FISEVIER

# Ends against the middle: Determining public service provision when there are private alternatives 

Dennis Epple ${ }^{\text {a }}$, Richard E. Romano ${ }^{\text {b.* }}$<br>"Carnegie Mellon University, Pittsburgh. PA 15213, USA<br>${ }^{6}$ Department of Economics, University of Florida. Gainesville. FL 32611, USA

Received April 1994; final version received July 1995


#### Abstract

Public provision of a service coexists with private market provision. The quality of public provision is determined by majority vote. Preferences are not single peaked owing to the presence of private alternatives. We identify two cases. In one, majority voting equilibrium always exists and the median-income voter is pivotal. In the other, a necessary condition for equilibrium identifies the pivotal voter who must have income below the median. When equilibrium exists, a coalition of middleincome households who consume the public alternative will be opposed by a coalition of rich and poor households, with the rich choosing private consumptivn.


Keywords: Voting; Education; Public provision
JEL classification: D72; H42; H52; I21

## 1. Introduction

Many publicly provided goods have privately available counterparts, including education, health, crime prevention, postal service, sanitation, and transportation. Issues related to such dual provision systems are increasingly in the forefront of policy debate. In education, mechanisms such as vouchers

[^0]are proposed to stimulate competition between public and private providers. In health care, many countries provide a specified level of care at public expense, and individuals are required to pay a premium for additional services.

Each dual-provision good or service has unique attributes. In health care, individuals may consume simultaneously both publicly and privately provided services. In education, by contrast, a given student in a given year typically consumes only the public or only the private alternative. In transportation, users of private automobiles may benefit from an improved public bus system that they never use if the bus system reduces highway congestion. Despite these differences, dual systems of provision create a dilemma that is common to pubiic service providers unless the public and private alternatives are perfect substitutes. On the one hand, the private alternative reduces the demand on the public system, thereby reducing its costs, to the benefit of users of the public system. On the other hand, the loss of clientele to the private sector can be expected to reduce public support for a high-quality public service, at least among those who do not use the public alternative. This is particularly true if those with the highest demand for quality are the first to opt out of the public system.

This dilemma has made dual provision the textbook example (Atkinson and Stiglitz, 1980, p. 303) of non-single-peaked preferences. Intuitively, non-single-peakedness occurs because, at low levels of public service quality, a household that prefers high-quality service may prefer the private alternative. Moderate increases in quality from 2 low base may make the household worse off because taxes rise while the increase in service quality is not sufficient to induce the household to consume the public alternative. Large increases in public service quality, by contrast, may make the household better off. The household may be induced to use the public alternative, and the increased tax cost of that alternative may be offset by the savings from forgoing the private service.

The implications of non-single-peakedness are that a voting equilibrium may not exist, and, if an equilibrium does exist, the standard approach (invocation of the median-voter theorem) does not generally apply to characterize that equilibrium. Since a majority-rule process is typically the simplest point of departure for characterizing the political process that determines the level of provision of public services, this has severely hampered modeling and policy analysis relating to dual provision issues.

Barzel (1973) pioneered the analysis of non-single-pcaked preferences for public education that arise when there is a private alternative, using a numerical example calibrated to actual data. In his example the richest of seven income segments opts for private education and favors zero public expenditure, and the majority choice is determined by an income segment below that containing the median-income household. Stiglitz (1974) first
detailed the theoretical problem of non-single-peaked oreferences in his comprehensive investigation of the demand for educatson in public and private school systems. Ireland (1990) characterizes many properties of dual-provision systems with vouchers, but he does not address the problem of endogenizing the level of public service provision. Glomm and Ravikumar (1996) are the first to endogenize public service provision in a model with a dual-provision system under majority rule. They do so by adopting specific functional forms for preferences and the di tribution of income.

In this paper we carry on the work of Barzel, and Blomm and Ravikumar. We generaiize their results in two respects. First, , se show that Glomm and Ravikumar's choice of utility function satisfies a single-crossing assumption. ${ }^{1}$ We then show that a majority voting equilibrium exists for any utility function that satisfies this single-crossing condition without restriction on the parametric form of the utility function and without any restriction on the distribution of income. With this single-crossing assumption, the medianincome voter is pivotal. Second, the appropriate single-crossing assumption depends on the properties of cicmand for the service in question. For some services (e.g. education), the opposite single-crossing assumption may be more appropriate.' Such an assumption is implicit in Barzel's example, and our findings for this case are consistent with his. We present a necessary condition for a majority-voting equilibrium for this alternative single-crossing condition. The median-income voter is pivotal only if, in equilibrium, no households choose the private alternative. It follows that the median-income voter is not pivotal in significant dual-provision cases, and we show that the level of public provision is generally below that preferred by the medianincome voter. Nioreover, in the result that motivates our title, we show that, if there is an equilibrium, a coalition of rich and poor prefer reduced public provision, while the middle class prefer an increase.

