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Source: *The Journal of Political Economy*, Vol. 100, No. 4 (Aug., 1992), pp. 818-834
Published by: The University of Chicago Press
Stable URL: http://www.jstor.org/stable/2138689
Accessed: 04/08/2010 08:44

Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality

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In this paper, we present an overlapping generations model with heterogeneous agents in which human capital investment through formal schooling is the engine of growth. We use simple functional forms for preferences, technologies, and income distribution to highlight the distinction between economies with public education and those with private education. We find that income inequality declines more quickly under public education. On the other hand, private education yields greater per capita incomes unless the initial income inequality is sufficiently high. We also find that societies will choose public education if a majority of agents have incomes below average.

I. Introduction

The main purpose of our study is to examine the implications of public investment in human capital on growth and the evolution of income inequality in an economy in which individuals have different income/skill levels. We concentrate on the formal schooling component of human capital investment as the engine of growth. We construct a model in which some decisions (such as time allocated to schooling) are made privately but others are made through majority

We would like to thank Bill Johnson, Alan Krueger, Jon Skinner, and seminar participants at the University of Virginia, the Federal Reserve Bank of Richmond, and the NBER Summer Institute for their comments. We would especially like to thank two anonymous referees for their helpful suggestions. Financial support from the Bankard Fund is gratefully acknowledged.

[*Journal of Political Economy,* 1992, vol. 100, no. 4]

voting (such as funding for schools). We then compare the results with those obtained from a model in which education is privately financed. Finally, we endogenize the choice of educational system.

Recent models of economic growth, such as Romer (1986) and Lucas (1988), emphasize investment in human capital as an important factor contributing to growth. These models generate persistent growth endogenously from the actions of individuals in the economy. To the extent that a significant component of human capital investment is formal schooling, these models do not account for the large involvement of the public sector in human capital investment.[1] Further, most models of long-run growth are representative agent models and, therefore, could not address any issues related to income distribution (an exception is Tamura [1991]).

In Section II, we construct an overlapping generations economy in which heterogeneous individuals live for two periods. Each agent's stock of human capital depends on the parent's stock of human capital, time spent in school, and the quality of schools. Each parent has a bequest motive and values the quality of education passed on to the offspring. Under the public education regime, a government levies taxes on the income of the old and uses tax revenues to provide "free" public education. The quality of public education is an increasing function of the tax revenues. The tax rate is determined endogenously by the old agents in each period through majority voting. This captures the idea that funding for public schools is typically an outcome of some political process.

In the private education regime, individuals allocate their income between the quality of education passed on to the offspring and their own consumption. In Section III, we define and solve for the competitive equilibrium for both education regimes.

The generations in our model are linked through two channels. The stock of human capital of parents affects their children's learning; the effects of this linkage are specific to the household. The second linkage between generations occurs through bequests; in our model, the bequest is the quality of education received by the children. In the public education regime, the latter linkage does not differ across agents of the same generation since school quality under the public education system is common to all agents.

We choose simple functional forms for preferences, technologies, and the initial income distribution. Preferences are logarithmic, production technologies are linear, and the learning technology is Cobb-

[1] In the United States, for instance, over the last 100 years the fraction of students at the elementary and secondary levels who attend public schools has never been below 86 percent (see *Digest of Education Statistics* [1989], U.S. Department of Education).

Douglas. We assume the initial income distribution to be lognormal. We have chosen this restrictive specification to highlight the influence of income distribution on economic growth and vice versa. Further, it also helps us highlight the distinction between economies in which the quality of education is determined through collective decisions and those in which the quality of education is chosen privately.

In Section IV, we compare the public and private education regimes when the population is homogeneous. In both regimes, a necessary condition for persistent growth is that the learning technology must exhibit nondecreasing returns to the quality of schools and the parental stock of human capital. Further, we find that per capita income under private education is higher than per capita income under public education, in each period.

In Section V, we examine the public and private education regimes for the heterogeneous population. Our results can be briefly summarized as follows: (i) Income inequality declines faster under public education than under private education. (ii) If two public education economies begin with the same per capita income but differ in income inequality, then the economy with lower inequality has higher per capita income in all future periods; this result holds for two private education economies under some additional restrictions. (iii) If the income inequality is sufficiently high, then the public education regime may yield higher per capita income for some future periods.

In Section VI, we let the old agents vote each period on whether the educational system should be private or public. We establish necessary and sufficient conditions under which a majority of old agents would prefer the public education system. Section VII contains the concluding remarks.

II. The Basic Framework

Consider an overlapping generations economy in which individuals live for two periods and die at the end of the second period. In the second period of life, each individual gives birth to another so that the population remains constant over time. Each generation consists of a continuum of agents. Agents within a generation are differentiated by the stock of human capital of their parents. At time $t = 0$, there is an initial generation of old agents in which the jth member is endowed with knowledge h_{j0}. Knowledge of the members of the initial generation is distributed according to the (probability) distribution function $G_0(\cdot)$. We restrict our attention to initial income distributions that are lognormal with parameters μ_0 and σ_0^2. In what follows, we suppress the index j to make the notation less cumbersome.

