Abstract

This paper investigates how neighbourhood effects interacting with income inequality affect poor people’s ability to access basic facilities like health care services, schooling, and so on. We model this interaction by integrating consumers’ income distribution with the spatial distribution of their location and explore the consequences of an increase in income inequality or variations in the neighbourhood characteristics on the welfare of the rich and poor in general, and their access to market in particular. We find that, in general, the impact will be non-monotonic owing to an interesting trade-off between the provision effect and the price effect. On the one hand, there is the positive ‘provision effect’: higher valuation of the rich attracts the supplier to enter into the neighbourhood, allowing the poor who live sufficiently close by to access the service. On the other hand, there is the negative ‘price effect’: the service provider charges a higher price higher is the income or larger is the proportion of the rich in the neighbourhood. In the extreme, there exists the possibility of complete exclusion of poor from the market: the service provider caters only to the rich and the poor has absolutely no market access. We have identified the higher income gap between rich and poor as the key factor that exposes the poor to this complete exclusion possibility.

Keywords: Inequality; Neighbourhoods; Market Access; Complete Exclusion.
1 Introduction

The key idea explored in this paper is the following. Though being poor in itself is a huge disadvantage, the situation might be influenced considerably by the type of neighbourhood the poor lives in. The reason is that private establishments like educational institutions, health care facilities or credit institutions take both the location and income mix of people into account while making strategic decisions like whether to enter into the neighbourhood at all, and, upon entry, what price and quality to choose for their products and services.\footnote{Contrary to the conventional belief, private establishments are a huge presence in the education and health care sectors of the less developed countries. In India Dreze and Sen (2002) estimate that, even by 1994, some 30% of all 6-14 year olds in rural areas were enrolled in private schools, while 80% or more attended private schools in urban areas, including low-income families. In the poor urban, periurban and rural areas surveyed by Tooley and Dixon (2006), the vast majority of school children were found to be in private schools: 75% in Lagos State, Nigeria, 65% in Ga, Ghana and in Hyderabad, India, and roughly 50% in Mahbubnagar, rural Andhra Pradesh, India. In Lahore, Pakistan, Alderman et al. (2003) estimates 51% of children from families earning less than $1 a day attend private schools, while Andrabi et al. (2010) reports that 35% of primary enrollment in Pakistan was in the private sector by 2000. Similarly on health, World Health Organization (2011) reports the following figures on private expenditure on health as a percentage of total health expenditure in 2009: Bangladesh 68%, Brazil 54%, Chile 53%, China 50%, Egypt 59%, Ghana 47%, Guatemala 63%, India 67%, Kenya 66%, Nigeria 64%, Pakistan 67%, Sierra Leone 93%.}

Is staying with the rich a virtue for the poor or is it a vice? Are the poor living in poor neighbourhoods better-off because living in a richer one costs too much? Or, are they significantly worse-off as they do not even have access to many basic facilities? These are the kinds of questions we are interested in exploring in this paper.

Answers to these questions depend not just on the costs relative to income, but also on the ease of access of the facilities. The reason is that certain goods and services are required at regular intervals so that distance becomes an important factor. In the less developed countries distance from schools is an important factor leading to high drop-out rates or low school enrollment.\footnote{There is strong empirical evidence showing that distance is a major predictor of school enrollment or drop-out rates in less developed countries; see, for example, Alderman et al. (2001), Andrabi et al. (2010), Colclough et al. (2000), Glick and Sahn (2006), Handa (2002), and Huisman and Smits (2009).} Similarly distance from the nearby health care facility is a major reason resulting in higher mortality of both mother and child during child birth in rural areas of developing countries.\footnote{Almost any study of health seeking behaviour in developing (and developed) countries finds some estimate of the distance or travel cost as an important and significant determinant of the choice of health care provider; see, for example, Acton (1975), Kessler and McClellan (2000), Kloos (1990), Stock (1983), and Tay (2003).} How readily a product or service is available is thus determined by the neighbourhood an individual lives in. So it is the interaction of the two, the individual’s...
income and his postcode, that determines his welfare.

As detailed in the following subsection, there is a substantial body of evidence showing how neighbourhood poverty affects poor people’s ability to access facilities such as health care and schooling. Although the evidence is compelling, there seems to be very little analytical research to understand how neighbourhood effects interacting with income inequality might affect poor people’s ability to access these basic facilities. This paper makes an early attempt to model this interaction by integrating consumers’ income distribution with the spatial distribution of their location and explores the consequences of an increase in income inequality or variations in the neighbourhood characteristics on the welfare of the rich and poor in general, and their access to market in particular.

We consider a homogeneous product or service (potentially) supplied to a neighbourhood by a single private establishment. The inequality-neighbourhood interaction is captured by the spatial structure where the neighbourhood is a linear city across which the consumers are uniformly distributed with rich and poor consumers living side by side. The preference structure reflects the higher willingness to pay of the richer consumers and the consumers’ reluctance to travel farther to access the product or service under consideration. The industrial structure is characterized by the presence of a fixed cost of production. The set-up is a two-stage game. In the first stage, the potential provider of the product or service decides whether to enter into the neighbourhood or not; upon entry, in the second stage, the provider chooses its price. In this set-up we explore the interaction of income inequality with the neighbourhood effects in determining the market outcomes and its consequences on the market access and welfare of the rich and poor.

We identify an interesting trade-off affecting the impact of income inequality or variations in the neighbourhood characteristics. On the one hand, the poor benefits from the presence of the rich: the higher willingness to pay for the service of the rich leads the service provider to enter into the neighbourhood, allowing the poor who live sufficiently close by to access the service. This is the positive ‘provision effect’. On the other hand, the higher the income or larger the proportion of rich, the higher is the equilibrium price, hurting the poor. This is the negative ‘price effect’. In the extreme, if income or proportion of the rich is high enough, the service provider completely abandons the poor and caters only to the rich. This is the negative ‘exclusion effect’. Interestingly, these neighbourhood externalities do not work only in one direction; that is, the presence of poor in the neighbourhood also generates similar effects on their richer neighbours.

For the market access and welfare of the poor we find that both the neighbourhood
characteristics – rich income and proportion of rich in the neighbourhood – work in the same direction: while the provision effect is positive, the exclusion and price effects are negative. For the rich we find that the two neighbourhood characteristics – poor income and proportion of poor in the neighbourhood – work almost in the opposite directions. While the provision effect is positive, the price effect of an increase in poor income is negative. On the other hand, the provision effects of an increase in the proportion of poor are negative, but the price effect is positive. Only the inclusion effect works in the same positive direction for both the neighbourhood characteristics.

The trade-off mentioned above generates similar (potential) non-monotonic impact arising from two variants of inequality – increasing rich income while keeping poor income and proportion of poor fixed, or increasing proportion of rich while keeping rich and poor incomes fixed. When either of these two variants of inequality increases we find that the rich and poor are affected almost similarly: the provision effect is positive while the exclusion and price effects are negative. In order to examine the role of inequality in its purest form, we also analyze the effect of a mean-preserving spread: keeping the proportions of rich and poor fixed we increase rich income together with a decrease in poor income such that the average income of the society remains unchanged. Here we find that an increase in mean-preserving spread affects market access and welfare of the rich and poor very differently. For the poor the overall effect is negative: while the provision effect is neutral, the exclusion effect is negative and, when the poor has a positive market access, the neutral price effect is dominated by the negative valuation effect (consumer’s valuation of the product increases with his income). For the rich while the provision effect is either positive or neutral, the exclusion effect is negative; also the positive valuation effect dominates either the neutral or the negative price effect.

We have also identified the possibility of complete exclusion of the poor from the market: a scenario where the service provider completely ignores the presence of the poor and chooses the price considering as if there are only rich individuals residing in the neighbourhood. Perhaps it is reasonable to expect that this can happen at a very low level of poor income. But, surprisingly enough, we find that even at moderate to high levels of income the poor people are not immune from this unfortunate possibility. We have isolated the higher income gap between the rich and poor as the key factor that exposes the poor to the complete exclusion possibility. We have also found that the poor are more likely to be completely excluded when they are a minority: the provider may completely ignore them even when the rich are not ultra rich just because the rich are more in number.
There exists a substantial body of literature addressing the effects of income inequality on a variety of socioeconomic outcomes. For example, higher inequality is found to be positively correlated with higher infant mortality (Waldman, 1992), lower economic growth (Alesina and Rodrik, 1994; Persson and Tabellini, 1994), violent crime (Fajnzylber et al., 2002), subversion of institutions (Glaeser et al., 2003), and so on. This paper complements this literature by exploring the impact of income inequality working through the trade-off of price and provision effects. Atkinson (1995) is the only work that we are aware of which investigates the implications of inequality operating through industrial structure. But while Atkinson (1995) examines how considerations of firm behaviour and industrial structure enter the determination of poverty, he does not consider the neighbourhood effects. The inequality-neighbourhood interaction is the key feature that gets highlighted in our paper.

The idea that people with higher income generally have higher willingness to pay and that firms do take this into account while making strategic decisions was developed by Gabszewicz and Thisse (1979) and extended by Shaked and Sutton (1982, 1983). Our specification allows consumers to differ with respect to both their income and location. The basic horizontal product differentiation model was introduced by Hotelling (1929) and later developed by Salop (1979). The literature on industrial organization that follows these seminal works (for example, Economides, 1993; Neven and Thisse, 1990) looks at product specifications combining both the vertical and horizontal characteristics. But, understandably, the industrial organization literature does not explore the implications of income inequality.

1.1 Motivational Evidence

As we have mentioned above, there exists substantive empirical literature that emphasizes the role of neighbourhood factors in affecting poor people’s ability to access various services such as health care and schooling. Let us consider health care first. An established body of studies has demonstrated that neighbourhood indicators of socioeconomic status predict individual mortality. For example, Stafford and Marmot (2003) and Yen and Kaplan (1999) find that low-income adults in advantaged neighbourhoods might experience a lower mortality risk than low-income adults in disadvantaged neighbourhoods because they benefit from the collective resources in their neighbourhoods. On the other hand, Roos et al. (2004), Veugelers et al. (2001) and Winkleby et al. (2006) show that low-income adults in advantaged neighbourhoods experience a higher risk of dying because of relative deprivation and/or low relative social standing. Analyzing a set of 85 developing country Demographic

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4For an extensive review of this literature see Atkinson and Bourguignon (2000).
and Health Surveys, Montgomery and Hewett (2005) find that both household and neighbour,
bhood living standards make a significantly important difference to health in the cities and
towns of developing countries. They report striking differentials in health depending 
on the region: poor city dwellers often face health risks that are nearly as bad as what is 
seen in the countryside and, sometimes, the risks are decidedly worse. For Rio de Janeiro, 
Brazil, Szwarcwald et al. (2002) find higher neighbourhood mean poverty and higher vari-
ance both act to increase infant mortality and adolescent fertility rates at the census tract 
level. In Delhi, India, Das and Hammer (2005) find that doctors located in the poorest 
neighbourhoods are one full standard deviation worse than doctors located in the richest 
neighbourhoods. In India, while the rural poor are underserved, at least they can access the 
limited number of government-supported medical facilities that are available to them; the 
urban poor fares even worse because they cannot afford to visit the private facilities that 
thrive in India’s cities (PriceWaterhouse Coopers, 2007).

Similarly on education, based on observed spatial variations in school performance and 
and drop-out rates, an extensive amount of research has identified that neighbourhood socio-
edconomic characteristics affect various aspects of educational outcomes. Compared to adults 
from wealthier neighbourhoods, those from relatively disadvantaged neighbourhoods tend 
to have lower test scores and grades (Dornbusch et al., 1991; Gonzales et al., 1996; Tur-
ley, 2003), a higher risk of dropping out of school (Aaronson, 1998; Brooks-Gunn et al., 
1993; Connell et al., 1995; Crane, 1991; Ensminger et al., 1996), a lower likelihood of post-
secondary education (Duncan, 1994), and complete fewer years of schooling (Corcoran et al., 
identifies positive and significant neighbourhood effects on school completion of children. 
Montgomery et al. (2005) find that educational attainment of poor children in urban Egypt 
and in the slums of Allahabad, India, depend not only on the standards of living of their own 
families, but also on the economic composition of their local surroundings. For a sample of 
rural households in Ethiopia, Weir (2007) finds that children’s schooling benefit significantly 
from the education of women in their neighbourhood.