The paper is organized as follows. Section 2 presents the model and theoretical results on majority-voting are developed. A computational counterpart to the theoretical model comprises Section 3. Using a constant elasticity of substitution (CES) utility specification and U.S. data on educational expenditure and the income distribution, we compute equilibria for a range of educational demand elasticities encompassing both cases of

[^1]single crossing. One purpose is to examine whether an equilibrium actually exists where it must be of the 'ends against the middle' variety. We find that the outcome that satisfies the necessary condition is, in fact, an equilibrium in all cases. Another purpose is to examine the consequences of much debated educational vouchers. ine interesting result is that, while private school enrollment is quite responsive to a voucher system, per student public expenditure is not.

## 2. Tiesoretical model and results

There are two goods, educational services and the numeraire commodity. We appeal to education in developing the theoretical results because it is an important example, but the analysis applies more generally. All households are assumed to have the same strictly increasing, strictly quasi-concave, and twice continuously differentiable utility function $U(x, b)$ over educational services, $x$, and the numeraire bundle, $b$. The following additional assumptions are imposed on preferences:

Assumption A1. Educational services are a normal (or superior good).
Assumption A2. For $x>0, b>0, \bar{x}$ and $\bar{b} \geqslant 0, U(x, b)>U(0, \bar{b})$ and $U(x, b)>U(\bar{x}, 0)$.

Assumption A1 is non-controversial and accords with all existing empirical evidence. Assumption A2 is for technical convenience.

The following property of indifference curves, a consequence of Assumption A1, will be used frequently. Subscripts on functions denote partial derivatives.

Diminishing marginal utility (DMU). Along an indifference curve, the marginal utility of the numeraire declines as the numeraire increases. That is, if $U\left(x_{1}, b_{1}\right)=U\left(x_{2}, b_{2}\right)$ and $b_{2}>b_{1}$, then $U_{2}\left(x_{1}, b_{1}\right)>U_{2}\left(x_{2}, b_{2}\right)$.

Proof. See Epple and Romano (1994).
Households differ in endowed income (i.e. numeraire commodity), $y$. The p.d.f. and c.d.f. of household income агe denoted $f(y)$ and $F(y)$, respectively, with support $[\underline{y}, \bar{y}] \in[0, \infty)$. We assume that $f(y)$ is continuous and positive over its support. We normalize the number of households to one and denote aggregate income by $Y \equiv \int_{\underline{v}}^{y} y f(y) \mathrm{d} y$, which is also then equal to mean income.

Educational services are produced from the numeraire commodity. One unit of publicly provided educational services is produced with one unit of the numeraire. All consumers of public school services obtain the same level of education services. Public school inputs are financed by a proportional tax, $t$, on income. Hence, the public school budget constraint is

$$
\begin{equation*}
t Y=N E, \tag{1}
\end{equation*}
$$

where $\hat{N}$ is the number of households using public schools, and $E$ is per household public school services. The level of public school expenditure is determined by majority vote of all households, whether or not they utilize public schools.

Private school services are provided by price-taking suppliers. The cost per unit of educational services provided by private schools is $p$ units of the numeraire. A household consuming private school services can choose as many units as it desires at price $p$ per unit. A household can consume either public or private school services, but not both. This follows the literature and is a good approximation for education. Epple and Romano (1996) añd Gouveia (1996) analyze the alternative where public consumption can be supplemented by private consumption, which may be a better approximation for some publicly provided services like health care.

A household that consumes private school services chooses $x$ to maximize $U(x, b)$ subject to the budget constraint $y(1-t)=p x+b$. Let

$$
\begin{equation*}
v(p, y(1-t))=\max _{\{x\rangle} U(x, y(t-1)-p x) \tag{2}
\end{equation*}
$$

be the indirect utility function of a household with income $y$ that chooses private schooling, and let $x^{*}(p, y(1-t))$ be the demand function for private educational services that solves the maximization problem in (2).

