pigouvian analysis. Consider a drop in the cost of a substitute to a particular input in production. If the direct effect of the cost reduction and the indirect effect (the negative price response by the input producers) are manifest at the same time (as in a static model) the direct effect dominates under standard assumptions. The price response by input producers dampens the intended effect of a subsidy on a substitute input but cannot completely offset it. Subsidies of alternatives to polluting inputs have their intended effect.

The markets for fuels and the effects on global warming are dynamic. Two features of real world dynamics are essential in understanding the impact of clean-energy innovation or subsidies. First, fossil fuels fuels such as oil, gas and coal are exhaustible resources with pricing that is inherently dynamic (Hotelling 1931). Second, the dynamic relationship between atmospheric carbon and global temperature includes positive feedback effects. As temperatures rise due to greater carbon concentration, reflective ice-fields melt and methane gas is released from melting permafrost (to take two examples), resulting in a higher rate of increase in greenhouse gases and global temperatures.

When alternatives to an exhaustible resource are subsidized, the indirect effect – the effect on consumption of the price response by existing producers – is necessarily manifest at an earlier time that the direct effect because of the exhaustibility of fossil fuel resources. The entire price path over time of fossil fuels drops, whether the resource is competitively supplied or owned by a cartel. Because of the positive feedback effect in the carbon-temperature interaction, the earlier release of carbon necessarily leads to a temperature path over time that must be higher initially and may be permanently higher. Clean-energy innovation may lead to a higher temperature forever, and even a small innovation may lead to a discretely higher steady-state temperature.

This paper explores the dynamics of innovation and global warming in the simplest model incorporating the exhaustible nature of carbon-based fuels and the positive feedback effects of the carbon-temperature relationship. Innovation in clean energy has a perverse impact on global warming that can be broken down into two components. The ex post effect of a new innovation that reduces the cost of a technology (or simply the subsidy of an existing technology) drives down the price path of the exhaustible resource, leading to earlier and more intensive carbon release and greater global warming because of the positive feedback effects as explained above. The ex ante ef-
fect of even the *prospect* of a successful innovation enhanced subsidies leads to a lower (and steeper) price path for the exhaustible carbon-based resource and earlier release of the carbon even in the event that the innovation does not materialize. With homogenous extraction costs across reserves of fossil fuels, the net impact of innovation in clean energy is to exacerbate global warming – unless the innovation is so dramatic (think: cold fusion) that the cost of the new technology is less that the extraction cost of the fossil fuel.

Allowing heterogeneous extraction costs in the theory eliminates the categorical prediction, and introduces instead a tradeoff. Successful innovations still lead to more intensive release of carbon earlier in the energy life-cycle. The detrimental impact of this early release is balanced against the gains from displacement of more of the highest cost fossil fuel resource pools with clean energy. The net impact of the ex post effect of an innovation is an empirical matter. The harmful ex ante effect of innovation remains: the prospect of clean energy innovation necessarily harms the environment. An unrequited love of energy technology – a high subjective probability of innovation combined with the bad luck of a poor realization – is guaranteed to worsen global warming, whatever the distribution of extraction costs.

In exploring the effects of innovation, I adopt a model with conventional assumptions of a frictionless exhaustible resource market and profit-maximizing firms. But it is important to recognize that the basic paradox depends only on the rationality of the owners of fossil fuel deposits. Suppose that tomorrow an innovation suddenly made clean energy available as a perfect substitute for all fossil fuels at a cost-equivalent of 50 dollars per barrel of oil. Conventional fossil fuels have an extraction cost less than 30 dollars per barrel (in enough quantity to meet projected demand to 2050). Owners of fossil fuels whatever the market conditions would rationally sell earlier at 49 dollars per barrel or less, rather than compete with the clean energy source at a price of 50 dollars. Fossil fuels would drop in price and continue to be the sole or dominant fuel until the deposits are exhausted. Because demand curve for energy is downward sloping, the effect of the innovation is extraction and release of the same stock of carbon at an earlier time path and with greater intensity. Some suppliers of carbon fuels may in reality be cash-constrained governments (Nigeria, Venezuela, Dubai) that would respond to a drop in price with greater output; this backwards-bending supply

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1Where GHG release is positively correlated with extraction costs across pools (e.g. extraction of oil from tar sands is both costly and high in the rate of carbon release) the benefits of clean energy innovation are higher; when the feedback effect in the GHG-carbon temperature relationship is strong, the detrimental effects are higher.

effect will only magnify the price drop and the early release of carbon. Given the positive feedback
dynamics of global warming, the earlier release of the carbon sets the temperature off on a higher path. The innovation will have beneficial effects in displacing non-conventional deposits with very high extraction costs – although even an innovation so dramatic as to allow a perfect substitute at a cost of 50 dollars per barrel would unlikely displace tar sands oil, for example – and in reducing the incentives for exploration. These effects are manifest in the future, however, and if the positive feedback effects in the carbon-temperature dynamic system are strong enough, the beneficial effects will not be enough to offset the early release of carbon.3

This paper has implications for global warming policy. Notwithstanding the conclusions outlined to this point, the main implication is not that innovation in clean energy may be harmful. This paper is not calling for a ban on clean-energy innovation. The important implication is about the other policy instrument in the battle against global warming: carbon pricing. Carbon pricing policy, whether through a carbon tax system or a cap-and-trade system, is vital – but not in the sense of an overall, time-invariant optimal carbon price that internalizes externalities. Carbon pricing is essential to counteract the negative features of innovation in clean energy. It is immediate from the theory that a carbon price ex ante, and an even higher carbon price ex post to innovation, can eliminate the negative and perverse features of innovation, since these features operate through the downward reaction of fossil fuel prices. A carbon pricing policy should be reactive to innovation successes. And the way in which carbon prices should respond might not be intuitive. When an innovation succeeds in giving clean energy producers a new advantage over conventional energy sources, a carbon price policy that offsets the negative impact of innovation is one that magnifies this advantage, by raising the tax on fossil fuel use. The role of reactive carbon prices is not to provide increased incentives for investment in innovation, which is exogenous in this model, but is simply to offset the perverse effect of realized innovation gains.