Note that while these empirical studies indicate that there exists causal relationship 
between neighbourhood factors and health/education indicators, the direction of the relation-
ship could go either way. This is indeed consistent with the theoretical results that we 
derive in our paper.

For instance, they find that in the slums of Nairobi rates of child mortality substantially exceed those 
found elsewhere in Nairobi; on the other hand, the slum residents are better shielded from risk than rural 
dwellers with respect to births attended by doctors, nurses and trained midwives.
The paper is organized as follows. Section 2 outlines the set-up and section 3 analyzes the equilibrium characterizing different possibilities. We use this characterization to examine the impact of variations in neighbourhood characteristics on the market access and welfare of the poor and the rich in sections 4 and 5 respectively. In section 6 we investigate the effects of income inequality. We highlight the possibility of complete exclusion of the poor in section 7. We discuss several interesting policy implications in section 8 and finally conclude in section 9.

2 The Model

2.1 The Set-up

Our model adapts the frameworks of Hotelling (1929) and Salop (1979). Consider a neighbourhood that can be represented as lying on a line segment of some finite length. Two types of consumers, rich and poor, are uniformly distributed along the length of the neighbourhood: there are $f$ proportion of poor with income $Y_P$ and $(1 - f)$ proportion of rich with income $Y_R$. Obviously $Y_R > Y_P$. The total number of consumers is normalized to 1.

There is a single private establishment located in the middle of this linear neighbourhood providing a homogeneous product or service. Examples of such establishments are private schools, hospitals, banks, and so on. For the sake of brevity let us refer to the establishment as a firm.

Each consumer buys either one unit of the homogeneous product or service from this single (monopolist) firm, or does not buy the product at all. Let $\theta Y$ be the gross utility a consumer with income $Y$ enjoys from consuming the product. Here $\theta > 0$ is a preference parameter indicating consumers’ valuation of the product. Since $\theta Y_R > \theta Y_P$, this formulation of gross utility captures the feature that willingness to pay is higher for the rich. This formulation also ensures that the preference is non-homothetic. Preference non-homotheticity and income heterogeneity imply that changes in prices may affect rich and poor consumers differently. This allows us to explore the role of income distribution.

A consumer located at a distance $x$ from the firm has to travel this distance to access the product or service, and he incurs a travel or transportation cost of $tx$. Of course he has to pay the price $p$ charged by the firm. Hence the net utility of a consumer with income $Y$.

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\(^6\)Our adaptation is similar to Bhaskar and To (1999, 2003) and Brekke et al. (2008).

\(^7\)It is well understood that when preferences are identical and homothetic, income distribution does not matter.
located at a distance $x$ from the firm and purchasing the product is given by

$$u(x, Y) = \theta Y - p - tx.$$  

If a consumer does not buy the product, his utility, the reservation utility, is $0$.

This formulation of the utility function helps to model the interaction of neighbourhood effects with income inequality in a simple and tractable way. While the gross utility captures the higher willingness to pay of the rich, the presence of travel cost reflects the disutility if the facility is not available nearby in the neighbourhood. Unlike the industrial organization literature where distance reflects horizontal product differentiation, we treat the distance literally as physical distance from the facility. For facilities like schools or hospitals the importance of distance or accessibility is undeniable.

Production entails a fixed cost. In order to produce any output at all, the firm must incur a fixed cost $F$. Further, there is a marginal cost of production, $c$, which is independent of output. Profit of the firm charging a price $p$ is then given by

$$\pi(p) = [p - c]D(p) - F,$$

where $D(p)$ denotes demand faced by the firm as a function of the price it charges. Given the spatial structure, we elaborate in the next subsection how demand depends on the firm’s choice of price, $p$.

The set-up is a two-stage game. In the first stage, the firm decides whether to enter or not. If it decides to enter then, in the second stage, the firm chooses its price.

### 2.2 General Demand Structure

Suppose the firm charges a price $p$. Since it is the only firm in the neighbourhood, demand from each type of consumer, rich or poor, is determined by the distance of the consumer indifferent between buying and not buying from the firm. Let $\delta_R$ and $\delta_P$ denote the distances of the rich and the poor consumers, respectively, who are indifferent between buying and not buying from the firm. Clearly $\delta_i$ is determined from $u(\delta_i, Y) = 0$, implying

$$\delta_i = \frac{\theta Y_i - p}{t}, \quad i = R, P.$$

It follows that the firm’s total demand from the rich is $2\delta_R$ while that from the poor is $2\delta_P$. Since there are $f$ proportion of poor and $(1 - f)$ proportion of rich, in general, the total demand facing the firm is given by

$$D(p) = f(2\delta_P) + (1 - f)(2\delta_R) = \frac{2}{t} \left[ \theta (fY_P + (1 - f)Y_R) - p \right].$$
Note that, in general, demand responds positively to the average income of the neighbourhood, \( fY_P + (1 - f) Y_R \). An increase in travel cost makes it costlier to access the facilities and reduces the demand as a result. The price response to demand is given by \( \frac{\partial D}{\partial p} = -\frac{2}{t} \). Note that an increase in travel cost reduces the price response to demand.

### 3 Equilibrium

There are three possibilities that can arise in equilibrium:

1. Possibility 1: Both the rich and the poor are served;
2. Possibility 2: Only the rich are served (the poor are completely excluded);
3. Possibility 3: The firm does not enter into the neighbourhood.

We normalize the firm’s profit under Possibility 3 to zero. We first determine the firm’s optimal prices and the maximum profits it earns under the two other possibilities. The firm obviously chooses the option that generates the highest profit. Hence, in what follows, we determine the parameter configurations under which each equilibrium possibility occurs.

Consider first Possibility 1 which occurs if the firm decides to serve both the rich and the poor. Then the total demand it faces is given by the general demand function derived above

\[
D(p) = f (2\delta_P) + (1 - f) (2\delta_R) = \frac{2}{t} [\theta (fY_P + (1 - f) Y_R) - p].
\]

The firm chooses its price \( p \) to maximize its profit

\[
\pi(p) = \frac{2}{t} [\theta (fY_P + (1 - f) Y_R) - p] \cdot (p - c) - F.
\]

It follows from the first-order condition of profit maximization\(^8\) that the optimal price is given by

\[
p^* \bigg|_{\text{Possibility 1}} = \frac{\theta (fY_P + (1 - f) Y_R) + c}{2}.
\]

Then, under Possibility 1, the maximum profit the firm earns is

\[
\pi^* \bigg|_{\text{Possibility 1}} = \frac{2}{t} \left[\frac{\theta (fY_P + (1 - f) Y_R) - c}{2}\right]^2 - F.
\]

\(^8\)Note that \( \frac{\partial^2 \pi}{\partial p^2} = -\frac{4}{t} < 0 \), implying that the profit function is locally concave in \( p \).
Next consider Possibility 2 which occurs if the firm decides to serve only the rich. Then it faces the following total demand
\[
D(p) = (1 - f) (2\delta_R) = 2 (1 - f) \left( \frac{\theta Y_R - p}{t} \right).
\]
From the first-order condition of profit maximization\(^9\) it follows that the optimal price is
\[
P^*|_{\text{Possibility 2}} = \frac{\theta Y_R + c}{2},
\]
so that the maximum profit the firm earns under Possibility 2 is
\[
\pi^*|_{\text{Possibility 2}} = \frac{2 (1 - f)}{t} \left( \frac{\theta Y_R - c}{2} \right)^2 - F.
\]
Possibility 1 will be an equilibrium outcome if and only if the following conditions are satisfied: \(\pi^*|_{\text{Possibility 1}} \geq \pi^*|_{\text{Possibility 2}}\), \(\pi^*|_{\text{Possibility 1}} \geq 0\), and \(\delta_P|_{\text{Possibility 1}} \geq 0\), that is, the market access of the poor is non-negative.\(^{10}\) The following proposition identifies the parameter values under which Possibility 1 is an equilibrium outcome.

**Proposition 1.** In equilibrium the firm charges a price
\[
P^*|_{\text{Possibility 1}} = \frac{\theta (f Y_P + (1 - f) Y_R) + c}{2}
\]
and serves both the rich and the poor if and only if the following conditions hold:
\[
\left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) \leq \left( 1 + \frac{1}{\sqrt{1-f}} \right) \left( \frac{\theta Y_P - c}{\sqrt{tF}} \right),
\]
and
\[
f \cdot \left( \frac{\theta Y_P - c}{\sqrt{tF}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) \geq \sqrt{2}.
\]
**Proof:** See Appendix, section 10.1.

The parameter combination \(\frac{\theta Y_R - c}{\sqrt{tF}}\) captures, in a nutshell, the valuation of the product relative to the costs: marginal, fixed and travel costs. So Proposition 1 says that, in equilibrium, the firm serves both the rich and the poor if and only if the average valuation of the product in the neighbourhood is high enough, and the income gap between rich and poor

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\(^9\)Note that \(\frac{\partial^2 \pi}{\partial p^2} = -\frac{4 (1 - f)}{t} < 0\), implying that the profit function is locally concave in \(p\).

\(^{10}\)Since \(\delta_R > \delta_P\) (as \(Y_R > Y_P\)), \(\delta_P \geq 0\) ensures that \(\delta_R > 0\).
is low enough. Figure 1 illustrates this proposition. Consider, for example, a specific value of $\frac{\theta Y_P - c}{\sqrt{tF}}$ marked as $\left(\frac{\theta Y_P - c}{\sqrt{tF}}\right)_{\text{Config}_2}$ in Figure 1. For this specific value of $\frac{\theta Y_P - c}{\sqrt{tF}}$, all the $\frac{\theta Y_R - c}{\sqrt{tF}}$ and $f$ combinations in between the solid blue curve and the green curve satisfy both the conditions in Proposition 1.

The equilibrium price reflects the fact that the firm serves both the rich and the poor: it is affected by both the rich income and the poor income. In particular the price is positively related to the average neighbourhood income, $fY_P + (1 - f)Y_R$. As expected, price increases with marginal cost. While the price is not affected by the fixed cost or the travel cost, they affect the relative valuation of the product. If these costs are too high in comparison with the average neighbourhood gross valuation, $\theta (fY_P + (1 - f)Y_R)$, condition (2b) may not hold so that the firm may make a net loss by serving both the rich and the poor in the neighbourhood.

Note that the poor has some market access only under Possibility 1. The following corollary, that follows from the two conditions under Proposition 1, establishes a lower bound on the poor’s income to have any market access.

**Corollary 1.** For the poor to have any market access it is necessary that the following condition holds:

$$\frac{\theta Y_P - c}{\sqrt{tF}} \geq \frac{\sqrt{2}}{(1 + \sqrt{1 - f})}.$$  

**Proof:** See Appendix, section 10.2.

In Figure 1 the upward sloping black curve shows this lower bound on the poor’s income. It is interesting to observe that this lower bound on the poor’s income decreases with the proportion of rich in the neighbourhood $(1 - f)$. On the other hand, this lower bound increases with all the cost parameters.

Now we examine when will Possibility 2 be an equilibrium outcome. That will be the case if and only if the following conditions are satisfied: $\pi^*_1|_{\text{Possibility 2}} \geq \pi^*_1|_{\text{Possibility 1}}$, $\pi^*_2|_{\text{Possibility 2}} \geq 0$, and $\delta_P|_{\text{Possibility 2}} < 0$, that is, the poor does not have any market access. The following proposition identifies the parameter values under which Possibility 2 is an equilibrium outcome.
Proposition 2. In equilibrium the firm charges a price

\[ p^\star_{\text{Possibility 2}} = \frac{\theta Y_R + c}{2} \quad (4) \]

and serves only the rich consumers (the poor are completely left out) if and only if the following conditions hold:

\[
\left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) > \left( 1 + \frac{1}{\sqrt{1-f}} \right) \left( \frac{\theta Y_P - c}{\sqrt{tF}} \right), \quad (5a)
\]

and

\[
\frac{\theta Y_R - c}{\sqrt{tF}} \geq \sqrt{\frac{2}{1-f}}. \quad (5b)
\]

Proof: See Appendix, section 10.3.

