Efficient control-test designs for diallel cross experiments

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EFFICIENT CONTROL-TEST DESIGNS FOR DIALLEL CROSS EXPERIMENTS

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SUMMARY. Diallel cross experiments for control versus test comparisons among the lines are studied under a completely randomized design model. A sufficient condition for designs to be A- and MV-optimal in these experimental situations are derived and it is seen that the class of Type-S designs yield efficient designs to estimate control versus test comparisons. Efficient designs with lower bound to the efficiencies are also tabulated within a practical range of parameters. The issue of efficient blocking of diallel crosses is also discussed.

Key words and phrases: Type-S design; control versus test comparison; A-optimality; inbred lines; efficiency.

1. Introduction

Diallel crosses are commonly used to study the genetic properties of inbred lines in plant and animal breeding experiments. Suppose there are $p+1$ inbred lines and let a cross between lines i and j be denoted by $(i, j), i < j = 0, 1, \ldots, p$. Suppose line 0 is a control or a standard line and lines $1, \ldots, p$ are test lines. Our interest lies in comparing the control line with the test lines with respect to their general combining ability effects.

Designs for diallel crosses, where the interest lies in all general combining ability pairwise comparisons among the lines, have been recently considered by several authors. Gupta and Kageyama (1994) proved that an unblocked complete diallel cross design is universally optimal (Kiefer, 1975). Optimal partial diallel crosses were studied by Mukerjee (1997) and Das, Dean and Gupta (1998) with respect to the A-, D-, E- and MS-optimality criterion.

It is noteworthy that although designs for varietal trials and factorial experiments have been extensively investigated in the literature over the past several decades, it was not until recently that some progress in the design of diallel cross experiments has been made. Designs for control versus test comparisons where the treatments form different levels of a factor have also been extensively investigated in the literature; see Majumdar (1996). The problem of deriving appropriate designs for diallel crosses being quite different from the set-up of designs for varietal trials and factorial experiments, here we continue the work of Gupta and Kageyama (1994) for studying optimal designs for control versus test comparisons among the lines with respect to their general combining ability effects. Recently Choi, Gupta and Kageyama (2002) introduced a class of designs, called the Type-S designs, for making control-test comparisons in a diallel cross experiment. They studied various properties of such Type-S designs. In Section 2 we derive sufficient conditions for designs to be ϕ-optimal for a fairly broad class of functions ϕ. As an important application we discuss A- and MV-optimal designs. For cases where the derived sufficient conditions are not applicable, some approximately optimal designs are suggested. In Section 3 we give a large number of highly A- and/or MV-efficient Type-S designs. It is observed that the Type-S designs are likely to be useful in practice as the A- and/or MV-efficiency of these designs is close to unity. Note that the A-efficiency (MV-efficiency) of an A-optimal (MV-optimal) design is unity. From practical considerations, it is useful to have a catalog of efficient designs. We present a comprehensive catalog of highly A- and/or MV-efficient Type-S designs in the practically useful ranges $3 \leq p \leq 30,\ n \leq p(p+9)$. The issue of efficient blocking of diallel crosses is also discussed in Section 3.

2. Optimal Designs

We consider diallel cross experiments involving $p+1$ inbred lines, giving rise to a total of $n_c = (p+1)p/2$ distinct crosses. Let a cross between lines i and j be denoted by $(i,j), i < j = 0, 1, \ldots, p$. Suppose line 0 is a control or a standard line and lines $1, \ldots, p$ are test lines. Our main interest lies in comparing the control line with the test lines with respect to their general combining ability effects. Let s_{dj} denote the total number of times that the jth line occurs in the crosses in the design d, $j = 0, 1, \ldots, p$. Further let $s_d = (s_{d0}, s_{d1}, \ldots, s_{dp})'$ and let n denote the total number of crosses in the design. Following e.g., Gupta and Kageyama (1994), the model under the completely randomized set-up, for a design d, is assumed to be