Carbon pricing is thus an important complementary instrument to clean energy innovation, being necessary even to ensure that the net impact of innovation is positive. This is once again the opposite of conventional wisdom. Much of the current thinking is that with carbon taxes seemingly impossible to implement given U.S. politics, innovation subsidies are becoming even more important. Discussion

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3In 2008, 43% of CO₂ emissions were produced from coal, 37% from oil and 20% from gas, according to the International Energy Agency (IEA) (2010), p. 8. Generation of electricity and heat was by far the largest producer of CO₂ emissions, responsible for 41% of world CO₂ emissions in 2008 (IEA 2010, p.8).
has shifted away from carbon pricing towards an increased emphasis on clean energy subsidies.\textsuperscript{4} In the popular press, clean energy subsidies have been touted as a superior instrument to carbon taxes and popular support is in favour of subsidies.\textsuperscript{5} All of this discussion presupposes that carbon pricing and clean energy subsidies are substitutes in the battle against global warming. This is a natural assumption, given that these are two instruments available to solve the same problem, but the assumption is wrong.

The economic theory in this paper draws on the literature on pricing in an exhaustible resource market subject to innovation in a backstop technology (Gallini, Lewis and Ware 1983, Dasgupta and Stiglitz 1981) as well as the literature on exhaustible resource pricing with heterogeneous extraction costs or reserve-dependent costs (see Devarajan and Fisher 1981). The theory here integrates economic model of resource extraction given innovation in a backstop technology, with the simplest of climate models allowing a positive feedback effect. The climate model describes the flows between two endogenous stocks of carbon – atmospheric carbon and carbon on the earth’s surface including the oceans. This type of model would be known in the climate dynamics literature (if this literature ever considered a theory so simple) as a "two-reservoir model". Carbon cycle models typically incorporate carbon and heat transfers between adjacent reservoirs in models with three reservoirs. The reservoir left out of the theory here is deep-ocean carbon.

The most prominent model integrating economics and climate dynamics is the Dynamic Integrated Model of Climate and the Economy (the DICE model) (Nordhaus, 2008). The latest version of this model does assume a backstop clean-energy technology, but assumes (1) a current Hotelling rent of $0.07 per ton of carbon for fossil fuels (Nordhaus, 2007, p.31), which implies a Hotelling rent of only $0.01 per barrel of crude oil;\textsuperscript{6} and (2) an extraction cost of zero (Nordhaus 2008, p.43). The effects analyzed here, in contrast, rely entirely on a large Hotelling rent. The DICE assumptions (1) and (2) imply a current price of crude oil that is close to zero, instead of the observed price of about 100 dollars per barrel.\textsuperscript{7} The point is that the interaction of innovation and carbon pricing policy is

\textsuperscript{4}Consider, for example, the recent joint call by the Brookings Institute and the American Enterprise Institute for an increase in clean energy investment from 4 billion to 25 billion annually. (Brookings Institution and A.E.I. 2010)

\textsuperscript{5}See, for example, the discussion in David Leonhardt, “A Climate Proposal Beyond Cap and Trade”, New York Times, October 12, 2010.

\textsuperscript{6}A barrel of oil weighs about 135 kg, of which 83% to 87% is carbon, meaning that a barrel has about \(0.85 \times 135 \times 0.001 = 0.115\) tonnes of carbon, which is \(1.1 \times 0.115 = 0.126\) tons. A rent of $0.07 per ton of carbon is \(0.07 \times 0.126 = \) about 1 cent per barrel.

\textsuperscript{7}A similar point could be made about coal.
entirely outside the focus of the DICE model. The Stern review of global warming policy (Stern, 2006) notes that the price reaction of the fossil fuel market may dampen the effects of policy of clean energy subsidies, but does not recognize the possibility of a negative net impact of innovation. Stern is solidly of the conventional view that the net effect of innovation in clean energy must be positive. Stern surveys the important positive feedback mechanisms in carbon models, but does not connect the feedback mechanisms to the endogenous price reaction of fossil fuels markets, which is the analytical focus in this paper. 8

While I have referred to optimal policies towards global warming, I focus in this paper entirely on the underlying positive economics rather than setting up an optimal planning problem. I present the theory in the next section of the paper. In the concluding section I discuss policy implications and argue for additional research on a set of open questions in the interaction of innovation and global warming, including the impact of multiple countries on the demand side of the energy market.

2 The Model

2.1 Background on the Assumptions

The first key assumption in our model is that fossil fuels, the source of new carbon introduced into the biosphere, are an exhaustible resource. This is obvious. Our second assumption is the global warming is a process with positive feedback. At least five feedback mechanisms have been identified by climate scientists. As ice sheets melt, the cooling effect that they produce by reflecting radiation away from the Earth is reduced. This means that higher temperatures lead to an increase in the rate of change of temperature (a positive-feedback process known as the ice-albedo effect). A second mechanism is that global warming could cause the death of vegetation in regions such as the Amazonian rainforests through reduced rainfall, leading to the release of CO2 to the atmosphere and to reductions in the absorption of CO2 by plants. Peter Cox et al. (2000) uncovered a third positive-feedback mechanism: that global warming can result in increased respiration from bacteria in the soil, releasing additional CO2. 9

8Sinn (2008) notes that that if suppliers of fossil fuels anticipate a future increase in carbon taxes in demanding countries, the price of fuel will drop and the release of carbon will be accelerated. This is an example of a short run perverse reaction to the announcement of a future change in government tax policy, a phenomenon common across announced changes in almost any kind of tax policies. The announcement of a future tax on any activity with long-run or medium-run private benefits always spurs a short run increase in the activity. The paradox analyzed in this paper is quite different.

