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On testing dependence between time to failure and cause of failure via conditional probability: An extension to missing data on cause of failure

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Abstract

We propose tests based on U-statistics to test for independence of the failure time and the cause of failure in the competing risks model when we all the causes of failure cannot be identified. The asymptotic distribution is normal in each case. Simulation studies look at power comparisons for the proposed tests for two families of distributions. The one-sided and the two-sided tests based on Kendall type statistic perform exceedingly well in detecting departures from independence.

Key words: competing risks, conditional probability, U-statistics, missing causes, dependence relation.

1 Introduction

We consider a competing risks set-up where a unit is subject to 2 risks and what is observable is the time to failure T and an identifier of the risk $\delta = j$ if the failure is due to the risk $j, j = 0, 1$. Let the joint distribution of (T, δ) be specified by the subsurvival functions $S_j(t) = P(T \geq t, \delta = j)$, or the subdistribution function given by $F_j(t) = P(T < t, \delta = j)$, $j = 0, 1$. The survival function and the distribution function of T are, respectively, given by

$$S(t) = P[T \geq t] = S_0(t) + S_1(t),$$

$$F(t) = P[T < t] = F_0(t) + F_1(t).$$

We assume that the subsurvival functions are continuous.

Let the conditional probability of failure due to the first risk given that there is no failure upto time t be given as

$$\Phi_1(t) = P[\delta = 1 \mid T \geq t] = S_1(t)/S(t),$$

whenever $S(t) > 0$. Cooke (1996) introduced these probabilities while studying failure and preventive maintenance in a censoring setting where the interest is in the distribution of the latent failure time which would have been observed in the absence of preventive maintenance. Another conditional probability of interest is

$$\Phi_0^*(t) = P(\delta = 0 \mid T < t) = F_1(t)/F(t).$$

Dewan *et al.* (2004) studied the dependence between the failure time T and the cause of failure δ in terms of the above two conditional probability functions. They showed that T and δ are independent if and only if $\Phi_1(t) = \Phi_1(0) = \phi(\text{say})$, for all t or equivalently $\Phi_0^*(t) = \Phi_0^*(0)$, for all t. Note that $\phi = P(\delta = 1)$. Under independence, T and δ can be studied separately since $S_j(t) = S(t)P[\delta = j]$. Thus, the hypothesis of equality of subsurvival functions, or that of equality of cause-spectific hazard rates reduces to testing whether $P[\delta = 1] = P[\delta = 0] = 1/2$, a Bernoulli trial with success probability half. Hence a two-dimensional problem reduces to one-dimensional problem.

Dewan *et al.* (2004) showed that T and δ are *positive quadrant dependent* (PQD) iff $\Phi_1(t) \geq \Phi_1(0)$, for all $t > 0$ or equivalently $\Phi_0^*(t) \geq \Phi_0^*(0)$, for all $t > 0$. They also considered various hypothesis testing problems of checking independence of T and δ against various alternatives specifying dependence structures. They derived U-statistics when the data on all n units, that is $(T_1, \delta_1), (T_2, \delta_2), \ldots, (T_n, \delta_n)$ are available. However, in many practical situations the experimenter may have information only on failure time for the individuals. It may not be possible to identify the cause of failure for all individuals due to financial or logistic reasons. Kodel and Chen (1987) considered an example from animal bioanalysis where all causes were not available. Lapidus *et al.* (1994) while studying motorcycle fatalities observed that 40 percent of the death certificates had no information on causes. Likelihood based estimation in case of missing causes of failure have been studied by Dinse (1982), Dewanji(1992), Goetghebeur and Ryan (1995), Dewanji and Sengupta (2003), Lu and Tsiatis (2005). Goetghebeur and Ryan (1990) considered a modified log rank test for competing risks with missing failure type. Miyawaka (1984) obtained maximum likelihood estimators and minimum variance unbiased estimators of the parameters of exponential distribution for the missing case. Kundu and Basu (2000) discussed approximate and asymptotic properties of these estimators and obtained confidence intervals.