$$Y_d = \mu 1_n + \Delta_{d1}\tau + \varepsilon, \tag{2.1}$$

where Y_d is the $n \times 1$ vector of responses, μ is the overall mean, 1_t is the $t \times 1$ column vector of 1's, $\tau = (\tau_0, \tau_1, \ldots, \tau_p)'$ is the vector of $p+1$ general combining ability effects and Δ_{d1} is the corresponding design matrix, that is, the (h,l)th element of Δ_{d1}, is 1 if the hth observation pertains to the lth line, and is zero otherwise, and ε is the $n \times 1$ vector of independent random errors with zero expectation and constant variance σ^2. The normal equations for estimating the vector of general combining ability parameters are then given by $C_d\tau = Q_d$ where $Q_d = \Delta'_{d1}(I_n - \frac{1}{n}1_n 1'_n)Y_d$,

$$C_d = G_d - \frac{1}{n}s_d s'_d \tag{2.2}$$

and $G_d = (g_{dii'}), g_{dii} = s_{di}$, and for $i \neq i', g_{dii'}$ is the number of times the cross (i,i') appears in the design. We denote an identy matrix of order t by I_t.

Now we can be more precise about what we mean by comparing test lines with a control line. In particular, because our primary goal is to determine which among the test lines might be better than the control, we would like to estimate the magnitude of each $\tau_i - \tau_0;\ i = 1, \ldots, p$, with as much precision as possible. In assigning crosses to experimental units, we have to make sure that the contrasts $\tau_i - \tau_0;\ i = 1, \ldots, p$, are estimable. A design d satisfying this later condition is said to be connected and we shall restrict our attention to such designs. It is known that a design d is connected if and only if its corresponding C-matrix C_d, as in (2.2), has rank p. Clearly there are a number of designs available for the situation being considered here and we want to choose one which is best in some sense. We shall use the notation $\mathcal{D}(p+1,n)$ to denote the set of all connected designs with p test lines, 1 control line and n crosses. For a class of designs $\mathcal{D}$, if $\hat{\tau}_{di}-\hat{\tau}_{d0}$ denotes the best linear unbiased estimator (BLUE) of $\tau_i-\tau_0$ using a design d, then a design is A-optimal for line-control contrasts if it minimizes $\sum_{i=1}^{p} Var(\hat{\tau}_{di}-\hat{\tau}_{d0})$ in $\mathcal{D}$. A design is MV-optimal for line-control contrasts if it minimizes $\max_{1\le i\le p} Var(\hat{\tau}_{di}-\hat{\tau}_{d0})$ in $\mathcal{D}$. A- and MV-optimality are statistically meaningful criteria in the present setup.

Let $P=(-1_p \quad I_p)$. Then the covariance matrix for the BLUE's $(\hat{\tau}_{d1}-\hat{\tau}_{d0},\hat{\tau}_{d2}-\hat{\tau}_{d0},\ldots,\hat{\tau}_{dp}-\hat{\tau}_{d0})$ of the line-control contrast is $\sigma^2 PC_d^- P'$. If one partitions C_d as:

$$C_d=\begin{pmatrix} c_{d00} & \gamma_d' \\ \gamma_d & M_d \end{pmatrix} \tag{2.3}$$

then it can be shown that (see Gupta, 1989),

$$(PC_d^- P')^{-1}=M_d, \tag{2.4}$$

i.e., M_d is the *Information matrix* for the line-control contrasts. Clearly an A-optimal design minimizes $tr(M_d^{-1})$ in $\mathcal{D}(p+1,n)$ and an MV-optimal design minimizes the maximum diagonal element of M_d^{-1} in $\mathcal{D}(p+1,n)$.

We now present results about what designs are ϕ-optimal for a fairly broad class of functions ϕ. As an important application we discuss A-optimal designs. We closely follow the techniques of Majumdar and Notz (1983) in arriving at the results. Let us start with an arbitrary design d in $\mathcal{D}(p+1,n)$. Using Kiefer's (1975) technique of averaging, we obtain

$$tr(PC_d^- P')\ge tr(P\bar{C}_d^- P'), \tag{2.5}$$

where $\bar{C}_d=\frac{1}{p!}\sum_\pi \pi C_d \pi'$, the summation taken over all $(p+1)\times(p+1)$ permutation matrices π that correspond to permutations of the p test treatments only. If we partition $\bar{C}_d$ as in (2.3), then we see that $\bar{M}_d=(P\bar{C}_d^- P')^{-1}$ is a completely symmetric matrix. In general, there may be no design in $\mathcal{D}(p+1,n)$ for which $\bar{M}_d$ is the Information matrix for the line-control contrasts. If there is such a design, then for this design, call it d^*, $M_{d^*}=\bar{M}_{d^*}$ is completely symmetric and γ_{d^*} (see (2.3)) is a vector with all entries equal. That is, d^* belongs to a class of designs, called *Type-S designs*, introduced by Choi, Gupta and Kageyama (2002).

