FEATURE INTERVIEWS

"Venturing Into Something New"

Ravindra Bapat Interviewed by A.K. Lal[1], Manjunath Prasad[2], and Sukanta Pati[3]

Ravindra Bapat

Q - When and how did you get interested in mathematics?

Bapat - During my school days, I liked mathematics as a subject but I thought it was something very natural, that everybody would get it easily, and I found it somewhat predictable. Perhaps it had something to do with the school syllabus not being very challenging. In that sense, I did not have to make any special effort to study the subject and never really made an attempt to sincerely study until a few days before the examination. This continued throughout the initial high school classes; maybe in the last one or two years I put in some effort. I liked geometry and had the sense that I liked mathematics but there wasn't any strong inclination towards mathematics. Later on, after going to college, I found mathematics interesting but found pure mathematics a little too abstract. So, for my bachelor's, I didn't choose pure mathematics as the main subject but rather chose statistics because I liked mathematical statistics. I realize now that I do not have the feeling for data that is required to do statistics. But I continued with statistics as a subject even for my master's and the combination of statistics and mathematics was something that I found interesting.

While starting research towards my Ph.D., again initially I was just trying to do something in game theory and a linear complementarity problem because of the interest of my advisor; but then, within the first one or two semesters, I started liking linear algebra. I was attracted by very simple statements which connected together and formed somewhat of a broad theory; for example, I was fascinated by the various characterizations of positive definite matrices. That certainly was the point when I got interested in matrix theory as a subject rather than mathematics as a whole. And then I started to study nonnegative matrices. At that time the book of Berman and Plemmons on nonnegative matrices had come out and we started doing some work on the van der Waerden conjecture which was a major open problem at that time. My thesis was about some attempt towards the conjecture. Permanents of nonnegative matrices continued to be my main interest for several years.

Q - As per our information, after your undergraduate degree, you joined a famous management institute in Mumbai. At what point did you decide to leave it and why?

Bapat - There were two instances in my academic career when I strayed away from the path of academics or mathematics as a subject. The first instance was when I finished the first two years of college and appeared for the entrance examination of the Indian Institute of Technology, which continues even now to be an attraction for students in India, because of either parental pressure or pressure from the community and friends. I did succeed in the examination in the sense that I was called for the interview and was sure to get a seat. However, while studying for the examination, I realized that I did not want to go for engineering and decided not to appear for the interview. It was a very difficult decision at that time, because admission to the institute is considered very prestigious. Thus I did not go for engineering but instead went for the bachelor of science.

The second instance came when I finished the bachelor of science and there was an option to join the Bajaj Institute of Management, a very prestigious management institute where the admission was considered very difficult, but I could get admission on the basis of my bachelor's score. I, in fact, joined the institute and completed one semester and during that semester I realized this is not what I wanted to do, because completing the course would have meant immediately going for employment in some commercial place or industry, and I did not want to do that so early. So, after one semester, I decided to leave the management institute and then it was too late to join the regular master's programme at Bombay University. But I found a way out as the Indian Statistical Institute had an examination which is called the Statisticians' Diploma where one studies and writes the exam, and it is equivalent to the first year of a master in science. I decided

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to write that examination. I studied for that exam for about two or three months and completed all the ten papers and thus became eligible for admission into the second year of the master of statistics at the Indian Statistical Institute in Delhi.

Q - Did you choose your advisor or did your advisor choose you when you were a student at the Indian Statistical Institute?

Bapat - In a way it was mutual. The Delhi campus of the Indian Statistical Institute started in 1975–76 and our class was the first batch. It was a very exciting place. Professor C. R. Rao was in the institute, S. K. Mitra was there, very famous statisticians like Godambe and Khatri used to visit and there were very famous economists like T. N. Srinivasan, Kirit Parikh and so on and we took courses from some of them. I remember that professor Godambe gave some seminars, maybe three or four, on sampling theory, at the end of which we were told that it was actually a course and were asked to write an exam. So things were quite informal. We also took a course from Prof K. R. Parthasarathy. Professor T. E. S. Raghavan was on sabbatical from the University of Illinois at Chicago during that period and I took two courses from him in Statistical Decision Theory and Game Theory. I liked his approach as well as the area and hinted to him that I would be interested in following him to the University of Illinois at Chicago for graduate studies. I did not apply to any other university, which was unusual, as places such as Berkeley and Stanford used to be, and still are, the top preferences of students going to the USA. At the same time, Raghavan was also looking for a prospective student and backed me completely. He even wrote a personal check of 20 dollars which was the application fee, as getting foreign exchange involved a lot of formalities at that time.

Q - Which is (are) the result(s) of yours that you like most? And how do you see its (their) development?