In this paper, we extend some of the tests based on U-statistics for testing independence of T and δ to a situation when the causes of failure are not observed for all individuals under consideration. Let (T_i, δ_i), $i = 1, \ldots, n$ be the competing risks data available on n individuals. Here, we consider a situation when δ_i may not be observed always *i.e.,* it may be missing for some units. Let O_i be an indicator variable which takes value one if δ_i is observed and zero if δ_i is missing. Let $p = pr(O_i = 1)$ be the probability that δ is observed. We assume that δ_i are missing at random and hence O_i is independent of (T_i, δ_i) . The fact that the cause of failure will be observed or not will have no bearing on the actual cause. Similar assumptions are made by Rubin (1976) and Lu and Tsiatis (2005).

In section 2 we consider the problem of testing

$$H_0 : T\ and\ \delta \text{ are independent}$$

against various alternative hypotheses characterising the dependence structure of T and δ, which are:

$$\begin{aligned} H_1 &: T \text{ and } \delta \text{ are not independent} \\ H_2 &: T \text{ and } \delta \text{ are positive quadrant dependent.} \end{aligned}$$

We extend the U-statistics proposed in Dewan *et al.* (2004) to the case when $\delta' s$ are not observed for all the units. In section 3 we carry out simulation studies for comparing the power of the tests for two families of distributions. In section 4 we conclude with some remarks.

2 Extension of U-statistics

2.1 General dependence between T and δ

First we consider the problem of testing H_0 against H_1. Recall that a pair (T_i, δ_i) and (T_j, δ_j) is a concordant pair if $T_i > T_j,\ \delta_i = 1, \delta_j = 0$ or $T_i < T_j,\ \delta_i = 0, \delta_j = 1$ and is a discordant pair if $T_i > T_j,\ \delta_i = 0, \delta_j = 1$ or $T_i < T_j,\ \delta_i = 1, \delta_j = 0$. The U-statistic proposed by Dewan *et al.* (2004) based on the idea of concordance and discordance pairs is

$$U_k = \frac{1}{\binom{n}{2}} \sum_{1 \le i < j \le n} \psi_k(T_i, \delta_i, T_j, \delta_j),$$

where

$$\psi_k(T_i, \delta_i, T_j, \delta_j) \;=\; \begin{cases} 1 & \text{if } T_i > T_j,\ \delta_i = 1, \delta_j = 0 \\ & \text{or } T_i < T_j,\ \delta_i = 0, \delta_j = 1 \\ -1 & \text{if } T_i > T_j,\ \delta_i = 0, \delta_j = 1 \\ & \text{or } T_i < T_j,\ \delta_i = 1, \delta_j = 0 \\ 0 & \text{otherwise.} \end{cases}$$

If δ is missing for some units then $\psi_k(T_i, \delta_i, T_j, \delta_j)$ cannot be defined for all pairs. In the following table m indicates that δ is not observable and ? indicates the cases when ψ_k is not defined.

Table 1: Values taken by the kernel ψ_k for various combinations of the pairs (T_i, δ_i) and (T_j, δ_j)

(δ_i, δ_j)	(1,1)	(1,0)	(1,m)	(0,1)	(0,0)	(0,m)	(m,1)	(m,0)	(m,m)
$T_i > T_j$	0	1	?	-1	0	?	?	?	?
$T_i \le T_j$	0	-1	?	1	0	?	?	?	?