9provide cite
The fourth positive-feedback mechanism is the release of GHG’s (mainly methane) from the tundra in the arctic, mainly Eastern Siberia. A well-known study in 2007, led by University of Alaska’s International Arctic Research Centre and the Russian Academy of Sciences, demonstrated a strong potential for this mechanism. The fifth positive feedback is from the evaporation of water and the accumulation of water vapour in the upper atmosphere. Water vapour is accumulated in greater amounts in a warmer atmosphere, as we know from basic physics. This leads to a higher rate of temperature increase because water vapour itself is a powerful greenhouse gas. The power of positive feedback mechanisms is high. Stern (2006: p.3) summarizes the scientific evidence on these effects as indicating that they would likely amplify warming by 1-2 degrees Celsius by 2100, against an estimated increase of at least 3 degrees in GHG emissions remained at current levels.

I incorporate in the model the property that the strength of the positive feedback mechanisms is small at low global temperatures but, over some region, is increasing in temperature.

2.2 Assumptions

Economic Assumptions: I consider a market for energy that can be supplied by fossil fuels, an exhaustible resource, or by an existing backstop technology at known cost. Let $s_t$ be the flow of resource extraction at time $t$, $q(p)$ be the (stationary) demand for energy, $p_t$ be the price of energy and $b_0$ be the cost per unit of producing energy with the existing backstop technology. The stock of resource extracted up to time $t$ is $x_t = \int_0^t s_t \, dt$. The resource market and the backstop technology are each supplied by a competitive market, in an economy with a constant rate of interest, $r$. I allow for heterogeneity in the costs of extraction across resource pools, capturing this in an increasing function $c(x)$ that gives the marginal cost of extraction when $x$ has been extracted already; alternatively, in a distribution $Q(c)$ that gives the amount of fossil fuel of marginal cost less than or

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10 As stated in a the National Science Foundation press release at the time of this study, “Release of even a fraction of the methane stored in the shelf could trigger abrupt climate warming.” Since the study in 2007, some estimates of the amount of trapped methane have more than doubled. The authors of a study published in 2009 in Global Biogeochemical Cycles (GB2023, doi:10.1029/2008GB003327) report (in Science Daily July 6, 2009) “We now estimate the deposits contain over 1.5 trillion tons of frozen carbon, about twice as much carbon as contained in the atmosphere”, said Dr. Charles Tarnocai, Agriculture and Agri-Food Canada, Ottawa, and lead author. Dr Canadell. All evidence to date shows that carbon in permafrost is likely to play a significant role in the 21st century climate given the large carbon deposits, the readiness of its organic matter to release greenhouse gases when thawed, and the fact that high latitudes will experience the largest increase in air temperature of all regions.”

11 There are other positive feedback mechanisms, including mechanisms on the demand side of the energy market by which higher temperatures lead to greater derived demand for energy through higher demand for air-conditioning and vehicles.
equal to $c$.\textsuperscript{12}

Innovation in new energy technology is represented by the probability of discovery of a new backstop technology with cost $b < b_0$. There is only one possible new technology and once it is discovered, no future innovation is possible. The probability of discovery of the new technology in any small interval of time, $dt$, is $\rho dt$. The probability $\rho$ is exogenous to the model (and can be interpreted as an instrument of government policy). We develop and compare the equilibrium path of variables, $s_t$, under the scenarios that innovation is not possible (the cost of the backstop remaining at $b_0$); that innovation is possible but not realized prior to exhaustion of the fossil fuel; and that innovation occurs at some date $t$.\textsuperscript{13}

**Physics Assumptions:** We are concerned with the interaction of the fossil fuels market with global warming dynamics and in particular with the possibility of positive feedback effects, as discussed above. I adopt the simplest possible carbon cycle model allowing positive feedback. I represent a state of the environment with two state variables: $g_1^t$, the total volume of GHG’s in the atmosphere and $g_2^t$, the total carbon and other GHG-potential elements (which we shall refer to simply as “carbon”) in the earth’s surface, including the oceans. Both of these variables we should think of as measured in the same units, e.g., CO$_2$-equivalents in terms of their GHG impact. Temperature, $h$, is a function $h = H(g^1)$ of atmospheric GHG’s. The environmental state variables are linked by the two relationships: greenhouse gases $g^1$ are recaptured (transformed to $g^2$) at a constant rate $c$. Recapture of carbon to the earth’s surface is through absorption by plants and incomplete re-release into the atmosphere when plants die and decay. (Antweiler (2010) adopts an estimate of the recapture rate at about 2 percent per year; Uzawa (2003) assumes a higher but, Antweiler argues, unreasonable rate.) GHG’s are released from the earth’s surface at a rate that is an increasing function $m(h_t)$ of the current temperature. This is the positive feedback effect.

The market equilibrium is linked to the global warming dynamics by an assumption that extraction $s_t$ results in a flow of GHG’s to the atmosphere at the rate $as_t$ for some constant $a$. There are, in sum, there are six state variables in the model: $x, g^1, g^2, h; a$ binary variable $I \in \{0, 1\}$ indicating

\textsuperscript{12}I interpret $c(x)$ as incorporating the cost of discovery of new resource pools. I thus set aside the uncertainty in the outcome of exploration.

\textsuperscript{13}I model the connections between markets and carbon emissions; and between carbon emissions and temperature; but I do not need to refer to the relationship between temperature and damages. Incorporating damages would only magnify the costs of an early release of carbon, because of discounting. I also set aside evolution of technology in other dimensions, such as carbon emission mitigation. Again, allowing for an exogenous trend in improvements in this technology would add to the cost of early release of carbon.
whether or not the new innovation has been discovered; and a variable $\tau$ indicating the time passed since the date of innovation (conditional upon $I = 1$).

The six-state-variable model is tractable, and in fact will be reduced to a single differential equation describing the evolution of the environment in the post-extraction phase. Two natural separabilities allow this tractibility. The first is a separability between the market variables, $(x_t, I_t)$ and the carbon/climate variables $(g^1_t, g^2_t, h_t)$. The market equilibrium is not affected by the evolution of $(g^1_t, g^2_t, h_t)$, because of an assumption that global warming is a pure externality (and because we are ignoring demand-side feedback mechanisms, such as the greater demand for air conditioning as temperature rises). We can therefore develop the market equilibrium completely without reference to the environment. Then we can develop the global warming dynamic system taking the evolution of $x$ as exogenous since $s_t$ is unaffected by the evolution of the other state variables. The second natural separability is over time. At some (random) date $T$, when extraction is complete, the entire system can be described in terms of only two state variables $g^1$ and $g^2$. The fossil fuel market simply disappears, taking with it three state variables, $x_t, I_t$ and $\tau$. At the random date $T$, the market leaves the environment endowed with a total amount of carbon, $g^1 + g^2$, and a particular mix of this carbon between the atmosphere and the earth’s surface, $g^1/g^2$. Both dimensions of the endowment matter and in some cases innovation in clean energy will reduce the total amount of terminal carbon in the environment but – because of the market equilibrium reaction to innovation in the form of earlier extraction, combined with the positive feedback effect – lead to a greater fraction of the system carbon in the atmosphere. The dynamic system may be tipped into an eventual resting point where this fraction, and the earth’s temperature, are permanently higher.