Definition 2.1. A design $d\in\mathcal{D}(p+1,n)$ is called a Type-S design if there are positive integers g_0 and g_1, such that:

$$\begin{aligned} g_{dii'} &= g_1 \text{ for } i,i'=1,\ldots,p\ (i\neq i') \\ g_{d0i} &= g_0 \text{ for } i=1,\ldots,p. \end{aligned}$$

We denote a Type-S design with parameters p, g_0 and g_1 by $S(p,g_0,g_1)$.

For a Type-S $S(p,g_0,g_1)$ design d Choi, Gupta and Kageyama (2002) have shown that the following hold.

$s_{d0}=pg_0$,

$s_{di} = g_0 + (p-1)g_1$ for $i = 1, \ldots, p,$

$n = (s_{d0} + ps_{d1})/2,$

$Var(\hat{\tau}_{di} - \hat{\tau}_{d0}) = \frac{f\{a_1-(p-2)b_1\}\sigma^2}{(a_1+b_1)p(p-1)g_0g_1},\ i = 1, \ldots, p,$

$Cov(\hat{\tau}_{di} - \hat{\tau}_{d0},\ \hat{\tau}_{di'} - \hat{\tau}_{d0}) = \frac{fb_1\sigma^2}{(a_1+b_1)p(p-1)g_0g_1},\ i \neq i' = 1, \ldots, p,$

where

$$\begin{array}{rcl} f & = & p\{2g_0 + (p-1)g_1\}, \\ a_1 & = & (p-1)\{g_0 + (p-1)g_1\}\{2g_0 + (p-2)g_1\}, \\ b_1 & = & 2g_0g_1(p-2) + (p-1)(p-2)g_1^2 + 2g_0^2. \end{array} \tag{2.6}$$

We would require the following three lemmas for our main result.

Lemma 2.1. If $d \in \mathcal{D}(p+1, n)$ then $\bar{M}_d$ has eigenvalues $\mu(d1),\ \mu(d2) = \cdots = \mu(dp)$ with

$$\begin{array}{rcl} \mu(d1) & = & \frac{s_{d0}(n-s_{d0})}{np}, \\ \mu(d2) & = & \frac{np(2n-s_{d0})-p\sum_{i=1}^{p} s_{di}^2 - s_{d0}(n-s_{d0})}{np(p-1)}. \end{array}$$

Proof. From (2.2) and (2.3), the entries of M_d are

$$m_{di_1i_2} = \begin{cases} s_{di_1} - s_{di_1}^2/n & (i_1 = i_2) \\ g_{di_1i_2} - s_{di_1}s_{di_2}/n & (i_1 \neq i_2) \end{cases}$$

and the sum of the entries in the ith row (or ith column) $\sum_{i_1=1}^{p} m_{di_1i_2} = -g_{d0i_2} + s_{d0}s_{di_2}/n$. Thus it is straightforward to check that $\bar{M}_d = aI_p + b1_p1_p'$ with $b = \frac{1}{p(p-1)}\sum_{1\leq i_1\neq i_2\leq p} m_{di_1i_2}$ and $a = \frac{1}{p}\sum_{i_2=1}^{p} m_{di_2i_2} - b$. The result now follows, after some computations, from the well known fact that $aI_p + b1_p1_p'$ has eigenvalues a with multiplicity $p-1$ and $a + bp$ with multiplicity 1.

Lemma 2.2. Suppose ϕ is a convex real-valued possibly infinite function on the set of all $p \times p$ non-negative definite matrices and ϕ is invariant under permutations, i.e., if π_t is a permutation matrix of order p, $\phi(\pi_t M_d \pi_t') = \phi(M_d)$. Then for $d \in \mathcal{D}(p+1, n)$, $\phi(\bar{M}_d) = \phi(M_d)$.