Proposition 2 says that, in equilibrium, the firm serves only the rich consumers and completely excludes the poor if the income gap between rich and poor is high enough and the relative valuation of the product is high enough for the rich. In Figure 1 the red curve demonstrates the lower bound on rich income given by the second condition in Proposition 2. It demonstrates further that for the specific value of \( \frac{\theta Y_R - c}{\sqrt{tF}} \) marked as \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right)_{\text{Config2}} \), all the \( \frac{\theta Y_R - c}{\sqrt{tF}} \) and \( f \) combinations above the solid blue curve satisfy both the conditions of Proposition 2.

Since the firm serves only the rich, the equilibrium price is not affected by either the poor income or the proportion of poor in the neighbourhood. Instead they affect condition (5a). For example, if the poor income is high or the poor are relatively more in number, then condition (5a) might get reversed. Similar to Possibility 1, the fixed cost or the travel cost does not affect the equilibrium price but influences the profitability condition (5b). If the gross valuation of the rich, \( \theta Y_R \), is not high enough as compared to these costs, then the firm may make a net loss by serving even the rich community only.

While propositions 1 and 2 define parameter values under which possibilities 1 and 2, respectively, are equilibrium outcomes, there exists the third possibility that the firm does not even enter into the neighbourhood. In order to get the complete picture, the following proposition provides a complete characterization of the equilibrium under all possible parameter configurations. This characterization is illustrated in Figure 1 by plotting the different thresholds for poor and rich incomes for different values of \( f \), the proportion of poor in the neighbourhood.
Proposition 3. The equilibrium can be characterized by the different threshold values of poor income complemented with the rich and/or average neighbourhood incomes as follows.

- **Configuration 1:** \( \frac{\theta Y_P - c}{\sqrt{1F}} < \frac{\sqrt{2}}{1 + \sqrt{1-J}} \) and
  
  1(a) \( \frac{\theta Y_R - c}{\sqrt{1F}} < \frac{2}{1-J} \): the firm does not enter into the neighbourhood;
  
  1(b) \( \frac{\theta Y_R - c}{\sqrt{1F}} \geq \frac{2}{1-J} \): the firm charges the price \( p^* \) |Possibility 2 \), serves only the rich.

- **Configuration 2:** \( \frac{\sqrt{2}}{1 + \sqrt{1-J}} \leq \frac{\theta Y_R - c}{\sqrt{1F}} \leq \sqrt{2} \) and
  
  2(a) \( \frac{\theta Y_R - c}{\sqrt{1F}} < \frac{2}{1-J} \): the firm charges the price \( p^* \) |Possibility 1 \), serves both the rich and the poor if \( f \cdot \left( \frac{\theta Y_R - c}{\sqrt{1F}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{1F}} \right) \geq \sqrt{2} \); otherwise, the firm does not enter into the neighbourhood;
  
  2(b) \( \frac{\theta Y_R - c}{\sqrt{1F}} \geq \frac{2}{1-J} \): The firm charges the price \( p^* \) |Possibility 1 \), serves both the rich and the poor if \( \left( \frac{\theta Y_R - c}{\sqrt{1F}} \right) \leq (1 + \frac{1}{\sqrt{1-J}}) \left( \frac{\theta Y_P - c}{\sqrt{1F}} \right) \); otherwise, the firm charges the price \( p^* \) |Possibility 2 \), serves only the rich.

- **Configuration 3:** \( \frac{\theta Y_P - c}{\sqrt{1F}} > \sqrt{2} \) and
  
  3(a) \( \left( \frac{\theta Y_R - c}{\sqrt{1F}} \right) \leq \left( 1 + \frac{1}{\sqrt{1-J}} \right) \left( \frac{\theta Y_P - c}{\sqrt{1F}} \right) \): The firm charges the price \( p^* \) |Possibility 1 \), serves both the rich and the poor consumers;
  
  3(b) \( \left( \frac{\theta Y_R - c}{\sqrt{1F}} \right) \geq \left( 1 + \frac{1}{\sqrt{1-J}} \right) \left( \frac{\theta Y_P - c}{\sqrt{1F}} \right) \): The firm charges the price \( p^* \) |Possibility 2 \), serves only the rich consumers.

**Proof:** See Appendix, section 10.4.

Figure 1 illustrates this characterization. For any \( f \), if \( \frac{\theta Y_P - c}{\sqrt{1F}} \) and \( \frac{\theta Y_R - c}{\sqrt{1F}} \) are below the black and the red curves respectively, the firm does not enter into the neighbourhood. That is, if the poor are too poor and the rich are also not rich enough for the firm to recover its fixed cost, then the firm does not enter into the neighbourhood. If instead, \( \frac{\theta Y_R - c}{\sqrt{1F}} \) is above the red curve so that the firm can recover its fixed cost, it enters into the neighbourhood but serves only the rich – the poor are completely left out. Thus, as long as \( \frac{\theta Y_R - c}{\sqrt{1F}} \) is less than the black curve, that is, the poor are too poor, they do not have any market access.
Figure 1: Characterizing the Equilibrium
In Figure 1, \( \left( \frac{\theta Y_p - c}{\sqrt{1 + F}} \right)_{\text{Config} 2} \) is chosen such that \( \frac{\sqrt{2}}{1 + \sqrt{1 - f}} \leq \left( \frac{\theta Y_p - c}{\sqrt{1 + F}} \right)_{\text{Config} 2} \leq \sqrt{2} \) for \( f \leq f_2 \), that is, the poor has a moderate income. For any specific value of \( f \leq f_2 \), if \( \frac{\theta Y_p - c}{\sqrt{1 + F}} \) is below the red curve, then the firm serves both the rich and the poor only if \( \frac{\theta Y_p - c}{\sqrt{1 + F}} \) is above the green curve; otherwise the firm does not enter into the neighbourhood. This illustrates one key neighbourhood externality identified in the model that the poor benefits from the presence of the rich: the higher valuation of the rich leads the firm enter into the neighbourhood, allowing the poor who live sufficiently close to the firm to access the product. This is the positive ‘provision effect’. On the other hand, if \( \frac{\theta Y_p - c}{\sqrt{1 + F}} \) is above the red curve, then the firm serves both the rich and the poor only if \( \frac{\theta Y_p - c}{\sqrt{1 + F}} \) is below the blue curve; otherwise, if the rich are richer, then the firm abandons the poor and caters only to the rich. This illustrates the other side of the picture: if the rich income is high enough (as compared to the poor income), the poor faces complete exclusion. This is the negative ‘exclusion effect’. In between these two extremes, that is, in between the green and blue curves, as rich income increases equilibrium price also increases (refer to equation (1)), hurting the poor. This is the negative ‘price effect’.

Similarly, \( \left( \frac{\theta Y_p - c}{\sqrt{1 + F}} \right)_{\text{Config} 3} \) is chosen in Figure 1 such that \( \left( \frac{\theta Y_p - c}{\sqrt{1 + F}} \right)_{\text{Config} 3} > \sqrt{2} \), that is, the poor income is reasonably high. Here for any specific value of \( f \), the firm serves both the rich and the poor only if \( \frac{\theta Y_p - c}{\sqrt{1 + F}} \) is below the dashed blue curve; for any value of \( \frac{\theta Y_p - c}{\sqrt{1 + F}} \) higher than that the firm serves only the rich and the poor are completely left out. Thus although the presence of the firm in the neighbourhood is not a worry any more, the poor may still face complete exclusion if the rich income is relatively high.

In what follows we use this equilibrium characterization to analyze the impact of income inequality, neighbourhood effects and own income effects on the market access and welfare of the rich and poor living in the neighbourhood. Before we start the discussion, a general comment on the organization of the discussion below is in order. In all the comparative static exercises that follow we distinguish between two kinds of effects: the threshold effects and the within-thresholds effects. Threshold effects examine how a change in any parameter affects the thresholds that separate the scenarios between access and no access. Threshold effects are of two types. (a) The provision effect examines how a change in any parameter affects the firm’s decision to enter into or exit from the neighbourhood. (b) The inclusion/exclusion effect examines the firm’s decision to include or exclude the poor from accessing the product or service under consideration. On the other hand, the within-threshold effects are the impact of parameter changes on the magnitude of market access and welfare of the rich and poor when they already have access. Here we will see the tension between the valuation
4 Market Access and Welfare of the Poor

In this section we analyze how the market access and welfare of the poor get affected by their own income and by their neighbourhood characteristics.

The poor has market access only under equilibrium Possibility 1. There the aggregate market access of the poor community as a whole is

\[ A_P = 2 \delta_P |_{\text{Possibility 1}} = \frac{2}{t} \left( \theta Y_P - p^* |_{\text{Possibility 1}} \right) = \frac{2}{t} \left[ \theta Y_P - \frac{\theta (f Y_P + (1-f) Y_R) + c}{2} \right]. \tag{6} \]

To calculate the the aggregate consumer surplus of the poor community as a whole note that the surplus to a poor consumer located at a distance \( x \) from the firm is \( \theta Y_P - p^* |_{\text{Possibility 1}} - tx \).\(^{11}\) Since only the poor consumers located at a distance up to \( \frac{\theta Y_P - p^* |_{\text{Possibility 1}}}{t} \) on either side of the firm have market access, while all the poor consumers located at distance beyond this does not have market access, the aggregate consumer surplus of the poor is

\[ CS_P = 2 \int_0^{\theta Y_P - p^* |_{\text{Possibility 1}} \over t} \left[ \theta Y_P - p^* |_{\text{Possibility 1}} - tx \right] dx = \frac{\left( \theta Y_P - p^* |_{\text{Possibility 1}} \right)^2}{t} \tag{7} \]

4.1 Own Income Effects

We examine how market access and welfare of the poor vary as their own income increases. Consider the threshold effects first. Recall from Corollary 1 that for the poor to have any market access it is necessary that \( \frac{\theta Y_P - c}{\sqrt{1-t}} \geq \frac{\sqrt{2}}{1 + \sqrt{1-t}} \), and it becomes easier to meet this threshold as poor income increases. To see that the provision effect is positive consider Configuration 2(a) with \( f \cdot \left( \frac{\theta Y_P - c}{\sqrt{1-t}} \right) + (1-f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{1-t}} \right) < \sqrt{2} \) so that the firm does not enter into the neighbourhood. In such a situation, other things (for example, \( Y_R \) or \( f \)) remaining the same, an increase in \( Y_P \) so as to revert this inequality makes it possible for the firm to enter into the neighbourhood and charge \( p^* |_{\text{Possibility 1}} \). Then the poor earns a positive market access and consumer surplus. We also find a positive inclusion effect. Consider Configuration 3(b) or Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{1-t}} \right) > \left( 1 + \frac{1}{\sqrt{1-t}} \right) \left( \frac{\theta Y_P - c}{\sqrt{1-t}} \right) \) so that the firm serves only

\(^{11}\)Recall that the reservation utility of the consumer is 0.
the rich and the poor are completely excluded. In situations like these, an increase in \( Y_P \) so as to revert this inequality makes it possible for the firm to charge \( p^* \)\textsubscript{Possibility 1} and include the poor into the service.

Next consider the \textit{within-thresholds effects}. When the poor do have market access, it follows from equations (6) and (7) that

\[
\frac{\partial A_P}{\partial Y_P} = \frac{\theta}{t} (2 - f) > 0,
\]

and

\[
\frac{\partial CS_P}{\partial Y_P} = \frac{\theta}{t} \left[ \theta Y_P - \frac{\theta (f Y_P + (1-f) Y_R) + c}{2} \right] (2 - f) > 0.
\]

There are two effects at work. First is the direct positive \textit{valuation effect} working through the preference parameter \( \theta \). Second effect is the indirect negative effect working through the increase in \textit{price} as poor income increases. It turns out that the positive valuation effect dominates the negative effect from increase in price.