2.3 Market Equilibrium:

2.3.1 Equilibrium with no innovation

The case of no innovation serves as a benchmark. Hotelling (1931) offered a now-famous arbitrage condition for competitive prices in a market for exhaustible resources with zero extraction costs and a constant interest rate, $r : \hat{p}_t / p_t = r$. In a market where extraction cost is positive at constant at a cost $c$, the arbitrage condition becomes $\frac{\partial(p_t-c)}{\partial t} / (p_t - c) = r$, i.e. the rent rises at the rate of interest. Only under this condition will a competitive supply be positive at all dates until exhaustion of the resource.

In a setting of heterogeneous extraction costs, the competitive equilibrium involves the extraction
of resources pools in order of extraction costs, from lowest to highest cost pools. Letting $c'(\tilde{x})$ represent \( \frac{\partial c(x)}{\partial x} \) evaluated at \( \tilde{x} \), the arbitrage condition becomes the following (with the right hand side expressed in several ways):

\[
\frac{\partial (p_t - c(x_t))}{\partial t} = r - \frac{\partial c(x_t)}{\partial t} = r - \frac{c'(x_t) \cdot \partial x_t}{p_t - c(x_t)} = r - \frac{c'(x_t) \cdot q(p_t)}{p_t - c(x_t)}
\]

The opportunity cost of delaying extraction by a small interval, \( dt \), is no longer simply the present value of the equilibrium rent at \( t + dt \) when costs rise with depletion; the opportunity cost is reduced by the savings in extraction costs at time \( t + dt \) that the delay would bring. The price must reflect this reduced opportunity cost, leading to the second term on the first right-hand side of equation (1).  

Extraction of fossil fuels will stop at the date \( T \) where the cost of extraction equals the backstop technology cost. In sum, the equilibrium with no innovation is \([p_t, s_t; T]\) that solves (1) over \([0, T]\) with boundary condition \( p_T = c(x_T) = b_0; \) \( x_t = \int_0^t s_t d\tau \) and \( s_t = q(p_t) \); and \( s_t = 0 \) for \( t > T \). To think about the equilibrium conceptually (or to solve this set of conditions numerically), one would use a backwards solution. Note that stock of the resource satisfying \( c(x) = b_0 \), i.e. the stock \( x = c^{-1}(b_0) \), is the stock of the resource that will be extracted. Set the price \( p_T \) at \( b_0 \) and move backwards in time, setting price \( p_t \) according to the differential equation (1), and the extraction rate according to \( s_t = q(p_t) \), until the cumulative amount extracted is \( c^{-1}(b_0) \). This gives the "shape" of the equilibrium price path, which can be moved until the starting point is the date \( t = 0 \).

### 2.3.2 Equilibrium with innovation

When the probability of innovation, \( \rho \), is positive, the equilibrium price path \( p_t \) is stochastic. The sample paths that we must characterize to set out the equilibrium conditions are the pre-innovation paths \( p^n_t \) and \( s^n_t \) for price, quantity and cumulative extraction, with extraction terminated at date \( T^n \) if there is no discovery; and for each possible discovery date \( \tau \in [0, T] \) of the new backstop technology, the paths \( \hat{p}_t(\tau) \) and \( \hat{s}_t(\tau) \) between \( \tau \) and the exhaustion date \( \hat{T}(\tau) \) contingent upon discovery at date \( \tau \). We denote \( x^n(t) \) and \( \hat{x}_t(\tau) \), for \( t \geq \tau \), as the cumulative resource extracted along the pre-innovation and post-innovation paths.

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Note that in the Hotelling model or in the model with a constant extraction cost, arbitrage equates the value of extracting a unit of resource at the current date \( t \) with the present value of extracting the unit at any other date. With resource-dependent costs, these values are equated only between date \( t \) and the next instant, date \( t + d_t \), i.e. is the first-order condition for the optimal timing of extraction for the owner of a unit of the resource of particular extraction cost.
The date of discovery is the only random variable in this stochastic system of equilibrium paths. The equilibrium paths ex post to discovery, \( \hat{p}_i(\tau) \) and \( \hat{s}_i(\tau) \), are identical to the model above without innovation, except of course that the terminal price is lower. That is, the ex post market price path satisfies conditions identical to those set out in the case of no innovation except that the boundary condition becomes \( \hat{p}_{\tau}(\tau) = c(x_{\tau}(\tau)) = b \), where \( b < b_0 \).

The price path \( p^n_t \) is the equilibrium sample path until either discovery or the cumulative resource extracted reaches the level \( x = c^{-1}(b_0) \), whichever comes first. To derive the differential equation characterizing \( p^n_t \), note that at time \( t \), competitive equilibrium requires that the future value at time \( t + dt \), for a vanishingly small time period \( dt > 0 \), of the rent from extracting at time \( t \) must equal the opportunity cost of waiting until \( t + dt \). Under risk-neutrality, this condition is

\[
(1 + r dt)[p^n_t - c(x^n_t)] = \rho dt[\hat{p}_i(t) - c(\hat{x}_i(t))] + (1 - \rho dt)[p^n_{t+dt} - c(x^n_{t+dt})] - c'(x^n_t) \cdot \partial x^n_t / \partial t \cdot dt
\]

Taking the limit of this expression as \( dt \to 0 \) yields the condition (see Appendix 1):

\[
\frac{\partial(p^n_t - c(x_t))}{\partial t} = r + \rho \left[ 1 - \frac{\hat{p}_i(t) - c(x_t)}{p^n_t - c(x_t)} \right] - \frac{\partial c(x_t)}{\partial t} \cdot \frac{c(x_t)}{c(x_t)}
\]

The rate of change of rents along the pre-innovation path must equal the interest rate plus the expected rate of capital loss on deposits, minus the rate of cost savings from delay.