All individuals born at $t = 0, 1, 2, \ldots$ have identical preferences over leisure when young, consumption when old, and the bequest left to their offspring. Formally, the preferences of an individual j born at time t are represented by $\ln n_t + \ln c_{t+1} + \ln e_{t+1}$, where n_t is leisure at time t, c_{t+1} is consumption at time $t + 1$, and e_{t+1} is the quality of schools at time $t + 1$. The bequest in our model is the quality of schools.[2]

Individuals are endowed with one divisible unit of time in their youth. Young individuals at time t allocate n_t units of their endowment toward leisure at time t and devote the remaining $1 - n_t$ units toward accumulating human capital according to

$$h_{t+1} = \theta(1 - n_t)^{\beta} e_t^{\gamma} h_t^{\delta}, \quad \theta > 0, \tag{1}$$

where h_t is the stock of human capital of the corresponding parent. We assume that $\beta, \gamma, \delta \in (0, 1)$ so that all factors exhibit diminishing returns. At time $t + 1$, an individual's income is the same as his human capital h_{t+1}.

Our assumption that the quality of schools is an argument in the learning technology is consistent with Card and Krueger (1992), who provide estimates of the effects of school quality measured by student/teacher ratio, the average term length, and the relative pay of teachers on the rate of return to education for men born in the United States between 1920 and 1949. They find that men educated in states with high average school quality have a higher return to additional years of schooling.[3] On the theoretical side, our assumption is similar to the learning technology in Lucas (1988). A discrete-time version of his technology may be written as $h_{t+1} - h_t = \delta(1 - n_t)h_t$, where δ can be interpreted as the quality of education.

The use of parental knowledge as an input in the learning function is consistent with a number of studies. Coleman et al. (1966), for instance, found a positive correlation between parental education and performance on standardized tests.

Under the public education regime, each individual's earnings at

[2] There are at least three formulations of intergenerational altruism in the distribution and growth literature. First, members of the current generation could value the utility level achieved by their descendants (e.g., Loury 1981). Second, they could value the allocations of their descendants (e.g., Kohlberg 1976). Third, members of the current generation may value the wealth they pass on to their descendants (e.g., Banerjee and Newman 1989). In the first two formulations, members of each generation take as given the optimal decision rules of their offspring. In the third formulation, individuals simply allocate their wealth optimally between their own consumption and the bequests to the offspring. The equilibrium concept is significantly simpler in the third approach.

[3] See Hanushek (1986) for a review of the evidence on the impact of various measures of quality of education (inputs) on test scores and graduation rates (educational outputs).

time $t + 1$ are taxed at the rate τ_{t+1}. Total tax revenues determine the quality of public schools at time $t + 1$ according to

$$E_{t+1} = \tau_{t+1} H_{t+1}, \tag{2}$$

where $H_{t+1} \equiv \int h_{t+1} dG_{t+1}(h_{t+1})$. We use E to denote the quality of schools in the public education regime to emphasize the fact that all individuals face the same quality and that it is outside the control of one agent. However, the quality is endogenously determined in each period through majority voting. Thus the only difference between agents born at time t is the skill of their parents, which is an input to their learning technology.

The private education regime differs from the public education regime only in the determination of the quality of education e_{t+1}. Each individual in the private education regime allocates his income h_{t+1} between own consumption, c_{t+1}, and the quality of education, e_{t+1}, for the offspring, that is, $c_{t+1} + e_{t+1} = h_{t+1}$. Note that all variables including the quality of education are individual-specific in the private education regime.

III. Equilibrium

In the public education regime, we solve individual j's optimization problem in two steps. First, we solve for optimal effort, consumption, and human capital investment; that is, individual j's problem is to choose n_t and c_{t+1} to maximize

$$\ln n_t + \ln c_{t+1} + \ln E_{t+1}$$

subject to

$$c_{t+1} = (1 - \tau_{t+1}) h_{t+1},$$
$$h_{t+1} = \theta(1 - n_t)^{\beta} E_t^{\gamma} h_t^{\delta},$$

given E_t, h_t, E_{t+1}, and τ_{t+1}.

The quality of schools and the parent's human capital at time t are already determined in the beginning of the period. In the next step, we solve for the agent's preferred tax rate by maximizing

$$\ln[(1 - \tau_{t+1}) h_{t+1}] + \ln \tau_{t+1} H_{t+1},$$

where H_{t+1} is the mean income at time $t + 1$. In the optimization problem above, we have implicitly assumed that the young agent at time t cannot commit himself to a tax rate at time $t + 1$. Note that the old agent's choice of tax rate does not alter his income but affects the fraction of income he can consume.