### 4.2 Neighbourhood Effect: Increase in Rich Income

We consider two aspects of neighbourhood effect on the poor: effects of rich income and effects of proportion of the rich in the neighbourhood. In this subsection we examine how market access and welfare of the poor vary as the rich income increases.

We find that the \textit{provision effect} of an increase in rich income is positive. Consider once again Configuration 2(a) with \( f \cdot \left( \frac{\theta Y_P - c}{\sqrt{tF}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) < \sqrt{2} \) so that the firm does not enter into the neighbourhood and the poor has no market access. Other things remaining the same, if \( Y_R \) increases enough so as to revert this inequality, then the firm enters into the neighbourhood and charges \( p^* \)\textsubscript{Possibility 1}. Now the poor (along with the rich) starts earning a positive market access and consumer surplus. On the other hand, an increase in rich income may generate a negative \textit{exclusion effect}. To see that consider Configurations 2(b) and 3(a). The poor has positive market access and consumer surplus as long as \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) \leq \left( 1 + \frac{1}{\sqrt{tF}} \right) \left( \frac{\theta Y_P - c}{\sqrt{tF}} \right) \). Other things remaining the same, if \( Y_R \) increases enough so as to revert this inequality, then the firm abandons the poor completely to cater only to the rich and the poor loses their entire market access and consumer surplus.

Within the thresholds when the poor have market access, we have

\[
\frac{\partial A_P}{\partial Y_R} = -\frac{\theta (1 - f)}{t} < 0,
\]

and

\[
\frac{\partial CS_P}{\partial Y_R} = -\frac{\theta}{t} \left[ \theta Y_P - \frac{\theta (f Y_P + (1-f) Y_R) + c}{2} \right] (1 - f) < 0.
\]
As \( Y_R \) increases, equilibrium price also increases. Other things remaining the same, this increase in price reduces the poor’s market access and consumer surplus.

Figure 2 illustrates the effect of rich income on the poor fixing \( f \) at \( f_1 \) and \( \frac{\theta Y_{P-c}}{\sqrt{tF}} \) at \( \frac{\sqrt{2}}{1+\sqrt{1-f_1}} < \left( \frac{\theta Y_{P-c}}{\sqrt{tF}} \right)_{\text{Config}2} < \sqrt{2} \) so that we are under Configuration 2. The poor has positive market access and consumer surplus only when the rich income \( \left( \frac{\theta Y_{R-c}}{\sqrt{tF}} \right) \) is in between the heights of the points \( R_1 \) and \( R_2 \). The positive provision effect is illustrated by the point \( R_1 \). Till the rich income is below the height of \( R_1 \), the firm does not enter into the neighbourhood and neither the poor nor the rich has any market access. As the rich income increases to \( R_1 \), the firm enters and the poor (along with the rich) starts enjoying a positive market access and consumer surplus. The negative exclusion effect is illustrated by the point \( R_2 \). The poor enjoys a positive market access and consumer surplus till the rich income increases to \( R_2 \). Any further increase in the rich income instigates the firm to exclude the poor completely. As the rich income increases from \( R_1 \) to \( R_2 \), market access and consumer surplus of the poor steadily decline due to the increase in price.

### 4.3 Neighbourhood Effect: Increase in Proportion of the Rich

Now we consider the second aspect of the neighbourhood effect on the poor – the effects of an increase in the proportion of rich consumers in the neighbourhood.

Examining Configuration 2(a) or Configurations 2(b) and 3(a) it is easy to find that the effects of an increase in the proportion of rich work in the same way as the effects of an increase in the rich income. That is, an increase in the proportion of rich generates a positive provision effect but a negative exclusion effect on market access and consumer surplus of the poor. Further it generates an additional positive effect: it becomes easier for the poor to overcome the income lower bound to have any market access, \( \frac{\theta Y_{P-c}}{\sqrt{tF}} \geq \frac{\sqrt{2}}{1+\sqrt{1-f}} \), as the proportion of rich \((1-f)\) increases.

The within-threshold effects are also similar. We have

\[
\frac{\partial A_P}{\partial (1-f)} = \frac{\theta (Y_P - Y_R)}{t} < 0,
\]

and

\[
\frac{\partial CS_P}{\partial (1-f)} = \frac{\theta}{t} \left[ \theta Y_P - \frac{\theta (f Y_P + (1-f) Y_R) + c}{2} \right] (Y_P - Y_R) < 0.
\]

An increase in the proportion of rich increases the equilibrium price which in turn reduces the poor’s market access and consumer surplus.
Figure 2: Neighbourhood Effects on Market Access and Welfare of the Poor
The effects of an increase in the proportion of rich in the neighborhood on the poor is illustrated in Figure 2 by fixing $\frac{\theta Y_p - c}{\sqrt{tF}}$ at $(\frac{\theta Y_p - c}{\sqrt{tF}})_{Config2}$ and $\frac{\theta Y_p - c}{\sqrt{tF}}$ at the height of the point $R_2$. The figure shows that the poor has positive market access and consumer surplus only when the proportion of rich is in between $(1 - f_3)$ and $(1 - f_1)$. The positive provision effect is illustrated by the point $R_2'$ with $(1 - f_3)$ as the corresponding proportion of rich, while the negative exclusion effect is illustrated by the point $R_2$ with corresponding proportion of rich $(1 - f_1)$. As the proportion of rich increases from $(1 - f_3)$ to $(1 - f_1)$, market access and consumer surplus of the poor steadily decline due to the increase in price.

The following proposition summarizes the effects on market access and welfare of the poor.

**Proposition 4. (Effects on Market Access and Welfare of the Poor)**

(a) *Own Income Effects:* Both the threshold effects of an increase in poor income – the provision effect and the inclusion effect – are positive. Within the thresholds, when the poor has positive market access, the positive valuation effect of an increase in poor income dominates the negative price effect so that the overall effect is again positive.

(b) *Neighbourhood Effects:* Both the neighbourhood characteristics – rich income and proportion of rich in the neighbourhood – work in the same direction. While the provision effect is positive, the exclusion effect of a change in the neighbourhood characteristic is negative. Within the thresholds, when the poor has positive market access, an increase in either rich income or proportion of rich in the neighbourhood leads to a negative price effect.

### 5 Market Access and Welfare of the Rich

The rich has market access under both equilibrium possibilities 1 and 2. Under equilibrium Possibility 1, the aggregate market access of the rich community as a whole is

$$A_R|_{Possibility\ 1} = \frac{2}{t} \left( \theta Y_R - p^* \right)|_{Possibility\ 1} = \frac{2}{t} \left[ \theta Y_R - \frac{\theta (f Y_P + (1 - f) Y_R) + c}{2} \right],$$

while their aggregate consumer surplus is

$$CS_R|_{Possibility\ 1} = \left( \frac{\theta Y_R - p^*}{t} \right)|_{Possibility\ 1}^2 = \frac{1}{t} \left[ \theta Y_R - \frac{\theta (f Y_P + (1 - f) Y_R) + c}{2} \right]^2.$$
Under equilibrium Possibility 2, the aggregate market access of the rich community as a whole is
\[
AR_{\text{Possibility } 2} = \frac{2}{t} \left( \theta Y_R - p^*_{\text{Possibility } 2} \right) = \frac{2}{t} \left[ \theta Y_R - \frac{\theta Y_R + c}{2} \right],
\]
and their aggregate consumer surplus is
\[
CS_{R_{\text{Possibility } 2}} = \frac{\left( \theta Y_R - p^*_{\text{Possibility } 2} \right)^2}{t} = \frac{1}{t} \left[ \theta Y_R - \frac{\theta Y_R + c}{2} \right]^2.
\]

Since \( p^*_{\text{Possibility } 1} = \frac{\theta (f Y_P + (1 - f) Y_R) + c}{2} < \frac{\theta Y_R + c}{2} = p^*_{\text{Possibility } 2} \), we have \( AR_{\text{Possibility } 1} > AR_{\text{Possibility } 2} \) and \( CS_{R_{\text{Possibility } 1}} > CS_{R_{\text{Possibility } 2}} \) under the same parameter configurations. That is, the rich are better off in a neighbourhood where both the rich and the poor have market access as compared to the neighbourhood where the poor does not have access.

In this section we analyze how the market access and welfare of the rich get affected by their own income and by their neighbourhood characteristics.

### 5.1 Own Income Effects

We start with examining how market access and welfare of the rich vary as their own income increases.

Consider the *threshold effects* first. We find that the *provision effects* of an increase in rich income are positive. Consider Configuration 1(a), \( \frac{\theta Y_R - c}{\sqrt{1 - f}} < \frac{\sqrt{2}}{1 + \sqrt{1 - f}} \) and \( \frac{\theta Y_R - c}{\sqrt{1 - f}} < \frac{\sqrt{2}}{1 + \sqrt{1 - f}} \), so that the firm does not enter into the neighbourhood. If \( Y_R \) increases enough to revert the second inequality, then the firm enters and the rich starts enjoying positive market access and consumer surplus. Similarly the firm does not enter into the neighbourhood under Configuration 2(a) with \( f \cdot \left( \frac{\theta Y_R - c}{\sqrt{1 - f}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{1 - f}} \right) < \sqrt{2} \). Here also if \( Y_R \) increases enough to revert this inequality, then the firm enters and the rich (along with the poor) starts enjoying positive market access and consumer surplus. In contrast and interestingly, an increase in rich income may generate a negative *exclusion effect* on the rich. To see this consider Configuration 3(a) or Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{1 - f}} \right) \leq \left( 1 + \frac{1}{\sqrt{1 - f}} \right) \left( \frac{\theta Y_R - c}{\sqrt{1 - f}} \right) \), so that the rich is enjoying market access \( AR_{\text{Possibility } 1} \) and consumer surplus \( CS_{R_{\text{Possibility } 1}} \).

Now if \( Y_R \) increases enough so that this inequality gets reversed, then the firm caters only to the rich and the rich’s market access and consumer surplus get reduced to \( AR_{\text{Possibility } 2} < AR_{\text{Possibility } 1} \) and \( CS_{R_{\text{Possibility } 2}} < CS_{R_{\text{Possibility } 1}} \), respectively.

Within the thresholds, from the expressions for aggregate market access and consumer
surplus of the rich under equilibrium possibilities 1 and 2 given above, we derive
\[ \frac{\partial}{\partial Y_R} \left[ A_R \mid \text{Possibility } i \right] > 0, \text{ and } \frac{\partial}{\partial Y_R} \left[ CS_R \mid \text{Possibility } i \right] > 0, \ i = 1, 2. \]

Similar to the effects of poor income on market access and consumer surplus of the poor, there are two opposing effects at work, and the positive valuation effect dominates the negative price effect.

5.2 Neighbourhood Effect: Increase in Poor Income

We first consider the first aspect of the neighbourhood effect on the rich – the effects of an increase in income of the poor. We will establish the interesting result that both the threshold effects of an increase in the poor income on market access and consumer surplus of the rich are positive. To see that the provision effect is positive consider either Configuration 1(a) or Configuration 2(a) with \( f \cdot \left( \frac{\theta Y_P - c}{\sqrt{1 + f}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{1 + f}} \right) < \sqrt{2} \), so that the firm does not enter into the neighbourhood. If \( Y_P \) increases enough to revert this inequality, then the firm enters and the rich (along with the poor) starts enjoying positive market access and consumer surplus.