Note that when $T_i > T_j$ and $\delta_i = 1$, but δ_j is missing, $\psi_k(T_i, \delta_i, T_j, \delta_j)$ will take value 1 if $\delta_j = 0$ and value 0 if $\delta_j = 1$. Hence, in order to retrieve the best possible information we assign weight $(1 + 0)/2 = 1/2$ in this case. Similarly, when $T_i > T_j$ and $\delta_i = 0$, but δ_j is missing, $\psi_k(T_i, \delta_i, T_j, \delta_j)$ will take value -1 if $\delta_j = 1$ and value 0 if $\delta_j = 0$. Hence, we assign value $-1/2$ to the kernel in this case. Now, we redefine the kernel as follows

$$
\psi_{km}(T_i,\delta_i,O_i,T_j,\delta_j,O_j) = \begin{cases}
1 & \text{if } T_i > T_j,\ \delta_i = 1, \delta_j = 0, O_i = 1, O_j = 1 \\
 & \text{or } T_i < T_j,\ \delta_i = 0, \delta_j = 1, O_i = 1, O_j = 1 \\
1/2 & \text{if } T_i > T_j,\ \delta_i = 1, O_i = 1, O_j = 0 \\
 & \text{or } T_i < T_j,\ \delta_i = 0, O_i = 1, O_j = 0 \\
 & \text{if } T_i > T_j,\ \delta_j = 0, O_i = 0, O_j = 1 \\
 & \text{or } T_i < T_j,\ \delta_j = 1, O_i = 0, O_j = 1 \\
-1/2 & \text{if } T_i > T_j,\ \delta_i = 0, O_i = 1, O_j = 0 \\
 & \text{or } T_i < T_j,\ \delta_i = 1, O_i = 1, O_j = 0 \\
 & \text{if } T_i > T_j,\ \delta_j = 1, O_i = 0, O_j = 1 \\
 & \text{or } T_i < T_j,\ \delta_j = 0, O_i = 0, O_j = 1 \\
-1 & \text{if } T_i > T_j,\ \delta_i = 0, \delta_j = 1, O_i = 1, O_j = 1 \\
 & \text{or } T_i < T_j,\ \delta_i = 1, \delta_j = 0, O_i = 1, O_j = 1 \\
0 & \text{otherwise.}
\end{cases}
$$

Let U_{km} be the corresponding U-statistic. Then the expectations of U_{km} is given by

$$
\begin{aligned}
E(U_{km}) &= 2p^2\phi + 4p^2 \int_0^\infty S(t)dS_1(t), \\
&= 0,\ under\ \ H_0,
\end{aligned}
$$

and the asymptotic variance of U_{km} under H_0 is

$$
Var(U_{km}) = \frac{4}{3}p^2\phi(1-\phi) + \frac{1}{3}p(1-p).
$$

Note that when $p = 1$, the variance simplifies to $\frac{4}{3}\phi(1-\phi)$, which is given in Dewan *et al.* (2004).

Also, $E(U_{km}) \neq 0$ under H_1. From the results on U-statistics it follows that U_{km} has asymptotic normal distribution for large n.

Theorem 1 *Under H_0, $\sqrt{n}U_{km}$ converges in distribution to $N(0,\sigma^2)$ as $n \to \infty$, where $\sigma^2 = \frac{4}{3}p^2\phi(1-\phi) + \frac{1}{3}p(1-p)$.*

In practice, p, ϕ are generally unknown and can be replaced by their consistent estimators, $\sum_{i=1}^n I(O_i = 1)/n$, and $\sum_{i=1}^n I(\delta_i = 1)/n$, respectively. Hence, a test procedure for testing H_0 against H_1 is then: reject H_0 at $100\alpha\%$ level of significance if $\mid \sqrt{n}U_{km}/\hat{\sigma} \mid$ is larger than $z_{1-\alpha}$, cut-off point of standard normal distribution, where $\hat{\sigma}^2$ is a consistent estimator of σ^2 got by replacing p, ϕ by $\hat{p}, \hat{\phi}$.