Proof. This is Lemma 2.2 of Majumdar and Notz (1983).

Lemma 2.3. For given positive integers v and t, the minimum of $n_1^2 + n_2^2 + \cdots + n_v^2$ subject to $n_1 + n_2 + \cdots + n_v = t$, where n_i's are non-negative integers, is obtained when $t - v[t/v]$ of the n_i's are equal to $[t/v] + 1$ and $v - t + v[t/v]$ are equal to $[t/v]$, where $[z]$ denotes the largest integer not exceeding z. The corresponding minimum of $n_1^2 + n_2^2 + \cdots + n_v^2$ is $t(2[t/v] + 1) - v[t/v]([t/v] + 1)$.

From Lemmas 2.1, 2.2 and 2.3 we have

Theorem 2.1. Suppose ϕ is a real-valued possibly infinite function on the set of all $p \times p$ non-negative definite matrices satisfying $\phi(M_d) = \sum_{i=1}^{p} f(\mu_{di})$ where $\mu_{d1} \leq \mu_{d2} \leq \cdots \leq \mu_{dp}$ are the eigenvalues of M_d, f is a real valued possibly infinite function on the set of all non-negative numbers which is continuous on the set of all positive numbers, has $f' < 0$ and $f'' > 0$. Suppose there is a $d^* \in \mathcal{D}(p+1, n)$ such that M_{d^*} is completely symmetric and s_{d^*0} is the value of the integer s, $1 \leq s \leq n-1$, which minimizes

$$g(s; n, p) = f(\frac{s(n-s)}{np}) + (p-1)f(\frac{np(2n-s) - ph(s; n, p) - s(n-s)}{np(p-1)}), \qquad (2.7)$$

where $h(s; n, p) = px^2 + (2n-s-px)(2x+1)$ and $x = [\frac{2n-s}{p}]$, the largest integer not exceeding $\frac{2n-s}{p}$.

It is known that in our setup A-optimal designs are statistically very meaningful (see, Majumdar (1996)). In the notation of Theorem 2.1 A-optimality criteria means $\phi(M_d) = tr(M_d^{-1})$ and $f(\mu) = 1/\mu$. Equation (2.7) then becomes

$$g(s; n, p) = \frac{np}{s(n-s)} + \frac{np(p-1)^2}{np(2n-s) - ph(s; n, p) - s(n-s)}, \qquad (2.8)$$

with $h(s; n, p) = px^2 + (2n - s - px)(2x + 1)$ and $x = [\frac{2n-s}{p}]$.

The following result is a consequence of Theorem 2.1.

Theorem 2.2. Suppose Q is the value of the integer s, $1 \leq s \leq n-1$, which minimizes $g(s; n, p)$ as given in (2.8). Also suppose $d^* \in \mathcal{D}(p+1, n)$ is a Type-S design such that $s_{d^*0} = Q$. Then d^* is A-optimal over $\mathcal{D}(p+1, n)$.

The integer s which minimizes $g(s; n, p)$ can easily be found using a computer. As an example, $s = 30$ minimizes $g(s; 75, 10)$ and thus the Type-S $S(10, 3, 1)$ design is A-optimal over $\mathcal{D}(11, 75)$.

Remark 2.1. It follows from the fact, $\max_{1 \leq i \leq p} Var(\hat{\tau}_{di} - \hat{\tau}_{d0}) = \frac{1}{p} \sum_{i=1}^{p} Var(\hat{\tau}_{di} - \hat{\tau}_{d0})$ for a Type-S design d, that all Type-S designs that are A-optimal are also MV-optimal.

3. Efficient blocking and a catalog of efficient Type-S designs

The A-efficiency of a design $d \in \mathcal{D}$ for making test line-control comparisons is defined as $E_{Ad} = \frac{\sum_{i=1}^{p} Var(\hat{\tau}_{d_A i} - \hat{\tau}_{d_A 0})}{\sum_{i=1}^{p} Var(\hat{\tau}_{di} - \hat{\tau}_{d0})}$ where d_A is an A-optimal design in $\mathcal{D}$. Based on Theorem 2.2, a *lower bound* to the A-efficiency of a Type-S design d with parameters p, g_0, g_1 is given by