Equilibrium under Public Education

The equilibrium for the public education economy is a set of sequences $\{n_t\}_{t=0}^{\infty}$, $\{h_{t+1}\}_{t=0}^{\infty}$, $\{c_t\}_{t=0}^{\infty}$, $\{G_{t+1}(\cdot)\}_{t=0}^{\infty}$, $\{E_t\}_{t=0}^{\infty}$, and $\{\tau_t\}_{t=0}^{\infty}$ such that (i) n_t and c_{t+1} are the optimal choices of an agent born at time t whose parent's human capital is h_t; (ii) the human capital of each agent is determined by $h_{t+1} = \theta(1 - n_t)^{\beta}E_t^{\gamma}h_t^{\delta}$; (iii) given the distribution $G_t(\cdot)$ at time t, the distribution of income $G_{t+1}(\cdot)$ at time $t + 1$ is determined by the transformation of variables $h_{t+1} = \theta(1 - n_t)^{\beta}E_t^{\gamma}h_t^{\delta}$; (iv) the tax rate τ_t is preferred by a majority of old agents at time t; and (v) the quality of schools at time t is $E_t = \tau_t \int h_t dG_t(h_t)$.

It is easy to see that the time allocated to human capital investment by an individual born at time t is

$$1 - n_t = \frac{\beta}{1 + \beta}. \tag{3}$$

It is independent of the tax rate and the individual type because of the log preferences and Cobb-Douglas learning technology. The individual's stock of human capital at time $t + 1$ is independent of the tax rate τ_{t+1} but depends on his parent's stock of knowledge and is given by

$$h_{t+1} = \theta E_t^{\gamma}\left(\frac{\beta}{1 + \beta}\right)^{\beta} h_t^{\delta}. \tag{4}$$

Equation (4) describes the evolution of human capital for a given individual type. It is easy to see that each individual's preferred tax rate is given by

$$\tau_{t+1} = 1/2. \tag{5}$$

It is independent of individual income and constant over time because of log preferences over consumption and bequests.[4]

If human capital at time t is lognormally distributed with mean μ_t and variance σ_t^2, then human capital at time $t + 1$ is also lognormally distributed with mean μ_{t+1} and variance σ_{t+1}^2, where

$$\mu_{t+1} = \ln\left[\theta E_t^{\gamma}\left(\frac{\beta}{1 + \beta}\right)^{\beta}\right] + \delta\mu_t \tag{6a}$$

and

$$\sigma_{t+1}^2 = \delta^2\sigma_t^2. \tag{6b}$$

[4] If, for instance, preferences over bequests were $E^{1-\alpha}/(1 - \alpha)$, $\alpha \geq 0$, then the equilibrium tax rate would depend on per capita income and, hence, on time. The preferred tax rate τ must satisfy the first-order condition $1/(1 - \tau) = H^{1-\alpha}/\tau^{\alpha}$ so that τ is an increasing function of H if $1 - \alpha > 0$ and a decreasing function of H if $1 - \alpha < 0$.

With constant tax rates, equation (4) reduces to

$$h_{t+1} = \theta(\tfrac{1}{2})^{\gamma}\left(\frac{\beta}{1+\beta}\right)^{\beta}(H_t)^{\gamma}h_t^{\delta} \equiv A(H_t)^{\gamma}h_t^{\delta}, \tag{7}$$

where $A \equiv \theta(\tfrac{1}{2})^{\gamma}[\beta/(1+\beta)]^{\beta}$. For lognormal distribution, per capita income at time t is $H_t = \exp[\mu_t + (\sigma_t^2/2)]$, so that we can write

$$\mu_{t+1} = \ln(A) + \gamma\ln(H_t) + \delta\mu_t = \ln(A) + (\gamma+\delta)\mu_t + \frac{\gamma\sigma_t^2}{2}.$$

In the private education regime, the young individual chooses n_t, c_{t+1}, and e_{t+1} to maximize

$$\ln n_t + \ln c_{t+1} + \ln e_{t+1}$$

subject to

$$h_{t+1} = \theta(1-n_t)^{\beta}e_t^{\gamma}h_t^{\delta},$$

$$c_{t+1} = h_{t+1} - e_{t+1},$$

given e_t and h_t.

Equilibrium under Private Education

The equilibrium for the private education economy is a set of sequences $\{n_t\}_{t=0}^{\infty}$, $\{e_t\}_{t=0}^{\infty}$, $\{c_t\}_{t=0}^{\infty}$, $\{h_{t+1}\}_{t=0}^{\infty}$, and $\{G_{t+1}(\cdot)\}_{t=0}^{\infty}$ such that (i) n_t, c_{t+1}, and e_{t+1} are the optimal choices of an agent born at time t whose parent's human capital is h_t; (ii) the human capital of each agent is determined by $h_{t+1} = \theta(1-n_t)^{\beta}e_t^{\gamma}h_t^{\delta}$; and (iii) given the distribution $G_t(\cdot)$ at time t, the distribution of income $G_{t+1}(\cdot)$ at time $t+1$ is determined by the transformation of variables $h_{t+1} = \theta(1-n_t)^{\beta}e_t^{\gamma}h_t^{\delta}$.

Clearly, an agent born at time t will choose future consumption and quality of education to be $c_{t+1} = e_{t+1} = \tfrac{1}{2}h_{t+1}$. The quality is agent-specific: an agent with high income will bequeath high quality. The time allocated to human capital investment is then determined by the first-order condition $1/n_t = 2\beta/(1-n_t)$, which implies

$$1 - n_t = \frac{\beta}{\tfrac{1}{2}+\beta}. \tag{8}$$

Note that the time devoted to human capital accumulation is different in the two economies. In the private education regime, each agent accounts for the fact that an additional unit of time spent toward learning increases not only his earnings but also the bequests passed on to his offspring. In the public education regime, the latter benefit is not taken into account; each agent views his contribution to the quality of public education as negligible.