For computational purposes, it is necessary to express U_{km} as a function of ranks. Let $n_1 = \sum_{i=1}^n I(\delta_i = 1, O_i = 1)$, $n_2 = \sum_{i=1}^n I(\delta_i = 0, O_i = 1)$, and $n_3 = \sum_{i=1}^n I(O_i = 0)$ represent numbers of observations in three groups - causes are observed to be 1, causes are observed to be 0 and causes are not observed, respectively. Let the corresponding ordered times in each group be represented by $X_{(1)}, X_{(2)}, ..., X_{(n_1)}$, $Y_{(1)}, Y_{(2)}, ..., Y_{(n_2)}$, and $Z_{(1)}, Z_{(2)}, ..., Z_{(n_3)}$, respectively. Let R_i denote the combined rank of $X_{(i)}$ in the ordered arrangement of (n_1+n_2) samples of type X and Y, S_i denote the combined rank of $X_{(i)}$ in the ordered arrangement of (n_1+n_3) samples of type X and Z, and Q_i denote the combined rank of $Y_{(i)}$ in the ordered arrangement of (n_2+n_3) samples of type Y and Z. Then, the number of observations for which $\psi_{km}(.)$ takes value

$$\begin{aligned}
\#(+1) &= \sum_{i=1}^{n_1}(R_i - i)O_i\delta_i, \\
\#(-1) &= n_1n_2 - \sum_{i=1}^{n_1}(R_i - i)O_i\delta_i, \\
\#(+1/2) &= \sum_{i=1}^{n_1}(S_i - i)O_i\delta_i + n_2n_3 - \sum_{i=1}^{n_2}(Q_i - i)O_i(1-\delta_i), \\
\#(-1/2) &= n_1n_3 - \sum_{i=1}^{n_1}(S_i - i)O_i\delta_i + \sum_{i=1}^{n_2}(Q_i - i)O_i(1-\delta_i).
\end{aligned}$$

Thus, the expression U_{km} in terms of ranks is

$$\begin{aligned}
\binom{n}{2}U_k &= 2\sum_{i=1}^{n_1}(R_i - i) - n_1n_2 + \frac{1}{2}[2\sum_{i=1}^{n_1}(S_i - i) - 2\sum_{i=1}^{n_2}(Q_i - i) + n_2n_3 - n_1n_3] \\
&= 2\sum_{i=1}^{n_1}(R_i - i) + \sum_{i=1}^{n_1}(S_i - i) - \sum_{i=1}^{n_2}(Q_i - i) - n_1n_2 + \frac{n_2n_3}{2} - \frac{n_1n_3}{2}. \qquad (1)
\end{aligned}$$

2.2 Testing independence vs $PQD(\delta, T)$

Consider testing H_0 against H_2. The U-statistic proposed by Dewan *et al.* (2004) is

$$U_{PQD} = \frac{1}{\binom{n}{2}} \sum_{1\le i<j\le n} \psi_3(T_i, \delta_i, T_j, \delta_j),$$

where

$$\psi_{PQD}(T_i, \delta_i, T_j, \delta_j) = \begin{cases} 1 & \text{if } T_i > T_j,\ \delta_i = 1 \\ & \text{or } T_j > T_i,\ \delta_j = 1 \\ 0 & \text{otherwise.} \end{cases}$$

If δ is missing for some units then $\psi_{PQD}(T_i, \delta_i, T_j, \delta_j)$ cannot be defined for all pairs. The following Table 2 shows the pairs for which the kernel is defined completely and also the cases where it is not defined.

Table 2: Values taken by the kernel ψ_{PQD} for various combinations of the pairs (T_i, δ_i) and (T_j, δ_j)

(δ_i, δ_j)	(1,1)	(1,0)	(1,m)	(0,1)	(0,0)	(0,m)	(m,1)	(m,0)	(m,m)
$T_i > T_j$	1	1	1	0	0	0	?	?	?
$T_i \leq T_j$	1	0	?	1	0	?	1	0	?