$$e_{Ad} \; = \; g(Q; n, p)/B_d \; \leq \; E_{Ad} \qquad (3.1)$$

where $B_d = g(s_{d0}; n, p) = \frac{f\{a_1 - (p-2)b_1\}}{(a_1+b_1)(p-1)g_0 g_1}$; f, a_1 and b_1 being as in (2.6). The MV-efficiency (E_{Md}) of a design $d \in \mathcal{D}$ can be similarly defined by using the maximum variance of the $(\hat{\tau}_{di} - \hat{\tau}_{d0})$'s, instead of their sum, i.e., $E_{Md} = \frac{\max_{1 \leq i \leq p} Var(\hat{\tau}_{d_M i} - \hat{\tau}_{d_M 0})}{\max_{1 \leq i \leq p} Var(\hat{\tau}_{di} - \hat{\tau}_{d0})}$ where d_M is an MV-optimal design in $\mathcal{D}$. Again, based on Theorem 2.2, $E_{Md} \geq e_{Md} = e_{Ad}$ if d is a Type-S

design. High values of E_{Ad} and/or E_{Md} would indicate that the design d is highly efficient, and hence approximately optimal under A-optimality and/or MV-optimality.

For situations where blocking is necessary, one way to obtain an optimal block design is to construct, if possible, an *orthogonal* block design. Let $\mathcal{D}(p+1,b,k)$ be the class of diallel crosses involving p test lines, 1 control line and bk crosses arranged in b blocks each of size k. For any design $d \in \mathcal{D}(p+1,b,k)$, the model resembles (2.1) with $n = bk$. The only change is that an extra term $\Delta_{d2}\beta$ will now appear in the right-hand side of (2.1), where β is the vector of block effects and Δ_{d2} is the associated design matrix. As noted in Gupta and Kageyama (1994), the information matrix for τ, under d, is $A_d = G_d - k^{-1}N_dN_d'$, where G_d is defined just after (2.2) and $N_d = \Delta_{d1}'\Delta_{d2}$ is the $(p+1)\times b$ incidence matrix of lines versus blocks. For $d \in \mathcal{D}(p+1,b,k)$, let C_d be defined as in (2.2) with $n = bk$. Note that C_d is the information matrix for τ ignoring blocks. As noted in Gupta, Das and Kageyama (1995), for any $d \in \mathcal{D}(p+1,b,k)$, $C_d - A_d$ is nonnegative definite and furthermore, $C_d = A_d$ if and only if the condition

$$N_d = b^{-1}s_d1_b' \tag{3.2}$$

for orthogonal blocking holds. This has useful implications with respect to the construction of optimal or efficient block designs for diallel cross experiments. In an unblocked set-up suppose an A- and/or MV-efficient design d_a is available in $\mathcal{D}(p+1,bk)$. Let d_a be partitioned to yield a orthogonal block design $d' \in \mathcal{D}(p+1,b,k)$. Then d' is also A-efficient (MV-efficient) over $\mathcal{D}(p+1,b,k)$ since the A-efficiency (MV-efficiency) of $d' \in \mathcal{D}(p+1,b,k)$ is at least as large as e_{Ad_a} (e_{Md_a}).

Example 3.1. Suppose $p = 5, b = 5$ and $k = 6$. We have a Type-S $S(5,2,2)$ design d with $e_{Ad} = e_{Md} = 0.957$. For such a design $n = 30$. Thus partitioning the crosses of the design d into 5 groups (blocks) of size 6 each such that (3.2) is satisfied, we have the following orthogonal block design $d' \in \mathcal{D}(6,5,6)$ with $e_{Ad'} = e_{Md'} = 0.957$. (Rows are blocks)

$$\begin{array}{cccccc}
(1,4) & (2,5) & (3,1) & (4,2) & (5,0) & (3,0) \\
(2,3) & (3,4) & (4,5) & (5,1) & (1,0) & (2,0) \\
(2,5) & (3,1) & (4,2) & (5,3) & (1,0) & (4,0) \\
(3,4) & (4,5) & (5,1) & (1,2) & (2,0) & (3,0) \\
(5,3) & (1,2) & (1,4) & (2,3) & (4,0) & (5,0)
\end{array}.$$