The positive inclusion effects can be demonstrated in the following circumstances. First consider Configuration 1(b) so that the firm is serving only the rich who are getting market access \( A_R \mid \text{Possibility } 2 \) and consumer surplus \( CS_R \mid \text{Possibility } 2 \). If \( Y_P \) increases just enough so that we move to Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{1 + f}} \right) < \left( 1 + \frac{1}{\sqrt{1 + f}} \right) \left( \frac{\theta Y_R - c}{\sqrt{1 + f}} \right) \), then the firm switches to lowering price to \( p^* \mid \text{Possibility } 1 \) and serving both the rich and the poor with the rich’s market access and consumer surplus increasing to \( A_R \mid \text{Possibility } 1 > A_R \mid \text{Possibility } 2 \) and \( CS_R \mid \text{Possibility } 1 > CS_R \mid \text{Possibility } 2 \), respectively. Consider next Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{1 + f}} \right) > \left( 1 + \frac{1}{\sqrt{1 + f}} \right) \left( \frac{\theta Y_R - c}{\sqrt{1 + f}} \right) \) or Configuration 3(b), so that, once again, the firm is serving only the rich whose market access and consumer surplus are \( A_R \mid \text{Possibility } 2 \) and \( CS_R \mid \text{Possibility } 2 \), respectively. Now if \( Y_P \) increases enough so that this inequality gets reversed, then the firm switches to serving both the rich and the poor with the rich’s market access and consumer surplus increasing to \( A_R \mid \text{Possibility } 1 > A_R \mid \text{Possibility } 2 \) and \( CS_R \mid \text{Possibility } 1 > CS_R \mid \text{Possibility } 2 \), respectively.

\[ ^{12} \text{We have } \frac{\partial}{\partial Y_R} \left[ A_R \mid \text{Possibility } 1 \right] = \frac{\theta}{f} (1 + f) > 0, \text{ and } \frac{\partial}{\partial Y_R} \left[ CS_R \mid \text{Possibility } 1 \right] = \frac{\theta}{f} \left( Y_R - \frac{\theta (f Y_P + (1 - f) Y_R + c)}{2} \right) (1 + f) > 0, \text{ and } \frac{\partial}{\partial Y_R} \left[ A_R \mid \text{Possibility } 2 \right] = \frac{\theta}{f} > 0, \text{ and } \frac{\partial}{\partial Y_R} \left[ CS_R \mid \text{Possibility } 2 \right] = \frac{\theta}{f} \left[ \frac{\partial}{\partial Y_R} \left( Y_R - \frac{\theta Y_R + c}{2} \right) \right] > 0. \]
In contrast to the threshold effects, the *price effect* of an increase in poor income is negative when the firm serves both the rich and the poor.\footnote{We derive that } Under Possibility 1, as $Y_P$ increases, equilibrium price also increases. Other things remaining the same, this increase in price reduces the rich’s market access and consumer surplus as there is no valuation effect to counter this negative price effect. Finally, when the firm serves only the rich, market access and consumer surplus of the rich are independent of the poor income.

### 5.3 Neighbourhood Effect: Increase in Proportion of the Poor

Now we consider the second aspect of the neighbourhood effect on the rich – the effects of an increase in the proportion of poor in the neighbourhood. We will establish that these effects are almost opposite to the effects of an increase in the income of the poor. Thus these two aspects of the “neighbourhood effects” are almost opposed to each other for the case of market access and welfare of the rich. It is interesting to contrast this to the case of market access and welfare of the poor. There the two aspects of the “neighbourhood effects” – income of the rich and proportion of the rich – work in the same direction.

First consider the *threshold effects*. The following two scenarios demonstrate that, in contrast with the poor income, the *provision effects* of an increase in the proportion of poor are negative. First consider Configuration 1(b) with $\frac{\partial Y_P - c}{\sqrt{1-F}} < \frac{\sqrt{2}}{1+\sqrt{1-f}}$ and $\frac{\partial Y_P - c}{\sqrt{1-F}} = \sqrt{\frac{2}{1-f}}$, so that the firm is serving only the rich. Other things remaining the same, if the proportion of poor ($f$) increases so that the first inequality continues to hold whereas the second one becomes strictly less, then we move to Configuration 1(a) where the firm leaves the neighbourhood and even the rich does not have any market access. Consider next Configuration 2(a) with $f \cdot \left(\frac{\partial Y_P - c}{\sqrt{1-F}}\right) + (1-f) \cdot \left(\frac{\partial Y_P - c}{\sqrt{1-F}}\right) > \sqrt{2}$, so that the rich has market access $A_R|_{\text{Possibility 1}}$ and consumer surplus $CS_R|_{\text{Possibility 1}}$. If the proportion of poor ($f$) increases just enough to revert this inequality, then the firm leaves the neighbourhood and the rich loses their entire market access and consumer surplus.

Next we show that the *inclusion effect* of an increase in the proportion of poor is positive, similar to the effect of an increase in poor income. Consider Configuration 2(b) with $(\frac{\partial Y_P - c}{\sqrt{1-F}}) > (1 + \frac{1}{\sqrt{1-f}}) \left(\frac{\partial Y_P - c}{\sqrt{1-F}}\right)$ or Configuration 3(b), so that the firm is serving only the rich and they are enjoying market access $A_R|_{\text{Possibility 2}}$ and consumer surplus $CS_R|_{\text{Possibility 2}}$. Now if $f$ increases enough so that this inequality gets reversed, then the firm switches to serv-
ing both the rich and the poor with the rich’s market access and consumer surplus increasing to \( A_R|_{\text{Possibility 1}} > A_R|_{\text{Possibility 2}} \) and \( CS_R|_{\text{Possibility 1}} > CS_R|_{\text{Possibility 2}} \), respectively.

Within the thresholds, once again the price effect is in contrast with the effects of an increase in poor income when the firm serves both the rich and the poor.\(^{14}\) Under Possibility 1, as \( f \) increases, equilibrium price decreases. Other things remaining the same, this decrease in price increases the rich’s market access and consumer surplus. Under Possibility 2, when the firm serves only the rich, market access and consumer surplus of the rich are independent of the proportion of poor in the neighbourhood.

The following proposition summarizes the effects on market access and welfare of the rich.


(a) *Own Income Effects*: While the provision effect is positive, the exclusion effect of an increase in rich income is negative. Within the thresholds, when the rich has positive market access, the positive valuation effect of an increase in rich income dominates the negative price effect so that the overall effect is positive.

(b) *Neighbourhood Effects*: The two neighbourhood characteristics – poor income and proportion of poor in the neighbourhood – work almost in the opposite directions. While the provision effect is positive, the price effect of an increase in poor income is negative. On the other hand, the provision effects of an increase in the proportion of poor are negative, but the price effect is positive. Only the inclusion effect works in the same positive direction for both the neighbourhood characteristics.

## 6 Inequality and Market Access and Welfare

From our discussion in the last two sections we can derive the impact of inequality on market access and welfare of the rich and poor arising from two variants of inequality – increasing rich income while keeping poor income and proportion of poor fixed, or increasing proportion of rich while keeping rich and poor incomes fixed. For the poor both variants of inequality work in the same direction: it follows from Proposition 4(b) that while the provision effect is positive, the exclusion and price effects are negative. For the rich, for the first variant of

\[ \frac{\partial}{\partial f} \left[ A_R|_{\text{possibility 1}} \right] = \frac{\theta(Y_R - Y_P)}{t} > 0, \quad \text{and} \quad \frac{\partial}{\partial f} \left[ CS_R|_{\text{possibility 1}} \right] = \frac{\theta(Y_R - Y_P)}{t} > 0. \]

\(^{14}\)We derive that
inequality, it follows from Proposition 5(a) that the provision effect is positive, the exclusion effect is negative, and the within-threshold effect (positive valuation effect net of the negative price effect) is positive. For the second variant of inequality, it follows from Proposition 5(b) that the provision effect is positive, the exclusion effect is negative, and the price effect is negative. We summarize this observation in the following proposition.

**Proposition 6. (Effects of Inequality)**

For the two variants of inequality – increasing rich income while keeping poor income and proportion of poor fixed, or increasing proportion of rich while keeping rich and poor incomes fixed – the rich and poor are affected almost similarly: the provision effect is positive while the exclusion and price effects are negative. Only for the rich when the inequality increases by increasing the rich income, the positive valuation effect dominates the negative price effect.

But in both these two variants of inequality the society becomes richer. In order to capture the role of inequality in its purest form let us examine the effect of a mean-preserving spread: keeping $f$ fixed we increase $Y_R$ together with a decrease in $Y_P$ such that the average income of the society, $fY_P + (1 - f)Y_R$, remains fixed.

### 6.1 Mean-Preserving Spread and Market Access and Welfare of the Poor

We examine how market access and welfare of the poor vary with an increase in the mean-preserving spread as defined above.

First consider the *threshold effects*. We find that the *provision effects* are neutral. Consider first Configuration 2(a) with $f \cdot \frac{\partial Y_P - c}{\sqrt{1 + f}} + (1 - f) \cdot \frac{\partial Y_R - c}{\sqrt{1 + f}} < \sqrt{2}$ so that the firm does not enter into the neighbourhood. With a mean-preserving spread, since the average income of the society remains unchanged, this inequality remains unaltered. Thus a mean-preserving spread has no provision effect. The following circumstance shows that although a mean-preserving spread may lead the firm to enter, but it still does not serve the poor. Consider Configuration 1(a), $\frac{\partial Y_P - c}{\sqrt{1 + f}} < \frac{\sqrt{2}}{1 - f}$ and $\frac{\partial Y_R - c}{\sqrt{1 + f}} < \sqrt{\frac{2}{1 - f}}$, so that the firm does not enter into the neighbourhood. With mean-preserving spread, if $Y_R$ increases enough so as to revert the second inequality, then the firm enters the neighbourhood to serve only the rich.

On the other hand, a mean-preserving spread generates negative *exclusion effects*. Consider first Configuration 2(a) with $f \cdot \frac{\partial Y_P - c}{\sqrt{1 + f}} + (1 - f) \cdot \frac{\partial Y_R - c}{\sqrt{1 + f}} \geq \sqrt{2}$ so that the poor has positive market access and consumer surplus. With mean-preserving spread, if $Y_R$ increases
and $Y_P$ decreases enough so that we move to Configuration 1(b), then the firm abandons the poor completely to cater only to the rich, and the poor loses their entire market access and consumer surplus. Next consider Configurations 2(b) and 3(a). The poor has positive market access and consumer surplus as long as 

$$\left( \frac{y_{Y - c}}{\sqrt{1 - f}} \right) \leq \left( 1 + \frac{1}{\sqrt{1 - t}} \right) \left( \frac{y_{Y - c}}{\sqrt{1 - f}} \right).$$

With mean-preserving spread, if $Y_R$ increases and $Y_P$ decreases enough so as to revert this inequality, then the firm starts serving only the rich, and the poor loses their entire market access and consumer surplus.

Next consider the within-threshold effects. With mean-preserving spread, $p^*|_{\text{Possibility 1}}$ remains unchanged as the average income remains the same, but $Y_P$ decreases resulting in a decrease in both market access and consumer surplus of the poor (refer to the expressions in equations (6) and (7)).

### 6.2 Mean-Preserving Spread and Market Access and Welfare of the Rich

Now we analyze how an increase in the mean-preserving spread affects market access and welfare of the rich.

The following threshold effects for the rich follow from the same analysis of the threshold effects for the poor discussed above. From that analysis we can conclude that an increase in the mean-preserving spread generates either a positive or a neutral provision effect and negative exclusion effects on the market access and consumer surplus of the rich.

Within the thresholds when the rich have market access, we find that, in contrast to the poor, the effect on the rich is positive. Under Possibility 1, with mean-preserving spread, $p^*|_{\text{Possibility 1}}$ remains unchanged as the average income remains the same, but $Y_R$ increases resulting in an increase in both $A_R$ and $CS_R$. Under Possibility 2, since $A_R$ and $CS_R$ are independent of $Y_P$, the effect of mean-preserving spread is exactly the same as the effect of an increase in $Y_R$. We have seen in section 5.1 that both $A_R$ and $CS_R$ increases with an increase in $Y_R$. Hence both of them will increase with mean-preserving spread too.

The effects of an increase in mean-preserving spread on the market access and welfare of the rich and poor are summarized in the following proposition.

**Proposition 7. (Effects of a Mean-Preserving Spread)**

(a) **Market Access and Welfare of the Poor:** The provision effect of an increase in mean-preserving spread is neutral, but the exclusion effect is negative. Within the thresholds, when the poor has a positive market access, the neutral price effect is dominated by
the negative valuation effect. Thus the overall effect of an increase in mean-preserving spread on the market access and welfare of the poor is negative.

(b) Market Access and Welfare of the Rich: While the provision effect is either positive or neutral, the exclusion effect is negative. Within the thresholds, the positive valuation effect dominates either the neutral (under equilibrium Possibility 1) or the negative (under equilibrium Possibility 2) price effect.