The agent's stock of human capital at $t + 1$ in the private education regime is

$$h_{t+1} = Bh_t^{\gamma+\delta}, \tag{9}$$

where $B \equiv \theta(1/2)^{\gamma}[\beta/(1/2 + \beta)]^{\beta}$. Note that B is greater than A, defined in the public education regime. Again, if $\ln(h_t)$ is normally distributed with mean μ_t and variance σ_t^2, then $\ln(h_{t+1})$ is normally distributed with mean $\mu_{t+1} = \ln B + (\gamma + \delta)\mu_t$ and variance $\sigma_{t+1}^2 = (\gamma + \delta)^2\sigma_t^2$.

IV. Homogeneous Agents

In this section we compare the equilibrium paths of per capita income for the two education regimes when the initial generation is homogeneous; that is, the initial distribution of income is degenerate so that the per capita income at time t coincides with the representative agent's income. The purpose here is to abstract from distributional issues and compare the levels and growth rates of income in the two education regimes. To distinguish incomes in the two regimes we shall use superscript u for the public regime and superscript r for the private regime.

From (7) and (9), the evolution of income in the two regimes may be written as

$$h_{t+1}^u = A(h_t^u)^{\gamma+\delta} \tag{10}$$

and

$$h_{t+1}^r = B(h_t^r)^{\gamma+\delta}. \tag{11}$$

From these laws of motion, we first establish conditions for the existence and uniqueness of the (nontrivial) steady-state income/human capital and compare the steady states in the two economies.[5]

PROPOSITION 1. (*a*) If $\gamma + \delta \neq 1$, then there exists a unique steady state given by (i) $h_s^u > 0$ such that $h_{t+1}^u = h_s^u$ whenever $h_t^u = h_s^u$, (ii) $h_s^r > 0$ such that $h_{t+1}^r = h_s^r$ whenever $h_t^r = h_s^r$, and (iii) for $\gamma + \delta < 1$, $h_s^r > h_s^u$, and for $\gamma + \delta > 1$, $h_s^r < h_s^u$. (*b*) If $\gamma + \delta = 1$ and $A \neq 1$, then there does not exist a steady state for the public education economy. (*c*) If $\gamma + \delta = 1$ and $B \neq 1$, then there does not exist a steady state for the private education economy.

Proof. For part *a*, see figure 1*a* and *c*. Parts *b* and *c* follow directly from the laws of motion (10) and (11) (see fig. 1*b*). Q.E.D.

It should be clear from equations (10) and (11) that the evolution of human capital in both economies is similar to the capital accumulation

[5] By steady-state income we mean a level h^* such that $h_t = h^* \Rightarrow h_{t+1} = h^*$. For both economies there is a trivial steady state in which income is zero.

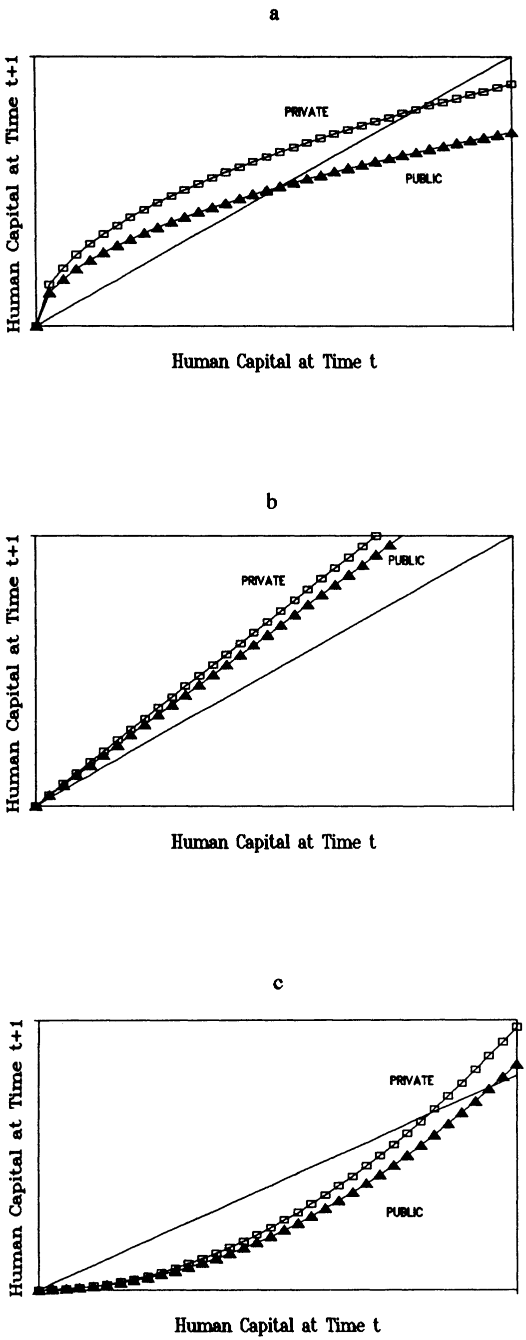

FIG. 1.—Human capital accumulation in a homogeneous household economy: *a*, decreasing returns; *b*, constant returns; *c*, increasing returns.

equation in the Cass-Koopmans framework. Thus, as figure 1 illustrates, when $\gamma + \delta < 1$, the steady state is globally stable in both economies and independent of the initial stock of human capital. Further, when $\gamma + \delta > 1$, the steady state is unstable in both economies; when $\gamma + \delta = 1$, a steady state typically will not exist.