I have characterized the equilibrium paths in three cases: where innovation is impossible; where innovation is possible but does not occur before the cost of extraction meets the cost of the existing backstop technology; and where innovation occurs at a realized date \( \tau \). The comparison of these prices paths is straightforward, given our characterizations. Because the system is Markov, the equilibrium values of the endogenous variables can be expressed as functions of the state variables: we let \( \hat{p}(x) \), \( p^n(x) \), \( \hat{p}(x; \tau) \) be the prices as functions of \( x \) conditional upon (respectively) no innovation possible; innovation possible but not realized, and innovation realized at date \( \tau \) years in the past.

From (1) and (2) and \( b < b_0 \), it follows directly that \( \hat{p}(x; \tau) < p^n(x) < p(x) \). It then follows from \( q'(p) < 0 \) that, at the same value of the state variable \( x \), the rates of change of \( x \) (the extraction rates) can be ordered under the three equilibrium paths as \( \dot{x} < \dot{x}^n < \dot{x} \). Now, at each of the

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From (1), the discovery of the new technology at a time \( t^* \in (t, t + dt) \) has two effects on the value of the stock of resource at time \( t + dt \): (a) a discontinuous capital loss, or drop in the market value of the resource at \( t^* \), which grows at a rate \( r \) between \( t^* \) and \( t + dt \); and (b) the impact of the difference in \( \partial x_t / \partial t \) between \( t^* \) and \( t + dt \). The second effect is given approximately by \( c'(x_{t^*}) \int_{t^*}^{t+dt} [\partial x_s / \partial s - \partial x_{t^*} / \partial s] ds \) which approaches zero as a first-order effect in \( dt \), as \( dt \to 0 \), whereas the first effect remains a discontinuous drop (both effects being conditional upon a successful innovation in \((t, t + dt)) \). We can therefore ignore the second effect, i.e. the impact of changes in \( \partial x_t / \partial t \) upon successful innovation, in deriving the arbitrage condition.

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respective extraction-termination dates, $T$, $T^*$ and $\tilde{T}(\tau)$, the state variable $x$ (in the respective equilibrium paths) takes the values $c^{-1}(b_0)$, $c^{-1}(b_0)$ and $c^{-1}(b)$, when $c$ is strictly increasing, with $c^{-1}(b) < c^{-1}(b_0)$ since $b < b_0$. Moving backwards in time from the termination dates, the ordering of the rates of change then allows us to order the termination dates. In sum,

**Proposition 1** When the current benchmark technology has a cost $b_0$ and innovation of a lower cost is governed by a stationary probability $p_{dt}$ of discovering a technology of lower cost $b$, the equilibrium extraction path has two phases: prior to discovery, the cumulative extraction path lies above the equilibrium path for the case where innovation is impossible. If discovery is possible but never realized, the same amount resource, $c^{-1}(b_0)$, is extracted as if innovation were impossible, but at earlier date and at a more intensive rate. If discovery is made, extraction is at a higher rate and terminates even earlier, but the total resource extracted is less: $c^{-1}(b) < c^{-1}(b_0)$.

Figure 1 illustrates typical paths for the state variable $x$, and Figure 2 represents price paths for the same realization of $\tau$, the one random variable in this stochastic system. Note that the price paths for the innovation-impossible case and the innovation-possible-but-not-realized case must cross.

When we take this pattern of extraction to the next section of the paper on global warming dynamics, the possibility of innovation in clean energy technology carries a cost and a benefit in terms of its impact on global warming. If innovation is possible and if it actually occurs, then the carbon associated with an amount of the resource $c^{-1}(b_0) - c^{-1}(b)$ is left in the ground rather than introduced into the environment. That is a benefit. But the carbon that is released is released earlier; temperatures always rise immediately upon discovery of the innovation and because of the positive-feedback nature of global warming may be on a permanently higher path even with the reduction in total carbon released into the atmosphere.

In the case where $c(\dot{x}_T) < b$, i.e. where the marginal extraction cost is less than even the cost of energy under the new technology (for example, where marginal extraction cost is constant at $c < b$), only the negative side of the tradeoff is present. The impact of the innovation is then to leave the same amount of fossil fuel extracted, and the same amount of carbon released, but at an earlier date and with greater intensity:

**Proposition 2** If $c(\dot{x}_T) < b$ then discovery leads to an earlier extraction of the same amount of the resource.
2.4 Global-warming dynamics

2.4.1 The carbon cycle, incorporating $\dot{x}_t$ as exogenous:

Our assumptions on the carbon cycle, set out above, are summarized in Figure 3. The extraction of fossil fuels provides an exogenous (to these dynamics) injection of carbon into the atmosphere at a rate $a \dot{x}_t$. The atmospheric carbon is absorbed by the earth’s surface at a constant rate $e$, but the rate of release of carbon from the surface to the atmosphere depends on the current temperature, through the function $m(h_t)$.