As in the earlier subsection, we define a modified kernel to take into account missing causes as follows.

$$\psi_{PQDm}(T_i, \delta_i, O_i, T_j, \delta_j, O_j) \;=\; \begin{cases} 1 & \text{if } T_i > T_j,\ \delta_i = 1, O_i = 1 \\ & \text{or } T_j > T_i,\ \delta_j = 1, O_j = 1 \\ 1/2 & \text{if } T_i > T_j,\ O_i = 0 \\ & \text{or } T_j > T_i,\ O_j = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let U_{PQDm} be the corresponding U-statistic. Then the expectations of U_{PQDm} are given by

$$\begin{aligned} E(U_{PQDm}) &= 2p \int_0^\infty S_1(t)dF(t) + \frac{(1-p)}{2} \\ &= p\phi + \frac{(1-p)}{2}, \quad under \;\; H_0, \end{aligned}$$

and $E(U_{PQDm}) > p\phi + \frac{(1-p)}{2}$ under H_2. The asymptotic variance of U_{PQDm} under H_0 is

$$Var(\sqrt{n}U_{PQDm}) = \frac{4}{3}p^2\phi(1-\phi) + \frac{1}{3}p(1-p).$$

Note that when $p = 1$, the variance simplifies to $\frac{4}{3}\phi(1-\phi)$, which is given in Dewan *et al.* (2004).

THEOREM **2** *Under* H_0, $\sqrt{n}(U_{PQDm} - E(U_{PQDm}))$ *converges in distribution to* $N(0, \sigma^2_{PQDm})$, *where* $\hat{\sigma}^2_{PQDm} = \frac{4}{3}p^2\hat{\phi}(1-\hat{\phi}) + \frac{1}{3}\hat{p}(1-\hat{p})$, *as* $n \to \infty$.

We reject the null hypothesis for large values of $Z = \sqrt{n}(U_{PQDm} - E(\hat{U_{PQDmm}}))/\hat{\sigma}(U_{PQDm})$, where $E(\hat{U_{PQDm}})$ and $\hat{\sigma}(U_{PQDm})$ are obtained by replacing ϕ and p by their empirical estimators. As mentioned earlier, p can be replaced by its consistent estimator $\sum_{i=1}^n I(O_i = 1)/n$.

Let R_i^* denote the rank of T_i in the ordered observations $(T_1, T_2, ..., T_n)$. Then, it is easy to see that

$$\binom{n}{2} U_{PQDm} = \sum_{i=1}^n (R_i^* - 1)O_i\delta_i + \frac{1}{2}\sum_{i=1}^n (R_i^* - 1)(1 - O_i). \tag{2}$$

Large values of the statistic are significant.

Note that a one-sided test based on U_{km} can also be used for testing H_0 against H_2 since $E(U_{km}) \geq 0$ under H_2. In fact, this one-sided test uses data on both the T and δ in each pairwise comparison while U_{PQDm} uses only information on (T, δ) from one and T from the other in a pairwise comparison. As noted by Dewan *et al.* (2004) U_k is four times more efficient than U_2 (which is U_{PQD} here). In the next section we compare the empirical powers of these three tests and we will refer one-sided test based on U_{km} as U_{km1}.

3 Simulation study

We apply the proposed tests to simulated data from two parametric families of distributions and evaluate empirical powers.

Example 1: Parametric family of distributions of Dewan *et al.* (2004)
Consider the parametric family of distributions proposed in Dewan *et al.* (2004)

$$\begin{aligned} F_1(t) &= \phi F^a(t) \\ F_0(t) &= F(t) - \phi F^a(t), \end{aligned}$$

where $1 \leq a \leq 2$, $0 \leq \phi \leq 0.5$ and $F(t)$ is a proper distribution function. Note that $P(\delta = 1) = \phi$ and

$$\Phi_1(t) = \frac{\phi(1 - F^a(t))}{1 - F(t)}$$

which is an increasing function of t. For $a = 1$, $\Phi_1(t) = \phi$, that is, T and δ are independent and for $1 < a \leq 2$, $\Phi_1(t) > \phi$, that is T and δ are PQD , and hence H_2 holds. Let $F(t) = 1 - \exp\{-\lambda t\}$ be the overall distribution function. We simulated random samples by varying n, p, and (λ, a, ϕ) from the above distribution. Empirical level of significance and power were calculated by using 1000 replications for each combination of n, and (λ, a, ϕ). Table 3a gives the empirical level of significance and Table 3b give the empirical power of the three tests based on U_{km}, U_{km1} and U_{PQDm}.