In the next three propositions we compare the public and private education economies. To make the comparisons legitimate, we shall assume that both private and public education economies start off with the same (positive) level of initial income h_0. In proposition 2, we establish that the private education economy has a higher income level than the public education economy in every time period. In proposition 3, we show that every generation is better off in the private education economy. In proposition 4, we compare the long-run growth rates.

PROPOSITION 2. $h_t^r > h_t^u$ for all $t > 0$.

Proof. See figure 1.

PROPOSITION 3. $V_t^r > V_t^u$ for all $t \geq 0$, where V_t^r is the equilibrium level of utility of an agent born at time t in the private education economy and V_t^u is the corresponding utility in the public education economy.

Proof. See the Appendix.

PROPOSITION 4. (*a*) If $\gamma + \delta < 1$, then $\lim_{t\to\infty} (h_{t+1}^r/h_t^r) = \lim_{t\to\infty} (h_{t+1}^u/h_t^u) = 1$. (*b*) If $\gamma + \delta = 1$, then $h_{t+1}^r/h_t^r = B > A = h_{t+1}^u/h_t^u$ for all $t \geq 0$. (*c*) For $\gamma + \delta > 1$, (i) h_{t+1}^r/h_t^r is greater than one and increasing over time if $h_0 > h_s^r$ and (ii) h_{t+1}^u/h_t^u is greater than one and increasing over time if $h_0 > h_s^u$.

Proof. (*a*) See figure 1*a*. (*b*) When $\gamma + \delta = 1$, h_{t+1}^r/h_t^r is a constant equal to B for all $t \geq 0$. This is clearly greater than the gross growth rate in the public education economy, which equals A. (*c*) See figure 1*c*. Q.E.D.

The main reason why the evolution of income in the public education economy is different from that in the private education economy is that the time devoted to human capital accumulation is different in the two economies. Proposition 2 states that along the equilibrium path a private education economy yields higher income levels in all periods than a public education economy. As noted in Section III, the time devoted to human capital accumulation in a private education economy is higher than that in the public education economy, and hence incomes are higher. By continuity, it seems reasonable to expect that the result would hold if the population was "slightly" heterogeneous. However, if the population was sufficiently heterogeneous, then the public education economy may yield higher mean incomes for some future periods than a private education economy. We provide an example in the next section to demonstrate this result.

Proposition 3 states that the representative agent in the private education economy is better off than his counterpart in the public education economy. It is clear that the equilibrium allocations in the public education economy are feasible for the representative agent in the private education economy. By optimizing he can do better and attain a higher level of utility.

The key parameter that determines growth in our model is the sum $\gamma + \delta$. In both regimes, quality of education and human capital are the two channels through which accumulation takes place. Since our learning technology is Cobb-Douglas, income at time $t + 1$ depends critically on the sum of the exponents on quality and human capital at time t, that is, on $\gamma + \delta$.

The basic content of proposition 4 is that increasing returns are neither necessary nor sufficient for long-run growth in either economy. For the case $\gamma + \delta < 1$, the long-run growth rate is zero. This result is analogous to the zero net investment result in the Cass-Koopmans capital accumulation model. When $\gamma + \delta = 1$, the growth rate is constant and the magnitude of this constant depends on θ and β. For instance, if $\theta(1/2)^{\gamma}[\beta/(1 + \beta)]^{\beta} > 1$, then both economies will exhibit long-run growth. Again, if one examined linear technologies in the Cass-Koopmans framework, this result should be familiar (see Jones and Manuelli 1990). When $\gamma + \delta > 1$, we get unbounded growth depending on the initial conditions.

Two remarks are in order here. First, given the same initial conditions, propositions 2 and 4 suggest that the private education economy with homogeneous population achieves higher incomes and growth rates than the public education economy whenever $\gamma + \delta \geq 1$. Second, if a policy of mandatory schooling is enforced, then the allocations in the public education regime would be the same as in the private education regime. That is, if we set the time allocated to human capital investment equal to $\beta/(1/2 + \beta)$ in the public education regime, then the law of motion of human capital is identical in both regimes. Hence, the allocations in the two regimes must be the same.

V. Heterogeneous Agents

In this section we examine the heterogeneous agents case. As stated in Section II, the initial income distribution is assumed to be lognormal with parameters μ_0 and σ_0^2. The assumption helps us characterize the evolution of income inequality over time. It also makes the comparison between private and public education economies very convenient. In our model, income inequality at time t is naturally described by the parameter σ_t.[6] In proposition 5 below, we characterize the evolution of income inequality in both public and private education economies.