If for a Type-S design d, $e_{Ad} = e_{Md} = 1$ then the design is A- and MV-optimal. Using the expression given in (3.1), we have computed lower bounds to the A- and/or MV-efficiency of Type-S designs. In a Table, we present a catalog of highly A- and MV-efficient Type-S designs ($e \geq 0.950$) in the practically useful ranges $3 \leq p \leq 30, g_0 \leq 10, g_1 \leq 2$. Note that e is a *lower bound* to the A- and MV-efficiency of Type-S designs. The catalog of designs presented in the Table contains 322 designs in the ranges of parameters specified above. Out of these, 11 are A- and/or MV-optimal designs and are marked with a "$*$" in the table. The value of e is rounded to three decimal places.

Table

Catalog of A-efficient Type-S designs with $3 \le p \le 30,\ g_0 \le 10,\ g_1 \le 2$

No.	p	n	g_0	g_1	e	No.	p	n	g_0	g_1	e	No.	p	n	g_0	g_1	e
*1	3	6	1	1	1.000	47	9	108	4	2	0.984	93	13	130	4	1	0.999
*2	3	12	2	2	1.000	48	9	117	5	2	0.998	94	13	143	5	1	0.990
3	3	15	3	2	0.977	49	9	126	6	2	1.000	95	13	156	6	1	0.975
4	4	10	1	1	0.989	50	9	135	7	2	0.995	96	13	169	7	1	0.958
5	4	14	2	1	0.981	51	9	144	8	2	0.986	97	13	208	4	2	0.956
6	4	20	2	2	0.987	52	9	153	9	2	0.975	98	13	221	5	2	0.983
*7	4	24	3	2	1.000	53	9	162	10	2	0.963	99	13	234	6	2	0.995
8	4	28	4	2	0.979	54	10	65	2	1	0.977	100	13	247	7	2	1.000
9	5	15	1	1	0.959	*55	10	75	3	1	1.000	101	13	260	8	2	0.999
10	5	20	2	1	0.998	56	10	85	4	1	0.992	102	13	273	9	2	0.995
11	5	25	3	1	0.958	57	10	95	5	1	0.973	103	13	286	10	2	0.990
12	5	30	2	2	0.957	58	10	105	6	1	0.951	104	14	133	3	1	0.993
13	5	35	3	2	0.998	59	10	130	4	2	0.977	105	14	147	4	1	1.000
14	5	40	4	2	0.997	60	10	140	5	2	0.995	106	14	161	5	1	0.993
15	5	45	5	2	0.980	61	10	150	6	2	1.000	107	14	175	6	1	0.980
16	5	50	6	2	0.957	62	10	160	7	2	0.998	108	14	189	7	1	0.964
*17	6	27	2	1	1.000	63	10	170	8	2	0.992	109	14	252	5	2	0.978
18	6	33	3	1	0.980	64	10	180	9	2	0.983	110	14	266	6	2	0.993
19	6	48	3	2	0.987	65	10	190	10	2	0.973	111	14	280	7	2	0.999
*20	6	54	4	2	1.000	66	11	77	2	1	0.970	112	14	294	8	2	1.000
21	6	60	5	2	0.994	67	11	88	3	1	0.999	113	14	308	9	2	0.997
22	6	66	6	2	0.980	68	11	99	4	1	0.996	114	14	322	10	2	0.993
23	6	72	7	2	0.961	69	11	110	5	1	0.980	115	15	150	3	1	0.990
24	7	35	2	1	0.997	70	11	121	6	1	0.961	*116	15	165	4	1	1.000
25	7	42	3	1	0.992	71	11	154	4	2	0.970	117	15	180	5	1	0.995
26	7	49	4	1	0.963	72	11	165	5	2	0.991	118	15	195	6	1	0.984
27	7	63	3	2	0.974	73	11	176	6	2	0.999	119	15	210	7	1	0.970
28	7	70	4	2	0.997	74	11	187	7	2	1.000	120	15	225	8	1	0.955
29	7	77	5	2	0.999	75	11	198	8	2	0.995	121	15	285	5	2	0.974
30	7	84	6	2	0.992	76	11	209	9	2	0.989	122	15	300	6	2	0.990
31	7	91	7	2	0.979	77	11	220	10	2	0.980	123	15	315	7	2	0.998
32	7	98	8	2	0.963	78	12	90	2	1	0.963	*124	15	330	8	2	1.000
33	8	44	2	1	0.992	79	12	102	3	1	0.998	125	15	345	9	2	0.999
34	8	52	3	1	0.997	80	12	114	4	1	0.998	126	15	360	10	2	0.995
35	8	60	4	1	0.977	81	12	126	5	1	0.986	127	16	168	3	1	0.988
36	8	80	3	2	0.960	82	12	138	6	1	0.969	128	16	184	4	1	1.000
37	8	88	4	2	0.991	83	12	150	7	1	0.950	129	16	200	5	1	0.997
38	8	96	5	2	1.000	84	12	180	4	2	0.963	130	16	216	6	1	0.987
39	8	104	6	2	0.997	85	12	192	5	2	0.987	131	16	232	7	1	0.974
40	8	112	7	2	0.989	86	12	204	6	2	0.997	132	16	248	8	1	0.960
41	8	120	8	2	0.977	*87	12	216	7	2	1.000	133	16	320	5	2	0.970
42	8	128	9	2	0.964	88	12	228	8	2	0.998	134	16	336	6	2	0.988
43	9	54	2	1	0.985	89	12	240	9	2	0.993	135	16	352	7	2	0.996
44	9	63	3	1	1.000	90	12	252	10	2	0.986	136	16	368	8	2	1.000
45	9	72	4	1	0.986	91	13	104	2	1	0.956	137	16	384	9	2	0.999
46	9	81	5	1	0.963	92	13	117	3	1	0.995	138	16	400	10	2	0.997