Table 3a: Empirical level of significance of the three U-tests for a family of distributions of Dewan *et al.* **(2004)** $(\lambda, a) = (1, 1)$.

ϕ	p	$n = 25$			$n = 50$			$n = 100$		
		U_{km}	U_{km1}	U_{3m}	U_{km}	U_{km1}	U_{3m}	U_{km}	U_{km1}	U_{PQDm}
0.5	1	0.057	0.054	0	0.061	0.049	0.001	0.052	0.047	0.002
0.5	0.9	0.055	0.048	0.002	0.062	0.052	0.001	0.047	0.053	0.004
0.5	0.8	0.054	0.047	0.007	0.049	0.061	0.008	0.05	0.049	0.004
0.2	1	0.069	0.061	0.001	0.052	0.049	0.001	0.051	0.06	0.01
0.2	0.9	0.078	0.059	0.001	0.061	0.052	0.002	0.052	0.059	0.005
0.2	0.8	0.092	0.06	0.005	0.057	0.054	0.004	0.047	0.064	0.002

Table 3b: Empirical power of the three U-tests for a family of distributions of Dewan *et al.* **(2004)** $\lambda = 1$.

(a, ϕ)	p	$n = 25$			$n = 50$			$n = 100$		
		U_{km}	U_{km1}	U_{3m}	U_{km}	U_{km1}	U_{3m}	U_{km}	U_{km1}	U_{PQDm}
$(1.5, 0.5)$	1	0.421	0.546	0.049	0.690	0.812	0.201	0.942	0.974	0.575
$(1.5, 0.5)$	0.9	0.394	0.505	0.066	0.652	0.767	0.201	0.916	0.955	0.504
$(1.5, 0.5)$	0.8	0.365	0.489	0.079	0.609	0.711	0.208	0.894	0.938	0.453
$(1.8, 0.5)$	1	0.765	0.863	0.183	0.967	0.988	0.646	0.999	0.999	0.974
$(1.8, 0.5)$	0.9	0.713	0.802	0.181	0.947	0.975	0.545	0.998	0.999	0.917
$(1.8, 0.5)$	0.8	0.653	0.768	0.191	0.921	0.965	0.485	0.997	0.999	0.836

From the two tables it is clear that modified Kendall's statistic attains its level when roughly half of the failures are likely to be due to the first cause. The conclusions are valid even when 20 percent of the failure causes are not available. The power increases with increase in values of a and also with increase in sample size. The test has very good power even when $a = 1.5$. One should keep in mind the fact that the alternative of no independence is extremely general.

However, the test based on one sided version of the Kendall's τ performs much better than the test based on U_{PQDm}. The second test is extremely conservative. Dewan *et al.* (2004) observed that the test based on U_{PQD} is not efficient when $p = 1$, that is, none of the causes of failure are missing. The enteries for this test in the two tables confirm this observation. But given the fact that the level of significance attained is very low, it is able to detect alternatives reasonably well.

Example 2: Random sign censoring model (Cooke, 1996)

A random sign censoring (RSC), also known as an age-dependent censoring, is a model in which the lifetime of a unit (X) is censored by $Z = X - W\eta$, where W, $0 < W < X$ is the time at which a warning is emitted by the unit before its failure, and η is a random variable taking values $\{-1, 1\}$ and is independent of X. Hence $\eta = 1$ would lead to the censoring of the life time at $X - W$ giving $T = Z$ and $\delta = 0$ and $\eta = -1$ will lead to the observation of complete lifetime X, giving $T = X$ and $\delta = 1$. Assume that X has exponential distribution with parameter λ. In this case, $P(Z \geq t, Z < X) = P(X - W \geq t, \eta = 1)$ and $P(X \geq t, X < Z) = P(X \geq t)P(\eta = -1)$. This gives $\Phi_1(t) = P(T \geq t, \delta = 1)/P(T \geq t) = P(X \geq t, \eta = -1)/P(X \geq t, X - W\eta \geq t)$. When X is exponentially distributed with parameter λ and $W = aX, 0 < a < 1$,