The system characterizing the evolution of the four continuous state variables $(x_t, g^1_t, g^2_t, h_t)$ is given by the following:

\begin{align*}
\dot{g}^1_t &= a \dot{x}_t + m(h_t)(g^2_t) - e g^1_t \\
\dot{h}_t &= H(g^1_t) \\
\dot{g}^2_t &= e g^1_t - m(h_t)(g^2_t)
\end{align*}

Defining $F(g^1_t) \equiv m(H(g^1_t))$ allows us to reduce this system to the following couplet of dynamic equations:

\begin{align*}
\dot{g}^1_t &= a \dot{x}_t + F(g^1_t)g^2_t - e g^1_t \\
\dot{g}^2_t &= e g^1_t - F(g^1_t)g^2_t
\end{align*}

The positive feedback is captured in the function $F(\cdot)$. The strength of this function, i.e. $F'$, is key in determining whether earlier release of a given stock of carbon leads to permanently higher temperatures. “Earlier” and “higher” refer to partial orders over continuous extraction paths and temperature paths. For a given amount of fossil fuel $x$, and two different extraction paths of $x$, $s^1$ and $s^2$, with $\int_0^\infty s^i dt = x$, $i = 1, 2$, we say that $s^2$ is an earlier extraction path than $s^1$ if

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\[\text{Footnote: Obviously, this set of assumptions abstracts completely from a host of economy-climate interactions and the resulting model is by design illustrative rather than realistic. Nordhaus’ study, which opens with an statement of the value of simplicity and which is criticized by some climate scientists as being too simplistic, has 16 dynamic equations and 24 dynamic variables; our model will reduce climate dynamics to a single differential equation. More state variables would yield a more realistic description or computation. These would include a deep ocean carbon reservoir; the sensitivity of surface carbon to release (dependent on the history of atmospheric temperature, not just its current value); regionally variable temperature changes (the areas near the poles are forecast to experience a much higher rate of temperature increase), and so on. Energy exchanges across adjacent reservoirs, not just carbon exchanges, are incorporated in more complex climate models. Our assumption that the release of carbon into the atmosphere is proportional to surface carbon, rather than a more realistic general increasing function; the release of water vapour, for example, is not proportional to the existing stock of water on the surface. And a more realistic theory would treat separately two types of feedback effects: higher temperature causing release of additional GHG from the earth’s surface; and the reduction in the rate at which solar energy is reflected out of the atmosphere by ice-fields. Our formulation captures the first of these, but could be modified to include the second.}\]
higher temperature path provided the following relationship holds for all \( t \), with the inequality being strict for some set of \( t \). (This is analogous to first-order stochastic dominance.) And define one temperature path \( h^2 \) to be higher than a path \( h^1 \) if \( h^2_t \geq h^1_t \) with the inequality strict on a subset of positive measure.

It is clear that earlier emissions of a given stock of carbon does not always lead to a permanently higher temperature path. Suppose, for example, that the positive feedback mechanism is very weak, relative to \( e \), and compare the release of 99 percent of a stock of carbon in year 1 with the release of 99 percent of the stock in year 100. If, however, the feedback mechanism is strong enough, and the carbon on the earth’s surface (that is vulnerable to escape via the feedback mechanism) represented by \( g^2 \) is high enough, relative to the reabsorption rate \( e \), then an earlier release of carbon does lead to a higher temperature path:

**Lemma 3** In the dynamic system given by (3), let \( s^2 \) represent an earlier extraction path that \( s^1 \) (both paths being continuous). For common initial conditions \((g^1_0, g^2_0)\), the path \( s^2 \) leads to a higher temperature path provided the following relationship holds for \( s^1, s^2, \) and all paths in between (according to the “earlier” partial ordering) \( s^1 \) and \( s^2 \):

\[
\frac{\partial}{\partial g} F(g^1)g^2_t - eg^1_t = F'(g^1_t)g^2_t - e > 0
\]  

**Proof.** Consider a function \( \Delta = 1 \) on \((t_1, t_1+dt)\) and \(-1\) on \((t_2, t_2+dt)\), for \( t_1 < t_2 \) and vanishingly small \( dt \). Since the paths \( s^1 \) and \( s^2 \) are continuous and bounded, the paths are integrable. It therefore suffices to show that a move from \( s_1 \) to \( s_1 + \Delta \) results in a higher temperature path under (5) since the difference between \( s_1 \) and \( s_2 \) can be constructed from an infinite weighted sum of such differences. Let \( \tilde{g}^1 \) represent the path of atmospheric gases under the path \( s_1 + \Delta \). For any \( t \in (t_1+dt, t_2] \) \( \tilde{g}^1_t \approx g^1_t + 1 + \int_{t_1+dt}^t as^1_t + F(\tilde{g}^1_t)\tilde{g}^2_t - e\tilde{g}^1_t dt > g^1_t + 1 + \int_{t_1+dt}^t as^1_t + F(g^1_t)g^2_t - eg^1_t dt > g^1_t + \int_{t_1+dt}^t as^1_t + F(g^1_t)g^2_t - eg^1_t dt = g^1_t \)\(^{17}\). This inequality includes \( t = t_2 \). It then follows that for \( t > t_2 \) \( \tilde{g}^1_t \approx g^1_t + \int_{t_2+dt}^t as^1_t + F(\tilde{g}^1_t)\tilde{g}^2_t - e\tilde{g}^1_t dt > g^1_t + \int_{t_2+dt}^t as^1_t + F(g^1_t)g^2_t - eg^1_t dt > g^1_t + \int_{t_1+dt}^t as^1_t + F(g^1_t)g^2_t - eg^1_t dt = g^1_t \)

It follows immediately that under the “strong feedback condition” (5), if a sufficiently small proportion of total fuel deposits have extraction costs above \( b \), then the impact of a successful

\(^{17}\)The first inequality uses the fact that for any differential equation \( \ddot{y} = m(y) + a \) with boundary condition \( y_0 = \ddot{y} \) and \( m’ > 0, dy/d\dot{y} > 1 \).
innovation is a permanently higher temperature path.

**Proposition 4** Under (5):

(a) if \[ \left( Q(b_0) - Q(b) \right)/Q(b_0) \] is sufficiently small, where \( Q(c) \) is the distribution of deposits by extraction cost, then an innovation at any time leads to a higher temperature path.

(b) For an arbitrary (smooth) distribution \( Q(c) \), there is a time \( t^* < T^* \) such that innovation after time \( t^* \) leads to a higher temperature path compared to the case of no-innovation-possible, but a lower temperature path than in the case of innovation-possible-but-not-successful.

(c) if the innovation is unsuccessful as of time \( T^* \), the temperature path is higher than in the case of no-innovation-possible.