PROPOSITION 5. (*a*) In the public education economy, income inequality declines over time. (*b*) In the private education economy, income inequality declines over time if $\gamma + \delta < 1$, increases over time if $\gamma + \delta > 1$, and remains constant over time if $\gamma + \delta = 1$.

Proof. (*a*) As noted in Section III, in the public education economy, $\sigma_{t+1}^2 = \delta^2\sigma_t^2 < \sigma_t^2$ since δ is less than unity. (*b*) In the private education economy, $\sigma_{t+1}^2 = (\gamma + \delta)^2\sigma_t^2$, which is less than σ_t^2 if and only if $\gamma + \delta < 1$. Q.E.D.

Part *a* of proposition 5 follows directly from equation (7). Since $\delta < 1$, h_{t+1}/h_t is a decreasing function of h_t; that is, households with low incomes experience higher growth rates than households with high incomes so that income inequality declines over time. Thus income distribution in the long run is degenerate. The intuition for part *b* is similar since equation (9) has the same implications as (7). But note that even if $\gamma + \delta < 1$, income inequality in the private education economy does not decline as fast as in the public education economy.

The income convergence result in our public education economy is similar to that in Tamura (1991). In his model, the learning technology exhibits spillovers: each agent's stock of human capital tomorrow is not only a function of his private stock today but also a function of the average human capital stock of society today. In ours, all agents in the public education regime face the same quality that is a function of average income. In both models the growth rate of any agent's income is inversely related to the level of his income. Thus agents with income below the average grow faster than agents with income above the average. This is also the reason why we get income convergence in the private education economy when $\gamma + \delta < 1$.

The income convergence result also implies that the conditions for long-run growth in per capita income in the public education regime with heterogeneous agents are identical to those in the homogeneous agent economy (see proposition 4). It turns out that proposition 4 also holds for the private education economy with heterogeneous agents. This follows from equation (9). If $\gamma + \delta < 1$, then income distribution in the long run is degenerate so that the conditions for growth in per capita income are the same as those in the homoge-

[6] Alternatively, one could rank income distributions by Gini coefficients or Lorenz curves. For lognormal income distribution, the Gini coefficient depends only on σ (see McDonald and Ransom 1979, p. 1515). Further, ranking lognormal income distributions according to Gini coefficients is equivalent to ranking them according to the Lorenz criterion (see Aitchison and Brown 1969).

neous economy. If $\gamma + \delta = 1$, then all agents in the private education economy grow at the same constant rate, and hence the conditions hold for the "average" agent. If $\gamma + \delta > 1$, then, depending on initial conditions, the per capita income grows at an increasing rate without bounds.[7] Finally, note that along the balanced growth path ($\gamma + \delta = 1$) income inequality declines in the public education economy but stays the same in the private education economy.

The next proposition relates current income inequality to future levels of per capita income.

PROPOSITION 6. (*a*) Consider two distinct public education economies with the same mean income at time t, that is, $H_t = H'_t$. If $\sigma'_t > \sigma_t$, then $H_{t+1} > H'_{t+1}$. (*b*) Similarly, consider two distinct private education economies with $H_t = H'_t$ and $\sigma'_t > \sigma_t$. Then $H_{t+1} > H'_{t+1}$ if and only if $\gamma + \delta < 1$.

Proof. See the Appendix.

Proposition 6 states that if two public education economies start off with the same per capita income but different income distributions, then the economy with a lower income inequality will have a higher per capita income in the future. One way to see this is to think of the incomes in the two economies, h_t and h'_t, as random variables so that h'_t is a mean-preserving spread of h_t. The transformation of the random variable between periods t and $t + 1$ in each economy is $h_{t+1} = A(H_t)^{\gamma}h_t^{\delta}$. This transformation is concave since $\delta < 1$. Part *a* then is a direct application of theorem 2 in Rothschild and Stiglitz (1970). Similar reasoning can be used to establish part *b*. Although proposition 6 establishes an order relation between per capita income levels only for the next period, it should be clear that the relation holds for all future periods.

As noted at the end of Section IV, the private education economy yields higher growth and per capita incomes when the initial income inequality is low. We show through an example below that if the initial income inequality is sufficiently high, then the public education economy may yield higher per capita income for some future periods than the private education economy. Consider two economies with the same income distribution at time t and technology parameters satisfying $\gamma + 2\delta < 1$ so that $(\gamma + \delta)^2 < \gamma + \delta^2$. Note that

$$\ln(H^u_{t+1}) = \ln(A) + (\gamma + \delta)\mu_t + \frac{(\gamma + \delta^2)\sigma_t^2}{2}$$

[7] The formal statement of the proposition and proof may be obtained from the authors on request.

and

$$\ln(H_{t+1}^r) = \ln(B) + (\gamma + \delta)\mu_t + \frac{(\gamma + \delta)^2\sigma_t^2}{2}.$$

If σ_t^2 is sufficiently large, then it is easy to check that $H_{t+1}^u > H_{t+1}^r$.