Table (Contd.)

No.	p	n	g_0	g_1	e	No.	p	n	g_0	g_1	e	No.	p	n	g_0	g_1	e
139	17	187	3	1	0.985	186	20	480	5	2	0.954	233	24	372	4	1	0.992
140	17	204	4	1	0.999	187	20	500	6	2	0.977	234	24	396	5	1	1.000
141	17	221	5	1	0.998	188	20	520	7	2	0.990	235	24	420	6	1	0.998
142	17	238	6	1	0.989	189	20	540	8	2	0.997	236	24	444	7	1	0.993
143	17	255	7	1	0.978	190	20	560	9	2	1.000	237	24	468	8	1	0.984
144	17	272	8	1	0.965	191	20	580	10	2	1.000	238	24	492	9	1	0.975
145	17	289	9	1	0.951	192	21	273	3	1	0.974	239	24	516	10	1	0.965
146	17	357	5	2	0.966	193	21	294	4	1	0.996	240	24	696	6	2	0.967
147	17	374	6	2	0.985	194	21	315	5	1	1.000	241	24	720	7	2	0.982
148	17	391	7	2	0.995	195	21	336	6	1	0.996	242	24	744	8	2	0.992
149	17	408	8	2	0.999	196	21	357	7	1	0.988	243	24	768	9	2	0.997
150	17	425	9	2	1.000	197	21	378	8	1	0.978	244	24	792	10	2	1.000
151	17	442	10	2	0.998	198	21	399	9	1	0.967	245	25	375	3	1	0.964
152	18	207	3	1	0.982	199	21	420	10	1	0.955	246	25	400	4	1	0.991
153	18	225	4	1	0.999	200	21	525	5	2	0.951	247	25	425	5	1	0.999
154	18	243	5	1	0.999	201	21	546	6	2	0.974	248	25	450	6	1	0.999
155	18	261	6	1	0.992	202	21	567	7	2	0.988	249	25	475	7	1	0.994
156	18	279	7	1	0.981	203	21	588	8	2	0.996	250	25	500	8	1	0.986
157	18	297	8	1	0.969	204	21	609	9	2	0.999	251	25	525	9	1	0.977
158	18	315	9	1	0.956	205	21	630	10	2	1.000	252	25	550	10	1	0.967
159	18	396	5	2	0.962	206	22	297	3	1	0.972	253	25	750	6	2	0.964
160	18	414	6	2	0.982	207	22	319	4	1	0.995	254	25	775	7	2	0.981
161	18	432	7	2	0.993	*208	22	341	5	1	1.000	255	25	800	8	2	0.991
162	18	450	8	2	0.999	209	22	363	6	1	0.997	256	25	825	9	2	0.997
*163	18	468	9	2	1.000	210	22	385	7	1	0.990	257	25	850	10	2	0.999
164	18	486	10	2	0.999	211	22	407	8	1	0.980	258	26	403	3	1	0.962
165	19	228	3	1	0.979	212	22	429	9	1	0.970	259	26	429	4	1	0.990
166	19	247	4	1	0.998	213	22	451	10	1	0.959	260	26	455	5	1	0.999
167	19	266	5	1	0.999	214	22	594	6	2	0.972	261	26	481	6	1	0.999
168	19	285	6	1	0.993	215	22	616	7	2	0.986	262	26	507	7	1	0.995
169	19	304	7	1	0.984	216	22	638	8	2	0.995	263	26	533	8	1	0.988
170	19	323	8	1	0.972	217	22	660	9	2	0.999	264	26	559	9	1	0.979
171	19	342	9	1	0.960	218	22	682	10	2	1.000	265	26	585	10	1	0.970
172	19	437	5	2	0.958	219	23	322	3	1	0.969	266	26	806	6	2	0.962
173	19	456	6	2	0.979	220	23	345	4	1	0.993	267	26	832	7	2	0.979
174	19	475	7	2	0.992	221	23	368	5	1	1.000	268	26	858	8	2	0.990
175	19	494	8	2	0.998	222	23	391	6	1	0.998	269	26	884	9	2	0.996
176	19	513	9	2	1.000	223	23	414	7	1	0.991	270	26	910	10	2	0.999
177	19	532	10	2	0.999	224	23	437	8	1	0.982	271	27	432	3	1	0.960
178	20	250	3	1	0.977	225	23	460	9	1	0.972	272	27	459	4	1	0.988
179	20	270	4	1	0.997	226	23	483	10	1	0.962	273	27	486	5	1	0.999
180	20	290	5	1	1.000	227	23	644	6	2	0.969	274	27	513	6	1	1.000
181	20	310	6	1	0.995	228	23	667	7	2	0.984	275	27	540	7	1	0.996
182	20	330	7	1	0.986	229	23	690	8	2	0.993	276	27	567	8	1	0.989
183	20	350	8	1	0.975	230	23	713	9	2	0.998	277	27	594	9	1	0.981
184	20	370	9	1	0.964	231	23	736	10	2	1.000	278	27	621	10	1	0.972
185	20	390	10	1	0.952	232	24	348	3	1	0.967	279	27	864	6	2	0.960