2.4.2 Warming dynamics in the post-extraction phase:

Our formulation of the economic model as involving a backstop technology in all cases allows us to determine in a simple way the dynamics of global warming in the period from \( T \) onwards, when extraction has stopped. With no new carbon being ejected into the system in the post-extraction period, \( g_1^t + g_2^t \) is constant, equal to \( g_1^T + g_2^T \), which we denote as \( C \). It follows that for \( t > T \), \( g_1^t = -g_2^t \), from setting \( x_t = 0 \) in (4). Substituting \( g_2^t = C - g_1^t \) into the first equation of (4) allows us to express the post-extraction dynamics of the atmospheric GHG \( g_1^t \) (and therefore the dynamics of global temperature, via \( h = H(g_1^t) \)) in a single autonomous differential equation:

\[
\dot{g}_1^t = F(g_1^t)(C - g_1^t) - cg_1^t
\]  

for \( t \geq T \) with boundary condition given by the exogenous (to this system) value \( g_1^T \).

The equation (6) offers a particularly clear avenue for examining the effects on the path towards a steady state temperature. The market, under various policies or innovation events, leaves the post-extraction period with different values of \( C \) and \( g_1^T \); and the impact on the path of the policies can be examined through these two variables.

The function \( F \) is of course critical in this. The scientific basis for the feedback effect, reviewed earlier, justifies an assumption that the function is convex and nearly constant at low temperature values. The release of methane gas from the frozen tundra, for example, is will occur at a significant rate once temperature reaches a level somewhat higher than in the current climate.\(^{18}\) Given the

\(^{18}\)NTD: discuss
simplicity of our model assumptions, however, an unbounded rate of carbon release would lead to a steady state with no carbon on the earth’s surface, a physical impossibility in the real environment. Accordingly, we consider the case where the function $F$ is bounded with the rate $e$ falling between the lower and upper bound.

This leaves us with a sigmoid shape for $F$ as being most natural, as in Figure 4. $F$ leaves $g_1^t$ bounded between two values, $v$ and $w$. An example of such a function is the logistic function

$$F(x) = v + \left( \frac{1}{1 + e^{-kx}} \right) (w - v)$$

To understand the resulting dynamics, with $v < e < w$, note that for low values of $g_1^t$, $F(g_1^t) \approx v$. The differential equation (6) becomes approximately $\dot{g}_1^t = v(C - g_1^t) - eg_1^t$, which has a unique, stable steady state at $g_1^t = 0 \implies g_1^t = vC/(v + e)$. Similarly at high values of $g_1^t$, $F$ is approximately $w$ and there is a second steady state at approximately $g_1^t = wC/(w + e)$. This second steady state is at a higher level of $g_1^t$, and therefore a higher temperature, since $wC/(w + e) > vC/(v + e)$.

Figure 5 depicts equation (6) with $F$ taking on the logistic functional form, for particular values of the parameter $k$ (which measures the “steepness” of the logistic function at middle values) and values of $e$ and $C$. The intersections with the coordinate axis are the possible steady states of the system. As always, there is (generically) an odd number of steady states; for the parameter values in this figure, there are three steady states. The middle steady state is unstable. The arrows on the $g_1^t$ axis represent the phase space or phase line, which is only one dimension for this simple system.

We can summarize the steady state dynamics by depicting the phase diagram in the space $(g_1^t, C)$, where $C = g_1^t + g_2^t$, as in Figure 6. This is a degenerate phase diagram in the sense that $C$ is constant in the post-extraction period, but it is useful because a single point in this space represents the carbon endowment left to the environment by the market for fossil fuels under various realizations of the date of discovery of the new backstop technology.

An implication is that the different green energy policies, by endowing the environment with even small different starting points in the post-extraction period, may lead to sharply different global warming paths. Most significantly, in comparing two possible histories of innovation in the fossil fuels market, if one history leaves the environment with lower total carbon it may nonetheless lead to a higher steady state temperature if it leaves a higher proportion of the carbon in the

\[\text{For example, if } C = 100, v = 0.01 \text{ and } e = 0.02, \text{ this yields a steady state value of } 33.\]

\[\text{An approximate value for another locally stable steady state for } w = 0.06 \text{ and } e = 0.02, \text{ would be } 75.\]
atmosphere. An innovation that leads to some substitution from (high extraction cost) carbon fuels to clean energy, and therefore less total carbon injected into the carbon cycle, may nonetheless lead to a higher steady-state temperature because a higher proportion of this endowment, given the positive-feedback during the extraction period, will be in atmospheric carbon.

Figure 7 [in progress; via simulations] places on the same diagram the set of endowments of atmospheric carbon and total carbon that are left by the fossil fuels market, with particular values for demand, costs, and probability of innovation, indexed by the realized date of discovery. This diagram illustrates the basic paradox: the early discovery of a clean backstop technology may lead to a higher steady-state temperature – even higher than the steady state temperature reached when no innovation, and no substitution away from clean fuels, is possible.

3 Conclusion

Carbon pricing and clean energy subsidies are the two main instruments for tackling the problem of global warming. This paper has explored a consequence of subsidies that has received too little attention in the climate change policy literature: subsidies can easily make the global problem worse, through their impact on market prices of fossil fuels. When feedback effects in the carbon-temperature dynamics are strong enough, the perverse warming effect of subsidies – the greater intensity of fossil fuel consumption at an earlier time – will offset the beneficial effect, the displacement in the future of high-extraction-cost fossil fuels with subsidized clean energy.\textsuperscript{21}

Evidence of various types is consistent with this effect being significant. Hotelling rents are high in crude oil markets, with a price close to 100 dollars per barrel, and extraction costs of conventional crude being less than 30 dollars. The International Energy Agency estimates that more than 4 trillion barrels of oil are available at extraction costs of 60 dollars per barrel or less (about 1 trillion have been extracted to date).\textsuperscript{22} In other words, the stock of oil that would be consumed more

\textsuperscript{21}The Nordhaus (2008) critique of the subsidy approach to climate change policy is quite different:

Because of the political unpopularity of taxes, it is tempting to use subsidies for “clean” or “green” activities as a substitute for raising the price of carbon emissions. This is an economic and environmental snare to be avoided. The fundamental problem is that there are too many clean activities to subsidize. Virtually everything from market bicycles to nonmarket walking has a low carbon intensity relative to driving. There are simply insufficient resources to subsidize all activities that are low emitters.” (Nordhaus 2008: 21).