VI. Choice of Educational Regime

In the previous sections we exogenously imposed the educational system: either all agents attend public schools or all agents attend private schools. In this section we try to endogenize the choice of educational system. There are several ways to endogenize this choice: (i) parents pay taxes only if they send their children to public schools, (ii) all parents pay taxes but are free to send their children to private schools, and (iii) parents decide by majority vote whether the educational system should be private or public and no one can opt out.

In the first case, it is easy to see that no one would desire the public school system. The individual with the highest income can do better on his own and hence will not use the public schools. Once the richest individual opts out of the public education system, the second-richest individual has exactly the same incentives to opt out and the whole system unravels. For the second case, Stiglitz (1974) has shown that preferences over tax rates are not single-peaked. Hence, standard arguments do not guarantee the existence of a voting equilibrium.

We examine the third case in this section. In each period, the old generation decides by majority vote whether the educational system should be private or public. In the latter case it also decides the tax rate. The following proposition establishes necessary and sufficient conditions for a majority of old agents to choose the public education system at any time t.

PROPOSITION 7. A majority of old agents at time t would prefer public over private education if and only if $\sigma_t^2 > 0$.

Proof. Under public education, it is clear that the tax rate preferred by the majority of old agents is ½ and the indirect utility of an old agent with income h_t is

$$V_t^u = \ln(\tfrac{1}{2}h_t) + \ln(\tfrac{1}{2}H_t) = 2\ln\tfrac{1}{2} + \ln(h_t) + \mu_t + \frac{\sigma_t^2}{2}.$$

Under private education, the same agent's indirect utility is given by

$$V_t^r = 2\ln\tfrac{1}{2} + 2\ln(h_t).$$

Since the median voter is the agent with median income, we must have $\mu_t = \ln(h_t^m)$, where h_t^m is the median income at time t. Thus a majority of the old agents will have $V_t^u > V_t^r$ if and only if $\sigma_t^2 > 0$. Q.E.D.

The skewness of the income distribution is crucial to this result. In the public education regime, every parent's bequest depends on average income; in the private education regime, it depends on the parent's income. Since the median income is below the mean, majority voting results in public education.

VII. Conclusions

We have presented a model of endogenous economic growth with heterogeneous agents. We concentrate on the formal schooling aspect of human capital investment as the engine of growth in a model with simple functional forms for preferences, technologies, and income distribution. These functional forms help us obtain joint predictions on the growth of per capita income and the evolution of income distribution. We contrast two regimes of education: public schools, in which investment in the quality of schools is made through majority voting, and private schools, in which each household chooses its quality of education. We find that public education reduces income inequality more quickly than private education. On the other hand, private education yields higher per capita incomes unless the initial income inequality is sufficiently large. Finally, we endogenize the choice of education regime: if a majority of agents have income below average, then the vote is in favor of public education.

Appendix

Proof of Proposition 3

$$V_t^u \equiv \ln\left(\frac{1}{1+\beta}\right) + \ln[\tfrac{1}{2}A(h_t^u)^{\gamma+\delta}] + \ln[\tfrac{1}{2}A(h_t^u)^{\gamma+\delta}].$$

Substituting for A, we get

$$\begin{aligned} V_t^u &\equiv \ln\left(\frac{1}{1+\beta}\right) + 2\ln(\tfrac{1}{2}) + 2(\gamma+\delta)\ln(h_t^u) + 2\ln\left[\theta(\tfrac{1}{2})^{\gamma}\left(\frac{\beta}{1+\beta}\right)^{\beta}\right] \\ &\equiv 2\ln(\tfrac{1}{2}) + 2(\gamma+\delta)\ln(h_t^u) + 2\ln[\theta(\tfrac{1}{2})^{\gamma}\beta^{\beta}] - (1+2\beta)\ln(1+\beta). \end{aligned}$$

Similarly,

$$\begin{aligned} V_t^r &\equiv \ln\left(\frac{\tfrac{1}{2}}{\tfrac{1}{2}+\beta}\right) + 2\ln(\tfrac{1}{2}) + 2(\gamma+\delta)\ln(h_t^r) + 2\ln\left[\theta(\tfrac{1}{2})^{\gamma}\left(\frac{\beta}{\tfrac{1}{2}+\beta}\right)^{\beta}\right] \\ &\equiv 3\ln(\tfrac{1}{2}) + 2(\gamma+\delta)\ln(h_t^r) + 2\ln[\theta(\tfrac{1}{2})^{\gamma}\beta^{\beta}] - (1+2\beta)\ln(\tfrac{1}{2}+\beta). \end{aligned}$$

Clearly, $V_0^r > V_0^u$ if and only if

$$\begin{aligned}
&\ln(1/2) - (1 + 2\beta)\ln(1/2 + \beta) > -(1 + 2\beta)\ln(1 + \beta) \\
\Leftrightarrow &-\ln(2) - (1 + 2\beta)\ln(1 + 2\beta) + (1 + 2\beta)\ln(2) > -(1 + 2\beta)\ln(1 + \beta) \\
&\Leftrightarrow 2\beta \ln(2) > (1 + 2\beta)[\ln(1 + 2\beta) - \ln(1 + \beta)].
\end{aligned}$$

Now

$$\lim_{\beta\to 0} 2\beta \ln(2) = \lim_{\beta\to 0} (1 + 2\beta)[\ln(1 + 2\beta) - \ln(1 + \beta)],$$

both sides of the inequality are increasing in β, the right-hand side is convex in β, and, finally,

$$\lim_{\beta\to 1} 2\beta \ln(2) > \lim_{\beta\to 1} (1 + 2\beta)[\ln(1 + 2\beta) - \ln(1 + \beta)].$$

Thus the inequality above must be true. For $t > 0$, $h_t^r > h_t^u$ by proposition 2 so that $V_t^r > V_t^u$. Q.E.D.