7 Possibility of Complete Exclusion of the Poor

In this section we highlight the possibility of complete exclusion of the poor: the unfortunate scenario where the service provider completely ignores the presence of the poor and chooses the price considering as if there are only rich individuals residing in the neighbourhood.

Proposition 2 identifies that the firm excludes the poor customers completely when the relative income of the rich is high enough: \( \frac{\partial Y_R - c}{\partial Y_P - c} > 1 + \frac{1}{\sqrt{1 - f}} \). We can understand that this can happen at a very low level of poor income (Configuration 1(b)). But, surprisingly enough, Proposition 3 demonstrates that even at moderate to high levels of income the poor people are not immune from this unfortunate possibility (Configurations 2(b) with \( \left( \frac{\partial Y_R - c}{\sqrt{1 - f}} \right) > \left( 1 + \frac{1}{\sqrt{1 - f}} \right) \left( \frac{\partial Y_P - c}{\sqrt{1 - f}} \right) \), and Configuration 3(b)). Figure 1 can be used to illustrate this complete exclusion possibility for various levels of poor income. For any \( f \), if \( \frac{\partial Y_P - c}{\sqrt{1 - f}} \) is below the black curve while \( \frac{\partial Y_R - c}{\sqrt{1 - f}} \) is above the corresponding point on the red curve, this possibility occurs. For a moderate level of poor income \( \left( \frac{\partial Y_P - c}{\sqrt{1 - f}} \right)_{\text{Config}2} \), for any \( f \leq f_2 \), the possibility occurs when \( \frac{\partial Y_R - c}{\sqrt{1 - f}} \) is above the corresponding point on the solid blue curve. Unfortunately this complete exclusion of the poor can happen even when the poor income is high enough. In Figure 1 at a reasonably high level of poor income \( \left( \frac{\partial Y_P - c}{\sqrt{1 - f}} \right)_{\text{Config}3} \), for any \( f \), complete exclusion of the poor happens when \( \frac{\partial Y_R - c}{\sqrt{1 - f}} \) is above the corresponding point on the dashed blue curve. This complete exclusion possibility of the poor is similar to the findings in the literature that argues that income inequality may lead to social segmentation and club formation to ensure private provision of public services exclusively to the rich (see, for example, Graham, 1998; Jaramillo et al., 2003; Bhattacharya et al., 2012).

Implications of Poverty and Income Inequality:

Corollary 1 identifies the lower income threshold for the poor, \( Y_P \equiv \frac{1}{f} \left( c + \frac{\sqrt{21 - f}}{1 + \sqrt{1 - f}} \right) \), such that the poor are completely excluded if \( Y_P < Y_P \). Presence of this lower threshold for poor income emphasizes the implications of absolute levels of poverty in this structure.
It is interesting to observe that this lower income threshold for the poor decreases with the proportion of rich in the neighbourhood \((1 - f)\). This is an important positive neighbourhood externality identified in our model.

It is clear from the discussion above that it is the higher income gap between the rich and poor that is at the root of generating the possibility of complete exclusion of the poor. With a lower income gap the firm finds it optimal to extend service provision to at least some of the poor by lowering the price, while with a higher income gap it caters only to the rich keeping the price high enough. This result and mechanism is similar to the dilemma identified in Foellmi and Zweimuller (2006) where also non-homothetic preferences play an important role.

The Case of Minority Poor:

The poor are more likely to be completely excluded when they are a minority, that is, when \(f\) is low: firms may completely ignore the poor even when the rich are not ultra rich just because the rich are more in number. For example, in Figure 2, with the same income levels, \(\left(\sqrt{\frac{Y_p - c}{1 - f}}\right)_{\text{Config}2}\) for poor and just above \(R_2\) for rich, the complete exclusion possibility does not arise when the proportion of poor is \(f_3\); but it does arise when the proportion of poor is \(f_1\).

8 Discussion and Policy Considerations

Our analysis of the inequality-neighbourhood interactions can be summarized as follows. On the one hand, there is the positive ‘provision effect’: higher valuation of the rich attracts the supplier to enter into the neighbourhood, allowing the poor who live sufficiently close to the firm to purchase the product. On the other hand, there is the negative ‘price effect’: the service provider is tempted to charge a higher price higher is the income or larger is the proportion of the rich. In the extreme, if income or proportion of the rich is high enough, the service provider completely abandons the poor and caters only to the rich. This is the negative ‘exclusion effect’.

The externalities of richer neighbours upon poorer ones is an interesting and empirically quite relevant issue. While many developing countries, the giant emerging economies of Brazil, Russia, India and China, in particular, have experienced economic boom over the last couple of decades, rapidly widening income gap between the rich and the poor during the same period remains a continuous concern. Added to this is the conscious efforts in these
countries to move away from public provision of merit goods like health care or education (higher education, in particular) towards private provision. Our analysis points out that under private provision the externalities of richer neighbours upon poorer ones can be either positive (when the provision effect dominates, for example) or negative (when the price effect or exclusion effect dominates). For example, Estache et al. (2001) show that in the Latin American countries, in general, privatization of public utilities has led to an expansion of the outreach of such services, that is, the positive provision effect has dominated.

Importantly, we find that the trade-offs mentioned above generate (potential) non-monotonic impacts on market access and welfare of the poor arising from an increase in income inequality or variations in the neighbourhood characteristics. Thus any empirical testing of the effects of inequality or neighbourhood characteristics should allow for this non-monotone behaviour. Interestingly, studies by Feng and Yu (2007) and Li and Zhu (2006) lend strong empirical support to our findings. They establish a statistically significant inverted-U association between self-reported health status and neighbourhood level inequality using individual data from the China Health and Nutrition Survey (CHNS). It would be interesting to conduct similar analysis for other public goods recently put under private provision in many developing and emerging economies like higher education, public utilities, and so on.

Our model has several interesting policy implications in the context of the debates about social and public service delivery. In what follows we discuss two such policies: targeted price subsidy to the poor, and subsidy to the fixed entry cost.

### 8.1 Targeted Price Subsidy to the Poor

Since there are no issues of service quality or agency problems in our model, but there are distributional concerns and externality concerns, targeted price subsidy schemes should work quite effectively (see, for example, Blank, 2000). For example, Section 8 housing vouchers in the US provide low-income families with a subsidy that they can use for rental housing. Food Stamps provide a voucher for low-income families that can be used to purchase additional foods at the grocery store. The *Rashtriya Swasthya Bima Yojana* (National Health Insurance Plan) recently initiated by the Government of India allows the people below the poverty line to access any hospital, private or public, by paying only a nominal registration fee to join the insurance plan.

It is intuitive to see that targeted subsidized prices of these types increase the market access and welfare of the poor in our model. What is interesting is the finding that although targeted towards the poor, these types of price subsidies can have positive impact on the
market access and welfare of the rich also. The effects of a targeted price subsidy to the poor on the market access and welfare of the rich and poor are discussed in details in the Appendix (section 10.5) and summarized in the following proposition.

**Proposition 8. (Effects of a Targeted Price Subsidy to the Poor)**

(a) *Market Access and Welfare of the Poor*: The lower bound on poor income declines and both the threshold effects – the provision effect and the inclusion effect – are positive. Within the thresholds, when the poor has a positive market access, a net reduction in price\(^{15}\) leads to a positive price effect. Thus the overall effect of a targeted price subsidy on the market access and welfare of the poor is positive.

(b) *Market Access and Welfare of the Rich*: Although targeted towards the poor, both the threshold effects – the provision effect and the inclusion effect – of a targeted price subsidy on the market access and welfare of the rich are also positive. Within the thresholds, while an increase in price leads to a negative price effect under equilibrium Possibility 1, there is no such effect under equilibrium Possibility 2.

### 8.2 Subsidy to the Fixed Entry Cost

Since our model does not consider agency problems or issues concerning service quality, it does not call for a full-fledged government provision of the product or service under consideration. Instead, the government can encourage the private providers to enter into the neighbourhood by subsidizing the fixed entry cost. For example, providing the land at a subsidized price to encourage entry of private educational institutions or hospitals is a common practice.

In our model a subsidy to the fixed entry cost makes it easier for the firm to enter, and, in the process, increases market access and welfare of both the rich and poor. We analyze these effects in the Appendix (section 10.6) and summarize the results in the following proposition.

**Proposition 9. (Effects of a Subsidy to the Fixed Entry Cost)**

A subsidy to the fixed entry cost affects the market access and welfare of the rich and poor in a similar way. While the provision effect is positive, the inclusion/exclusion effect is either

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\(^{15}\)The equilibrium price increases (see in the Appendix that \(p^*_s|\text{Possibility } 1 > p^*_s|\text{Possibility } 1\)), but the increase is less than the amount of per unit subsidy received by the poor. Thus, the poor experiences a net reduction in price.
positive or neutral for both the rich and poor. The price effect is neutral as the subsidy leaves the equilibrium price unchanged.

9 Conclusion

The main contribution of this paper is to model the interaction between neighbourhood effects and income inequality in a simple and tractable way by integrating consumers’ income distribution with the spatial distribution of their location. While the basic analytical structure is adapted from the industrial organization literature (Hotelling, 1929; Salop, 1979; Bhaskar and To, 1999, 2003; Brekke et al., 2008), this literature does not explore the implications of income inequality. On the other hand, the literature on income inequality has not typically investigated the implications of inequality operating through industrial structure. This paper complements this literature by exploring the impact of income inequality working through the trade-off of price and provision effects.

Identifying this trade-off is one important contribution of this paper. On the one hand, there is the positive ‘provision effect’: higher valuation of the rich attracts the supplier to enter into the neighbourhood, allowing the poor who live sufficiently close by to access the service. On the other hand, there is the negative ‘price effect’: the service provider is tempted to charge a higher price higher is the income or larger is the proportion of the rich. In the extreme, if income or proportion of the rich is high enough, the service provider completely abandons the poor and caters only to the rich. This is the negative ‘exclusion effect’. These neighbourhood externalities work in the other direction also – the presence of poor in the neighbourhood also generates similar effects on their richer neighbours. We show that these trade-offs generate (potential) non-monotonic impacts on market access and welfare of the rich and poor arising from an increase in income inequality or variations in the neighbourhood characteristics.

As an added bonus, we identify the possibility of complete exclusion of one type of customer (the poor in our model) so far overlooked by the industrial organization literature: a scenario where the firms cater only to one type of customer (the rich) and the other type (the poor) has absolutely no market access. We have isolated the higher income gap between rich and poor as the key factor that exposes the poor to this complete exclusion possibility. The poor are also more likely to be completely excluded when they are a minority.
10 Appendix

10.1 Proof of Proposition 1

Possibility 1 is an equilibrium outcome if and only if \( \pi^*|_{\text{Possibility 1}} \geq \pi^*|_{\text{Possibility 2}} \), \( \pi^*|_{\text{Possibility 1}} \geq 0 \), and \( \delta_P|_{\text{Possibility 1}} \geq 0 \).

Using the expressions of \( \pi^*|_{\text{Possibility 1}} \) and \( \pi^*|_{\text{Possibility 2}} \) we find, after some algebraic manipulations, that

\[
\pi^*|_{\text{Possibility 1}} \geq \pi^*|_{\text{Possibility 2}} \quad \text{if and only if} \quad (\theta Y_R - c) \leq \left(1 + \frac{1}{\sqrt{1-f}}\right)(\theta Y_P - c).
\]

It follows similarly that

\[
\pi^*|_{\text{Possibility 1}} \geq 0 \quad \text{if and only if} \quad \frac{\theta Y_P - p^*|_{\text{Possibility 1}}}{t} \Leftrightarrow p^*|_{\text{Possibility 1}} \leq \theta Y_P.
\]

Finally, \( 0 \leq \delta_P|_{\text{Possibility 1}} = \frac{\theta Y_P - p^*|_{\text{Possibility 1}}}{t} \Leftrightarrow p^*|_{\text{Possibility 1}} \leq \theta Y_P \). Now, using the expression of \( p^*|_{\text{Possibility 1}} \) we derive

\[
\delta_P|_{\text{Possibility 1}} \geq 0 \quad \text{if and only if} \quad (\theta Y_R - c) \leq \left(1 + \frac{1}{1-f}\right)(\theta Y_P - c).
\]

Note that, for \( \pi^*|_{\text{Possibility 1}} \geq \pi^*|_{\text{Possibility 2}} \) we already have \( (\theta Y_R - c) \leq \left(1 + \frac{1}{\sqrt{1-f}}\right)(\theta Y_P - c) \).