$$\Phi_1(t) = (1 + p/(1-p)exp\{-\lambda t(a/(1-a))\})^{-1},$$

where $p = P(\eta = 1) = 1 - P(\eta = -1)$, leading to the increasing (and hence PQD) nature of $\Phi_1(t)$ in t. As a goes to zero $\Phi_1(t)$ goes to $(1-p)$, that is T, δ are independent. Hence, to evaluate the empirical level of significance we choose a very closen to zero.

The value a close to zero corresponds to independence of T and δ and $a > 0$ gives $\Phi_1(t)$ as an increasing function of t implying T and δ are PQD. For simulation purposes, we consider two values of $a = 0.00001$, and 0.5. Test based on U_{km} almost attains its level even for sample sizes as small as $n = 25$. This test has good power for $n = 100$. The test based on U_{km1} has a slightly higher level as well as higher power. But the test based on U_{PQDm} is a very conservative test. It has low power even for $n = 100$. One-sided test based on U_{km1} is definitely a better choice for detecting PQD alternatives.

Table 4: Empirical power of the three U-tests for random sign censoring model ($\lambda = 1$)

(a, ϕ)	p	$n = 25$			$n = 50$			$n = 100$		
		U_{km}	U_{km1}	U_{3m}	U_{km}	U_{km1}	U_{3m}	U_{km}	U_{km1}	U_{PQDm}
$(0.00001, 0.5)$	1	0.057	0.067	0.002	0.061	0.063	0	0.052	0.065	0
$(0.00001, 0.5)$	0.9	0.055	0.065	0.001	0.062	0.059	0.001	0.047	0.053	0
$(0.00001, 0.5)$	0.8	0.054	0.059	0.005	0.049	0.059	0.008	0.05	0.048	0.006
$(0.00001, 0.7)$	1	0.056	0.057	0	0.067	0.063	0	0.056	0.057	0
$(0.00001, 0.7)$	0.9	0.063	0.069	0.005	0.059	0.064	0.003	0.055	0.06	0.003
$(0.00001, 0.7)$	0.8	0.062	0.067	0.009	0.057	0.063	0.006	0.052	0.056	0.006
$(0.5, 0.5)$	1	0.352	0.474	0.029	0.577	0.702	0.11	0.85	0.923	0.381
$(0.5, 0.5)$	0.9	0.321	0.442	0.05	0.513	0.647	0.135	0.793	0.884	0.358
$(0.5, 0.5)$	0.8	0.286	0.41	0.066	0.473	0.609	0.121	0.751	0.848	0.306
$(0.5, 0.7)$	1	0.32	0.423	0.029	0.489	0.61	0.091	0.759	0.847	0.303
$(0.5, 0.7)$	0.9	0.266	0.378	0.045	0.449	0.564	0.105	0.705	0.811	0.249
$(0.5, 0.7)$	0.8	0.229	0.348	0.056	0.396	0.516	0.112	0.655	0.755	0.233

4 Discussion

If the causes of failure are missing then one should not ignore the corresponding failure times and apply the available tests with reduced sample size (which is random). The available information on failure times of individuals whose failure causes are not known also provides useful input regarding departures from independence of T and δ.

From the simulation studies it is clear that the two-sided Kendall's test is performing well for both the families of distributions for sample sizes as small as 25 and even when 20 percent of the causes of failure are not known. Even when all causes are known Dewan *et al.* (2004) observed that the test based on Kendall's τ is 4 times more efficient than the test based on U_{PQD}. Here too we recommend the use of the one-sided Kendall's τ for testing independence against PQD. One obvious reason is that Kendall's τ uses information on T, δ for both the individuals. Powers are higher in the case of first parametric family of distributions compared to RSC model for all sample sizes.

We are also looking at competing tests using the theory of enlarged filtration.

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