Proof of Proposition 6

a) $H_{t+1} = \exp[\mu_{t+1} + (\sigma_{t+1}^2/2)]$. Using the transformations $\mu_{t+1} = \ln(A) + (\gamma + \delta)\mu_t + (\gamma\sigma_t^2/2)$ and $\sigma_{t+1}^2 = \delta^2\sigma_t^2$, we get

$$H_{t+1} = \exp\left[\ln(A) + (\gamma + \delta)\mu_t + \frac{(\gamma + \delta^2)\sigma_t^2}{2}\right].$$

Thus $H_{t+1} > H'_{t+1}$ if and only if

$$(\gamma + \delta)\mu_t + \frac{(\gamma + \delta^2)\sigma_t^2}{2} > (\gamma + \delta)\mu_t' + \frac{(\gamma + \delta^2)\sigma_t'^2}{2}.$$

Now

$$\begin{aligned}
(\gamma + \delta)(\mu_t - \mu_t') &= \frac{(\gamma + \delta)(\sigma_t'^2 - \sigma_t^2)}{2} && \text{since } H_t = H_t' \\
&> \frac{(\gamma + \delta^2)(\sigma_t'^2 - \sigma_t^2)}{2} && \text{since } \delta < 1.
\end{aligned}$$

Hence, $H_{t+1} > H'_{t+1}$.

b) As in part *a*,

$$H_{t+1} = \exp\left[\ln(B) + (\gamma + \delta)\mu_t + \frac{(\gamma + \delta)^2\sigma_t^2}{2}\right].$$

Thus $H_{t+1} > H'_{t+1}$ if and only if

$$(\gamma + \delta)\mu_t + \frac{(\gamma + \delta)^2\sigma_t^2}{2} > (\gamma + \delta)\mu_t' + \frac{(\gamma + \delta)^2\sigma_t'^2}{2}.$$

Now

$$\begin{aligned}
(\gamma + \delta)(\mu_t - \mu_t') &= \frac{(\gamma + \delta)(\sigma_t'^2 - \sigma_t^2)}{2} && \text{since } H_t = H_t' \\
&> \frac{(\gamma + \delta)^2(\sigma_t'^2 - \sigma_t^2)}{2} && \text{if and only if } \gamma + \delta < 1.
\end{aligned}$$

Hence the result. Q.E.D.

References

Aitchison, John, and Brown, James A. C. *The Lognormal Distribution.* London: Cambridge Univ. Press, 1969.

Banerjee, Abhijit V., and Newman, Andrew. "Risk-bearing and the Theory of Income Distribution." Manuscript. Evanston, Ill.: Northwestern Univ., 1989.

Card, David, and Krueger, Alan B. "Does School Quality Matter? Returns to Education and the Characteristics of Public Schools in the United States." *J.P.E.* 100 (February 1992): 1–40.

Coleman, James S., et al. *Equality of Educational Opportunity.* Washington: Government Printing Office, 1966.

Hanushek, Eric A. "The Economics of Schooling: Production and Efficiency in Public Schools." *J. Econ. Literature* 24 (September 1986): 1141–77.

Jones, Larry E., and Manuelli, Rodolfo. "A Convex Model of Equilibrium Growth: Theory and Policy Implications." *J.P.E.* 98, no. 5, pt. 1 (October 1990): 1008–38.

Kohlberg, Elon. "A Model of Economic Growth with Altruism between Generations." *J. Econ. Theory* 13 (August 1976): 1–13.

Loury, Glenn C. "Intergenerational Transfers and the Distribution of Earnings." *Econometrica* 49 (July 1981): 843–67.

Lucas, Robert E., Jr. "On the Mechanics of Economic Development." *J. Monetary Econ.* 22 (July 1988): 3–42.

McDonald, James B., and Ransom, Michael R. "Functional Forms, Estimation Techniques and the Distribution of Income." *Econometrica* 47 (November 1979): 1513–25.

Romer, Paul M. "Increasing Returns and Long-Run Growth." *J.P.E.* 94 (October 1986): 1002–37.

Rothschild, Michael, and Stiglitz, Joseph E. "Increasing Risk: I. A Definition." *J. Econ. Theory* 2 (September 1970): 225–43.

Stiglitz, Joseph E. "The Demand for Education in Public and Private School Systems." *J. Public Econ.* 3 (November 1974): 349–85.

Tamura, Robert. "Income Convergence in an Endogenous Growth Model." *J.P.E.* 99 (June 1991): 522–40.