Since \( \sqrt{1-f} \geq (1-f) \) for \( 0 \leq f \leq 1 \), we have \( \frac{1}{1-f} \geq \frac{1}{\sqrt{1-f}} \). It follows that if we have \( (\theta Y_R - c) \leq \left(1 + \frac{1}{\sqrt{1-f}}\right)(\theta Y_P - c) \), then both the conditions – \( \pi^*|_{\text{Possibility 1}} \geq \pi^*|_{\text{Possibility 2}} \) and \( \delta_P|_{\text{Possibility 1}} \geq 0 \) – are satisfied.

Now Proposition 1 follows.

10.2 Proof of Corollary 1

Consider the two conditions specified in Proposition 1. The first condition, \( (\theta Y_R - c) \leq \left(1 + \frac{1}{\sqrt{1-f}}\right)(\theta Y_P - c) \), implies

\[
f(\theta Y_P - c) + (1-f)(\theta Y_R - c) \leq \left(1 + \sqrt{1-f}\right)(\theta Y_P - c).
\]

Combining this with the second condition, \( f \cdot \left(\frac{\theta Y_P - c}{\sqrt{tF}}\right) + (1-f) \cdot \left(\frac{\theta Y_R - c}{\sqrt{tF}}\right) \geq \sqrt{2} \), that is, \( \sqrt{2tF} \leq f(\theta Y_P - c) + (1-f)(\theta Y_R - c) \), we get

\[
\sqrt{2tF} \leq \left(1 + \sqrt{1-f}\right)(\theta Y_P - c),
\]

that is,

\[
\frac{\sqrt{2}}{(1+\sqrt{1-f})} \leq \frac{\theta Y_P - c}{\sqrt{tF}}.
\]
10.3 Proof of Proposition 2

Possibility 2 is an equilibrium outcome if and only if \( \pi^*_2 \geq \pi^*_1 \), \( \pi^*_2 \geq 0 \), and \( \delta_P^* < 0 \).

Using the expressions of \( \pi^*_1 \) and \( \pi^*_2 \) we find that

\[ \pi^*_2 \geq \pi^*_1 \quad \text{if and only if} \quad (\theta Y_R - c) \geq \left(1 + \frac{1}{\sqrt{1-f}}\right)(\theta Y_P - c). \]

We also find that

\[ \pi^*_2 \geq 0 \quad \text{if and only if} \quad \frac{\theta Y_P - c}{\sqrt{1-f}} \geq \frac{2}{1-f}. \]

Finally, \( \delta_P^* < 0 \) if and only if \( 2(\theta Y_P - c) < (\theta Y_R - c) \).

Note that, for \( \pi^*_2 \geq \pi^*_1 \) we already have \( (\theta Y_R - c) \geq \left(1 + \frac{1}{\sqrt{1-f}}\right)(\theta Y_P - c) \).

Since \( \left(1 + \frac{1}{\sqrt{1-f}}\right) \geq 2 \) for \( 0 \leq f \leq 1 \), it follows that if we have \( (\theta Y_R - c) \geq \left(1 + \frac{1}{\sqrt{1-f}}\right)(\theta Y_P - c) \), then both the conditions are satisfied.

Now Proposition 2 follows.

10.4 Proof of Proposition 3

First we make the following two observations that can be verified by simple algebra.

**Observation 1:** \( f \cdot \left(\sqrt{\frac{2}{1+f}}\right) + (1-f) \cdot \left(\sqrt{\frac{2}{1-f}}\right) = \sqrt{2}. \)

**Observation 2:** \( \left(1 + \frac{1}{\sqrt{1-f}}\right) \cdot \left(\sqrt{\frac{2}{1+f}}\right) = \sqrt{\frac{2}{1-f}}. \)

Now we use these two observations to establish the equilibrium outcomes under the three different parameter configurations.

**Configuration 1:** \( \frac{\theta Y_P - c}{\sqrt{1-f}} < \frac{\sqrt{2}}{1+\sqrt{1-f}} \)

It follows from Corollary 1 that Possibility 1 cannot occur in equilibrium.

Consider Configuration 1(a): \( \frac{\theta Y_P - c}{\sqrt{1-f}} < \frac{\sqrt{2}}{1+\sqrt{1-f}} \) and \( \frac{\theta Y_R - c}{\sqrt{1-f}} < \sqrt{2} \). It follows from Observation 1 that \( f \cdot \left(\frac{\theta Y_P - c}{\sqrt{1-f}}\right) + (1-f) \cdot \left(\frac{\theta Y_R - c}{\sqrt{1-f}}\right) < \sqrt{2} \iff \pi^*_1 < 0 \) (see the proof of Proposition 1). Also, \( \frac{\theta Y_R - c}{\sqrt{1-f}} < \frac{\sqrt{2}}{1-f} \iff \pi^*_2 < 0 \) (see the proof of Proposition 2). Both the maximized profits being negative, the firm does not enter into the neighbourhood.
Consider Configuration 1(b): $\frac{\partial Y_{p-c}}{\partial tF} < \frac{\sqrt{2}}{1+\sqrt{1-t}}$ and $\frac{\partial Y_{r-c}}{\partial tF} \geq \sqrt{\frac{2}{1-t}}$. Since $\frac{\partial Y_{p-c}}{\partial tF} < \frac{\sqrt{2}}{1+\sqrt{1-t}}$, it follows from Observation 2 that $\left(1 + \frac{1}{\sqrt{1-t}}\right) \left(\frac{\partial Y_{p-c}}{\partial tF}\right) < \sqrt{\frac{2}{1-t}}$. Since $\frac{\partial Y_{r-c}}{\partial tF} \geq \sqrt{\frac{2}{1-t}}$, it follows that both the conditions of Proposition 2 are satisfied implying that Possibility 2 is the only equilibrium outcome.

**Configuration 2:** $\frac{\sqrt{2}}{1+\sqrt{1-t}} \leq \frac{\partial Y_{p-c}}{\partial tF} \leq \sqrt{2}$

Consider Configuration 2(a): $\frac{\sqrt{2}}{1+\sqrt{1-t}} \leq \frac{\partial Y_{p-c}}{\partial tF} \leq \sqrt{2}$ and $\frac{\partial Y_{r-c}}{\partial tF} < \sqrt{\frac{2}{1-t}}$. Since $\frac{\sqrt{2}}{1+\sqrt{1-t}} \leq \frac{\partial Y_{p-c}}{\partial tF}$ and $\frac{\partial Y_{r-c}}{\partial tF} < \sqrt{\frac{2}{1-t}}$, it follows from Observation 2 that $\left(1 + \frac{1}{\sqrt{1-t}}\right) \left(\frac{\partial Y_{r-c}}{\partial tF}\right) < \left(1 + \frac{1}{\sqrt{1-t}}\right) \left(\frac{\partial Y_{p-c}}{\partial tF}\right)$.

Now if $f \cdot \left(\frac{\partial Y_{p-c}}{\partial tF}\right) + (1-f) \cdot \left(\frac{\partial Y_{r-c}}{\partial tF}\right) \geq \sqrt{2}$, then both the conditions of Proposition 1 are satisfied implying that Possibility 1 is the only equilibrium outcome. If, instead, $f \cdot \left(\frac{\partial Y_{p-c}}{\partial tF}\right) + (1-f) \cdot \left(\frac{\partial Y_{r-c}}{\partial tF}\right) < \sqrt{2} \Leftrightarrow \pi^*|_{\text{Possibility } 1} < 0$ (see the proof of Proposition 1). Also, $\frac{\partial Y_{r-c}}{\partial tF} < \sqrt{\frac{2}{1-t}} \Leftrightarrow \pi^*|_{\text{Possibility } 2} < 0$ (see the proof of Proposition 2). Both the maximized profits being negative, the firm does not enter into the neighbourhood.

Consider Configuration 2(b): $\frac{\sqrt{2}}{1+\sqrt{1-t}} \leq \frac{\partial Y_{p-c}}{\partial tF} \leq \sqrt{2}$ and $\frac{\partial Y_{r-c}}{\partial tF} \geq \sqrt{\frac{2}{1-t}}$. Since $\frac{\sqrt{2}}{1+\sqrt{1-t}} \leq \frac{\partial Y_{p-c}}{\partial tF}$ and $\frac{\partial Y_{r-c}}{\partial tF} \geq \sqrt{\frac{2}{1-t}}$, it follows from Observation 1 that $f \cdot \left(\frac{\partial Y_{p-c}}{\partial tF}\right) + (1-f) \cdot \left(\frac{\partial Y_{r-c}}{\partial tF}\right) \geq \sqrt{2}$. Now if $\left(\frac{\partial Y_{r-c}}{\partial tF}\right) \leq \left(1 + \frac{1}{\sqrt{1-t}}\right) \left(\frac{\partial Y_{p-c}}{\partial tF}\right)$, then both the conditions of Proposition 1 are satisfied implying that Possibility 1 is the only equilibrium outcome. If, instead, $\left(\frac{\partial Y_{r-c}}{\partial tF}\right) > \left(1 + \frac{1}{\sqrt{1-t}}\right) \left(\frac{\partial Y_{p-c}}{\partial tF}\right)$, then both the conditions of Proposition 2 are satisfied implying that Possibility 2 is the only equilibrium outcome.

**Configuration 3:** $\frac{\partial Y_{p-c}}{\partial tF} > \sqrt{2}$

Consider Configuration 3(a): $\frac{\partial Y_{p-c}}{\partial tF} > \sqrt{2}$ and $\left(\frac{\partial Y_{r-c}}{\partial tF}\right) \leq \left(1 + \frac{1}{\sqrt{1-t}}\right) \left(\frac{\partial Y_{p-c}}{\partial tF}\right)$. Here both the conditions of Proposition 1 are satisfied implying that Possibility 1 is the only equilibrium outcome.

Consider Configuration 3(b): $\frac{\partial Y_{p-c}}{\partial tF} > \sqrt{2}$ and $\left(\frac{\partial Y_{r-c}}{\partial tF}\right) > \left(1 + \frac{1}{\sqrt{1-t}}\right) \left(\frac{\partial Y_{p-c}}{\partial tF}\right)$. It follows that $\frac{\partial Y_{r-c}}{\partial tF} > \left(1 + \frac{1}{\sqrt{1-t}}\right) \sqrt{2} > \sqrt{\frac{2}{1-t}}$. Then both the conditions of Proposition 2 are satisfied implying that Possibility 2 is the only equilibrium outcome.

This completes the proof of Proposition 3.
10.5 Proof of Proposition 8

Let the poor receive a targeted price subsidy of $s$ per unit, that is, if the monopolist charges a price $p$, then a poor consumer pays the price $p - s$, while a rich consumer continues paying price $p$. The following proposition (that follows easily following the method outlined in section 3) summarizes how propositions 1 and 2 and corollary 1 get modified in the presence of this targeted price subsidy.

Proposition A.1.

Suppose the poor receives a targeted price subsidy of $s$ per unit.

(a) In equilibrium the firm charges a price

$$p^*_1|\text{Possibility 1} = \frac{\theta(fY_P + (1-f)Y_R) + fs + c}{2}$$

and serves both the rich and the poor if and only if the following conditions hold:

1. \( \left( \frac{\theta Y_R - c}{\sqrt{1-f}} \right) \leq \left( 1 + \frac{1}{\sqrt{1-f}} \right) \left( \frac{\theta Y_P + s - c}{\sqrt{1-f}} \right) \), and

2. \( f \cdot \left( \frac{\theta Y_P + s - c}{\sqrt{1-f}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{1-f}} \right) \geq \sqrt{2} \).