Table (Contd.)

No.	p	n	g_0	g_1	e	No.	p	n	g_0	g_1	e	No.	p	n	g_0	g_1	e
280	27	891	7	2	0.977	294	28	980	8	2	0.987	308	29	1073	9	2	0.993
281	27	918	8	2	0.988	295	28	1008	9	2	0.994	309	29	1102	10	2	0.998
282	27	945	9	2	0.995	296	28	1036	10	2	0.998	310	30	525	3	1	0.953
283	27	972	10	2	0.999	297	29	493	3	1	0.955	311	30	555	4	1	0.985
284	28	462	3	1	0.957	298	29	522	4	1	0.986	312	30	585	5	1	0.997
285	28	490	4	1	0.987	299	29	551	5	1	0.998	313	30	615	6	1	1.000
286	28	518	5	1	0.998	300	29	580	6	1	1.000	314	30	645	7	1	0.998
287	28	546	6	1	1.000	301	29	609	7	1	0.997	315	30	675	8	1	0.992
288	28	574	7	1	0.996	302	29	638	8	1	0.991	316	30	705	9	1	0.985
289	28	602	8	1	0.990	303	29	667	9	1	0.984	317	30	735	10	1	0.977
290	28	630	9	1	0.982	304	29	696	10	1	0.975	318	30	1050	6	2	0.953
291	28	658	10	1	0.974	305	29	986	6	2	0.955	319	30	1080	7	2	0.972
292	28	924	6	2	0.957	306	29	1015	7	2	0.974	320	30	1110	8	2	0.985
293	28	952	7	2	0.975	307	29	1044	8	2	0.986	321	30	1140	9	2	0.992
												322	30	1170	10	2	0.997

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