Bapat - I will mention a few. The first is a result with Sunder that the eigenvalues of the Schur product of a Hermitian matrix A and a correlation matrix (a positive definite matrix with ones on the diagonal) are majorized by the eigenvalues of A. It neatly generalizes Schur's classical result that the eigenvalues of a Hermitian matrix majorize its diagonal elements. We came to this result while attempting a conjecture in the book by Marshall and Olkin, which was also proved in the same paper. Another result I would mention is that if A is a symmetric, entrywise positive matrix with exactly one positive eigenvalue, then the entrywise reciprocal of A is positive definite. This result was developed while proving a conjecture due to Karlin and Rinott about characterizing a multinomial distribution with maximum entropy. I should also include the formula for the determinant of the q-distance matrix of a tree, obtained with Lal and Pati. Given a tree with n vertices, the q-distance between distinct vertices i and j at distance t, is defined as $d_q(i,j) = 1+q+q^2+\cdots+q^{t-1}$. The q-distance matrix D_q is the $n \times n$ matrix with (i,j)-element $d_q(i,j)$ if $i \neq j$, and with zeros on the diagonal. Then the determinant of D_q is given by $(-1)^{n-1}(n-1)(1+q)^{n-2}$. When $q = 1$, we recover a well-known formula of Graham and Pollak. The result is quite unexpected and provides a link with the Ihara zeta function, although we learnt about this connection much later. Finally, I will mention a recent result with Sivasubramanian that if D is the distance matrix of a tree with at least 3 vertices, then the number of negative eigenvalues of the Hadamard square $D \circ D$ equals the number of pendant vertices in the tree. It is a result with no explanation or intuition behind it. Some such recent results were invariably obtained by MATLAB experiments and guesswork.

Q - Are there any turning points in your academic career?

Bapat - The turning points are essentially the times when I visited other places and got introduced to a new area. After my Ph.D. I worked for a year at Northern Illinois University and returned to India in 1983. I joined the Department of Statistics at the University of Bombay. The first turning point was when I joined the Indian Statistical Institute in Delhi after two years. I met V. S. Sunder and we worked on majorization inequalities as well as permanents. Sunder moved to the Bangalore campus of the Institute and I visited him in 1989. There I got in touch with Bhaskara Rao and his student Manjunatha Prasad when I became interested in generalized inverses, an interest that continued for a long time. Another turning point was when I visited Miki Neumann for a year at the University of Connecticut. I worked with Neumann and Kirkland and that is when I picked up my interest in matrices and graphs, which continues even now. I should also mention my short visits during which I worked with Pauline van den Driessche, Dale Olesky, Stephane Gaubert, Marianne Akian, Devadatta Kulkarni and S. K. Jain when I benefited by venturing into something new.

Q - How come you wrote a mathematics book in Marathi?

Bapat - I grew up in a middle class, culturally strong Marathi neighborhood of Mumbai and had primary education in Marathi. I am not a big reader of English fiction or general literature but I have kept up with literature in Marathi to

some extent. When I want to write about nontechnical subjects or even explain technical material in a nontechnical way, I find it easier and enjoyable to do so in Marathi. I believe that Indians have been robbed of the pleasure of learning deep technical subjects in their mother tongue due to excessive dependence on English. So when I decided to write a book about mathematics with a recreational flavor, Marathi was the natural choice. It was a very pleasant experience and the book was well-received. Due to a government initiative, appropriate words have been coined in Indian languages for technical terms and are waiting to be used. One such word I like is "dyut-siddhanta" for game theory. "Siddhanta" means theory and "dyut" is the grand game of dice depicted in the *Mahabharata*.

Q - If you look at the present scenario of graduate education in India, then in most of the places the size of the class is quite large with the added problem of heterogeneity. In such a situation, what should be the ideal content of a 40-lecture introductory course on linear algebra?

Bapat - I will not be able to answer this question satisfactorily. But I will share some thoughts in general. I have served on several syllabus committees. Usually the discussion ends up in completely diverse views with a lot of ill feelings and with no signs of consensus. The amount of basic mathematics that we would like our students to know is just too vast. We would be deceiving ourselves if we claim that a few courses in a three year undergraduate or a two year graduate program can equip the students with the necessary amount of mathematics needed. I do not have a solution. One thought is to go back to the earlier days of studying books rather than subjects. We did not study probability, rather we studied Feller Volume I. That way you are sure which topics are covered and which aren't. We may compile a list of books that students must study. The list can be revised once in five years. So to partially answer your question, find a suitable introductory book on linear algebra and make the students master it to the best of their abilities. The syllabus will consist of only the name of the book, indicating the chapters/sections to be covered.

Q - You were one of the first researchers in India to work in spectral graph theory. Did you ever expect to see such a huge gathering of Indian researchers joining this topic?

Bapat - My first interest has always been linear algebra and matrix theory. I was drawn to graph theory due to the interesting properties of the matrices associated with a graph. So I do not really work in "spectral graph theory" but rather "linear algebraic graph theory." But this is a minor point. Broadly speaking, the area may still be called spectral graph theory. Traditionally, graph theory and linear algebra are taught at the master's level in many universities in India. So students already have the basic tools to work in spectral graph theory. Many researchers who initially worked in pure graph theory later took up the linear algebra connection since it leads to interesting problems. This may be a reason for the resurgence in the area. My students Lal and Pati got jobs in the Indian Institutes of Technology where they could attract students in the area. So I am happy if I contributed a little bit to this development. But I still do not see a substantial growth. One indicator would be the number of researchers with at least two papers in spectral graph theory listed in the Mathematical Reviews in the past five years. That number is still small.

Q - What are three important pieces of advice that you would like to give to young researchers in India?

Bapat - There used to be a time when I was enthusiastic about persuading young students to take up a career, research or related, in mathematics. After looking at the current situation I am no longer so. A student with some mathematical abilities is much better off entering the field of engineering or finance. So my first advice (though I don't like the word so much) would be to take up research in mathematics only if you are so passionate about it that you cannot possibly do anything else. The other two pieces of advice are the usual ones: Try to look for and formulate problems that are interesting and that you can possibly solve, and develop a set of techniques of your own, normally after reading the work of masters in the area, that you can effectively use.