(b) For the poor to have any market access it is necessary that the following condition holds:

$$\frac{\theta Y_P + s - c}{\sqrt{1-f}} \geq \frac{\sqrt{2}}{1+\sqrt{1-f}}.$$  

(c) In equilibrium the firm charges a price

$$p^*_2|\text{Possibility 2} = \frac{\theta Y_R + c}{2}$$

and serves only the rich consumers (the poor are completely left out) if and only if the following conditions hold:

1. \( \left( \frac{\theta Y_R - c}{\sqrt{1-f}} \right) > \left( 1 + \frac{1}{\sqrt{1-f}} \right) \left( \frac{\theta Y_P + s - c}{\sqrt{1-f}} \right) \), and

2. \( \frac{\theta Y_R - c}{\sqrt{1-f}} \geq \sqrt{\frac{2}{1-f}} \).

In what follows we use this proposition and the corresponding equilibrium characterization to analyze the effects of this targeted price subsidy on market access and welfare of the rich and poor.
10.5.1 Effects of Targeted Price Subsidy on Market Access and Welfare of the Poor

We show that, as expected, all the threshold effects are positive. The lower bound on poor income declines to \( Y^*_{P} = \frac{1}{\theta} \left( c + \frac{\sqrt{2fP}}{1+\sqrt{1-f}} - s \right) \); thus it becomes relatively easier for the poor people to access the service. To see that the provision effect is positive consider Configuration 2(a) with \( f \cdot \left( \frac{\theta Y_P - c}{\sqrt{tP}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) < \sqrt{2} \) so that the firm does not enter into the neighbourhood. In such a situation, a targeted price subsidy so as to revert this inequality to \( f \cdot \left( \frac{\theta Y_P + s - c}{\sqrt{tP}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) > \sqrt{2} \) makes it possible for the firm to enter into the neighbourhood and charge \( p^*_s \) Possibility 1 so that the poor earns a positive market access and consumer surplus.

The positive inclusion effects can be demonstrated in the following circumstances. First consider Configuration 1(b) so that the firm is serving only the rich and the poor are completely excluded. If a targeted price subsidy makes us move to Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) < \left( 1 + \frac{1}{\sqrt{1-f}} \right) \left( \frac{\theta Y_P + s - c}{\sqrt{tP}} \right) \), then the firm switches to lowering price to \( p^*_s \) Possibility 1 and serving both the rich and the poor. Consider next Configuration 3(b) or Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) > \left( 1 + \frac{1}{\sqrt{1-f}} \right) \left( \frac{\theta Y_P - c}{\sqrt{tP}} \right) \) so that the firm serves only the rich and the poor are completely excluded. In situations like these, a targeted price subsidy so as to revert this inequality to \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) < \left( 1 + \frac{1}{\sqrt{1-f}} \right) \left( \frac{\theta Y_P + s - c}{\sqrt{tP}} \right) \) makes it possible for the firm to charge \( p^*_s \) Possibility 1 and include the poor into the service.

Within the thresholds when the poor has market access under Possibility 1, the expressions for aggregate market access and consumer surplus of the poor with a targeted price subsidy are

\[
A^*_P = \frac{2}{t} \left( \theta Y_P - p^*_s \right) \text{Possibility 1} + s) = \frac{2}{t} \left[ \theta Y_P - \frac{\theta (fY_P + (1 - f)Y_R) + fs + c}{2} + s \right],
\]

and

\[
CS^*_P = \frac{\left( \theta Y_P - p^*_s \right) \text{Possibility 1} + s)^2}{t} = \frac{1}{t} \left[ \theta Y_P - \frac{\theta (fY_P + (1 - f)Y_R) + fs + c}{2} + s \right]^2.
\]

Clearly \( A^*_P > A_P \), and \( CS^*_P > CS_P \), that is, both the market access and consumer surplus of the poor are higher under the targeted price subsidy scheme.

10.5.2 Effects of Targeted Price Subsidy on Market Access and Welfare of the Rich

We will show that, although targeted towards the poor, the threshold effects of these price subsidies on the market access and welfare of the rich are also positive. The positive provision-
sion effect can be demonstrated by considering either Configuration 1(a) or Configuration 2(a) with \( f \cdot \left( \frac{\theta Y_P - c}{\sqrt{tF}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) < \sqrt{2} \), so that the firm does not enter into the neighbourhood. In such a situation, if a targeted price subsidy reverts this inequality to \( f \cdot \left( \frac{\theta Y_P + s - c}{\sqrt{tF}} \right) + (1 - f) \cdot \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) > \sqrt{2} \), then the firm enters and the rich (along with the poor) starts enjoying positive market access and consumer surplus.

For the positive inclusion effects consider first Configuration 1(b) so that the firm is serving only the rich who are getting market access \( A_{R\|Possibility\;1} \) and consumer surplus \( CS_{R\|Possibility\;1} \). If a targeted price subsidy makes us move to Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) < \left( 1 + \frac{1}{\sqrt{1-t}} \right) \left( \frac{\theta Y_P + s - c}{\sqrt{tF}} \right) \), then the firm switches to lowering price to \( p^*\|Possibility\;1 \) and serving both the rich and the poor with the rich’s market access and consumer surplus increasing to \( A_{R\|Possibility\;1} > A_{R\|Possibility\;2} \) and \( CS_{R\|Possibility\;1} > CS_{R\|Possibility\;2} \), respectively. Next consider Configuration 2(b) with \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) > \left( 1 + \frac{1}{\sqrt{1-t}} \right) \left( \frac{\theta Y_P + s - c}{\sqrt{tF}} \right) \) or Configuration 3(b), so that, once again, the firm is serving only the rich and they are enjoying market access \( A_{R\|Possibility\;2} \) and consumer surplus \( CS_{R\|Possibility\;2} \). Now if a targeted price subsidy reverts this inequality to \( \left( \frac{\theta Y_R - c}{\sqrt{tF}} \right) < \left( 1 + \frac{1}{\sqrt{1-t}} \right) \left( \frac{\theta Y_P + s - c}{\sqrt{tF}} \right) \), then the firm switches to serving both the rich and the poor with the rich’s market access and consumer surplus increasing to \( A_{R\|Possibility\;1} > A_{R\|Possibility\;2} \) and \( CS_{R\|Possibility\;1} > CS_{R\|Possibility\;2} \), respectively.

Within the thresholds, under equilibrium Possibility 1, the expressions for aggregate market access and consumer surplus of the rich with a targeted price subsidy are

\[ A_{R\|Possibility\;1} = \frac{t}{2} \left( \theta Y_R - p^*\|Possibility\;1 \right) = \frac{2}{t} \left[ \theta Y_R - \frac{\theta (fY_P + (1 - f)Y_R) + fs + c}{2} \right], \]

and

\[ CS_{R\|Possibility\;1} = \frac{\left( \theta Y_R - p^*\|Possibility\;1 \right)^2}{t} = \frac{1}{t} \left[ \theta Y_R - \frac{\theta (fY_P + (1 - f)Y_R) + fs + c}{2} \right]^2. \]

Clearly \( A_{R\|Possibility\;1} < A_{R\|Possibility\;1} \), and \( CS_{R\|Possibility\;1} < CS_{R\|Possibility\;1} \). Since \( p^*\|Possibility\;1 > p^*\|Possibility\;1 \), the within-threshold effects of these price subsidies on the market access and welfare of the rich are negative. Finally, It is easy to verify that market access and consumer surplus of the rich remains unaffected with this price subsidy under equilibrium Possibility 2.

This completes the proof of Proposition 8.

10.6 Proof of Proposition 9

Let the government subsidizes the entry cost so that the new entry cost is \( F' < F \). The following implications are easy to verify. (i) Equilibrium price remains unchanged under
both possibilities 1 and 2. (ii) The first condition in both propositions 1 and 2 (conditions (2a) and (5a), respectively) remains unaffected. (iii) In both propositions 1 and 2, the LHS of the second condition (conditions (2b) and (5b), respectively) falls so that it becomes easier for the firm to meet this profitability condition. (iv) The LHS of the inequality in Corollary 1 (condition (3)) falls so that the lower bound on poor income declines. Now we use these implications to analyze the effects of this targeted price subsidy on market access and welfare of the rich and poor.

10.6.1 Effects of Entry Cost Subsidy on Market Access and Welfare of the Poor

First consider the threshold effects. The lower bound on poor income declines to $Y_{PF} = \frac{1}{\theta} \left( c + \frac{\sqrt{2}t_F}{1 + \sqrt{1 - j}} \right)$; thus it becomes relatively easier for the poor people to access the service. The provision effect is also positive. Consider Configuration 2(a) with $f \cdot \left( \frac{\partial Y_{P-F}}{\sqrt{t_F}} \right) + (1 - f) \cdot \left( \frac{\partial Y_{R-F}}{\sqrt{t_F}} \right) < \sqrt{2}$ so that the firm does not enter into the neighbourhood. In such a situation, a subsidy to the fixed entry cost so as to revert this inequality to $f \cdot \left( \frac{\partial Y_{P-F}}{\sqrt{t_F}} \right) + (1 - f) \cdot \left( \frac{\partial Y_{R-F}}{\sqrt{t_F}} \right) > \sqrt{2}$ makes it possible for the firm to enter into the neighbourhood and charge $p^*|_{\text{Possibility 1}}$ so that the poor earns a positive market access and consumer surplus.

We find that the inclusion/exclusion effects are either positive or neutral. To see the positive inclusion effect consider Configuration 1(b), $\frac{\partial Y_{P-F}}{\sqrt{t_F}} < \frac{\sqrt{2}}{1 + \sqrt{1 - j}}$ and $\frac{\partial Y_{R-F}}{\sqrt{t_F}} \geq \frac{\sqrt{2}}{1 - j}$, so that the firm is serving only the rich and the poor are completely excluded. A subsidy to the fixed entry cost so that the the first inequality is reversed (while maintaining the second inequality) moves us to Configuration 2(b) with $\left( \frac{\partial Y_{R-F}}{\sqrt{t_F}} \right) < \left( 1 + \frac{1}{\sqrt{1 - j}} \right) \left( \frac{\partial Y_{P-F}}{\sqrt{t_F}} \right)$. Then the firm switches to lowering price to $p^*|_{\text{Possibility 1}}$ and serving both the rich and the poor. On the other hand, since the first condition in both propositions 1 and 2 (conditions (2a) and (5a), respectively) remains unaffected, it is easy to see that the inclusion/exclusion effect is neutral in all other configurations.

Within the thresholds, since the equilibrium price remains unchanged, market access and consumer surplus remain unaffected when the entry cost is subsidized.

10.6.2 Effects of Entry Cost Subsidy on Market Access and Welfare of the Rich

We find that the provision effects are positive. Consider first Configuration 1(a), $\frac{\partial Y_{P-F}}{\sqrt{t_F}} < \frac{\sqrt{2}}{1 + \sqrt{1 - j}}$ and $\frac{\partial Y_{R-F}}{\sqrt{t_F}} < \sqrt{\frac{2}{1 - j}}$; so that the firm does not enter into the neighbourhood. A subsidy to the fixed entry cost so that the second inequality is reversed moves us either to Configuration 1(b) or to Configuration 2(b) ensuring that the rich has positive market
access and consumer surplus. Provision effect of subsidized entry cost is also positive under Configuration 2(a) as discussed above in the context of market access and welfare of the poor.

Similar to the poor, for the rich also the inclusion/exclusion effects are either positive or neutral. As in the case of positive inclusion effect on the poor discussed above, a move from Configuration 1(b) to 2(b) increases the rich’s market access and consumer surplus to $A_R|_{\text{Possibility 1}} > A_R|_{\text{Possibility 2}}$ and $CS_R|_{\text{Possibility 1}} > CS_R|_{\text{Possibility 2}}$, respectively. The inclusion/exclusion effect is neutral in all other configurations.

Within the thresholds, as in the case of the poor, market access and consumer surplus of the rich also remain unaffected when the entry cost is subsidized as this subsidy does not affect equilibrium prices either under Possibility 1 or under Possibility 2.

This completes the proof of Proposition 9.
References