\textsuperscript{22}IEA (2005); see also Stern (2006) at 212. These are 2004 U.S. dollars.
rapidly and earlier in the event of even a strong innovation in clean energy (enough to yield a perfect substitute with a cost-equivalent of 60 dollars per barrel) is large; and the displacement of high-extraction-cost crude is in the distant future. Extraction cost data on coal, a major contributor to carbon emissions along with oil and gas, is less available but the doubling of coal prices in recent years suggests that scarcity rents are high for coal as well.

The policy implications of this paper are for carbon pricing (the instrument not being modelled). Carbon pricing that is responsive to innovation can eliminate the negative impact of clean-energy innovation.\textsuperscript{23} An optimal tax would magnify the cost advantage of any new clean-energy technology by raising carbon prices. The political feasibility of a policy that magnifies any negative impact on oil and gas industry profits of success in competing technologies is obvious. A general message of this paper, however, is that the route of subsidies may be easy politically but is not a solution at all.

Important aspects of the interaction of innovation and climate change dynamics have been set aside in this paper to allow a focus on the specific point being made here. The carbon cycle is more complex than the risibly simple dynamics assumed here. In terms of economics, the full range of relevant technologies and innovation include not just clean energy, but technologies to mitigate carbon emissions (such as carbon sequestration or cleaner automobile engines); demand-side innovation to make more efficient use of energy; and even technology to directly mitigate global warming (such as injecting reflective particles in the atmosphere).

If innovation in the full set of these technologies were incorporated in our model, the perverse impact of innovation on global warming would be limited to the clean-energy technology. But this would not be true if the model were extended further to incorporate multiple countries on the demand side. Innovation and high standards for energy efficiency in developed countries, for example, transfer consumption of fossil fuels to countries, via the depressing effect on prices, and if these other countries have less developed technology or lower emission standards, then the net impact of the innovation can be to worsen global warming. Like carbon pricing, innovation and standards require international cooperation even to have a guaranteed net beneficial impact.

More general changes in technology over time can add to the effect explained in this paper. An exogenous rate of improvement in mitigation technology adds to the damaging effect in our theory,\textsuperscript{23} I am not suggesting that the complete elimination of the negative-price effect on fossil fuels of innovation is optimal. Optimal carbon pricing in the presence of innovation remains an open question.
of the early release of carbon induced by clean-energy technology (and, as a matter of logic, could even replace our assumption of a positive feedback effect in the dynamics). The possible perverse or unintended effects of government subsidies to innovation are more complex than the single effect explored here. Portfolio effects across types of innovation are an additional source of unintended effects. The government subsidy of more basic research, traditionally justified by economists, may discourage private development of engineering solutions closer to application because of the risk that the applications may be displaced by superior technology. Clearly, economic theory can make a significant contribution towards understanding a wide range of problems in the interaction of innovation, incentives and government policies on climate change.

Appendix: Derivation of Equation (2)

The competitive arbitrage condition in the market with the possibility of innovation is the following (equation 2)

\[(1 + r dt)[p^n_t - c(x^n_t)] = \rho dt[(\dot{p}_t(t) - c(\hat{x}_t(t))] + (1 - \rho dt)[p^n_{t+dt} - c(x^n_{t+dt})] - c'(x^n_t) \cdot \partial x^n_t / \partial t \cdot dt\]

or

\[(rdt)[p^n_t - c(x^n_t)] = \rho dt[(\dot{p}_t(t) - c(\hat{x}_t(t))] + (1 - \rho dt)[p^n_{t} - c(x^n_{t})] + \frac{\partial}{\partial t}(p^n_t - c(x^n_t)) \cdot dt] - c'(x^n_t) \cdot \partial \hat{x}_t(t) / \partial t \cdot dt\]

Subtracting \([p^n_t - c(x^n_t)]\) from both sides, leads to

\[(rdt)[p^n_t - c(x^n_t)] = \rho dt[(\dot{p}_t(t) - c(\hat{x}_t(t))] + (1 - \rho dt)\frac{\partial}{\partial t}(p^n_{t} - c(x^n_{t})) \cdot dt] - c'(x^n_t) \cdot \partial \hat{x}_t(t) / \partial t \cdot dt\]

which in the limit as \(dt \to 0\) yields

\[r = \rho \left( \frac{\dot{p}_t(t) - c(\hat{x}_t(t))}{p^n_t - c(x^n_t)} - 1 \right) + \frac{\partial}{\partial t}(p^n_{t} - c(x^n_{t})) \cdot \frac{\partial \hat{x}_t(t)}{\partial t} - c'(x^n_t) \cdot \frac{\partial \hat{x}_t(t)}{\partial t}\]

or

\[\frac{\partial}{\partial t}(p^n_t - c(x^n_t)) = r + \rho \left( 1 - \frac{\dot{p}_t(t) - c(\hat{x}_t(t))}{p^n_t - c(x^n_t)} \right) + \frac{c'(x^n_t) \cdot \partial \hat{x}_t(t)}{p^n_t - c(x^n_t)}\]

\(^{24}\) Revkin (2010) reports

“Bill Gates, who had recently championed a big research push to advance non-polluting energy choices, has weighed in with some new thoughts after reading a critique of his thesis by Richard Rosen of the Tellus Institute. Rosen asserted that putting too much focus on research aimed at energy breakthroughs was “dangerous” because it might encourage people “to put off investment in the many good renewable technologies that we have today, in the hope that something dramatically better will come along in the future.”


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Figure 1: Sample Paths of Cumulative Fossil Fuels Extracted, $x_t$, under:
- No innovation possible
- Innovation possible, not realized
- Innovation at time $\gamma$
Figure 2: Equilibrium Price Paths for Fossil Fuels
Figure 3: the Carbon Cycle
Figure 4: Logistic Shape of F
Figure 5: Post-Extraction Dynamics of g1: Steady States and Phase Line
Figure 6: Phase Diagram Including C (conjectured; will be verified via simulation)