A household with income $y$ choosing public schooling obtains utility:

$$
\begin{equation*}
U(E, y(t-1)) \tag{3}
\end{equation*}
$$

Hence, the induced utility function of a household with income $y$ that can choose between public and private alternatives is

$$
\begin{equation*}
V(E, p, y(1-t))=\max [v(p, y(1-t)), U(E, y(1-t))] . \tag{4}
\end{equation*}
$$

The following observations are useful in sketching the indifference map in the ( $E, t$ ) plane corresponding to the utility function $V(\cdot)$. Note that continuity of $U(\cdot)$ implies continuity of $v(\cdot)$ and $V(\cdot)$. Assumption A2 implies that for a given tax rate $t \in[0,1$ ), all households consume positive private school services when the level of public school services, $E$, equals zero. Likewise, for a given $t$ and a household with a given income, there is a level of public schools services sufficiently large that the household will prefer public schools to private schools. In particular, this is clearly true if the public school offers a level of services $E$ as large or larger than the
household would buy if it utilized private schooling. Since the utility function (3) is continuously increasing in $E$, it follows that there is a unique positive value of $E$ such that, with a given $t$, the household is just indifferent between public and private schooling.

The locus of ( $E, t$ ) values that makes a household $y$ indifferent between public and private school satisfies $v(p, y(1-t))=U(E, y(1-t))$. Differentiation of this expression yields equality of the first two terms below:

$$
\begin{equation*}
\left.\frac{\mathrm{d} t}{\mathrm{~d} E}\right|_{v(\cdot)=U(\cdot)}=\frac{U_{2}(E, y(1-t))}{y\left[U_{2}(E, y(1-t))-U_{2}\left(x^{*}(\cdot), y(1-t)-p x^{*}(\cdot)\right)\right]}<0 \tag{5}
\end{equation*}
$$

Since $y(1-t)>y(1-t)-p x^{*}(\cdot)$, the inequality in (5) follows from DMU. We suppress $p$ as an argument and write the locus of $(E, t)$ pairs along which household $y$ is indifferent between public and private school as $\hat{E}(y(1-t))$. Expression (5) implies that $\hat{E}(\cdot)$ is downward sloping in the $(E, t)$ plane.

These observations imply that for any household, a typical indifference map in the ( $E, t$ ) plane will be as illustrated in Fig. 1. Let us consider first a typical indifference curve. For sufficiently low levels of public school services, i.e. for $E<\hat{E}$, the household will opt to use private schools. For values of $E$ such that the household uses private schooling, the household's utility is given by $v(p, y(1-t))$, which depends on $t$ but not $E$. Hence, in this range, a household's indifference curve in the ( $E, t$ ) plane is flat. For sufficiently high $E$, i.e. for $E>\hat{E}$, the household will use public schools and


Fig. 1
an indifference curve satisfies $U(E, y(1-t))=$ constant. Here $t$ increases necessitate increases in $E$ to maintain indifference. Using strict quasiconcavity of $U(x, b)$, the increasing portion of the indifference curve is a concave function in the ( $E, t$ ) plane. ${ }^{3}$

Utility is increasing in the southeasterly direction in Fig. 1. Since $\hat{E}(y(1-$ $t$ )) is downward sloping, the indifference map has the property, illustrated in Fig. 1, that the 'corner' shifts downward to the right as we look across different indifference curves in order of ascending utility.

Henceforth, for expositional convenience, we set the price per unit of private schooling, $p$, equal to 1 , and we suppress $p$ as an argument in utility and demand functions. All results below are valid for any $p>0$.

An analysis of voting requires comparing preference orderings across individuals. The properties that facilitate such comparisons are developed in several lemmas. Lemma 1 shows that the level of public expenditures necessary to induce a household to choose a public school increases with income. Specifically, for a given tax rate, the 'corners' of the indifference curves of individuals with differing incomes shift to the right in the ( $E, t$ ) plane as income increases.

Lemma 1. $\hat{E}(y(1-t))$ is increasing in $y$.

Proof. Differentiate and use (5).
Corollary 1. If at any ( $\tilde{E}, \tilde{t}$ ), household $y^{\prime}$ weakly prefers private to public schooling, then so do all households $y>y^{\prime}$, and if $y^{\prime}$ weakly prefers public to private schooling, then so do all households $y<y^{\prime}$.

Proof. Let $V^{\prime}$ be the indifference curve through $(\tilde{E}, \tilde{)})$ of a household $y^{\prime}$ that prefers private schooling to the public alternative $\tilde{E}$. The 'corner' of this indifference curve is to the right of point ( $\tilde{E}, \tilde{T}$ ), as illustrated by point $A$ in Fig. 2. By Lemma 1, any household with a higher income has an indifference curve through ( $\tilde{E}, \tilde{t}$ ) with a corner to the right of $A$. This is illustrated by point $B$ of the indifference curve $V^{\prime \prime}$ in Fig. 2, where $V^{\prime \prime}$ is the indifference curve of some household with $y^{\prime \prime}>y^{\prime}$. Thus, $y^{\prime \prime}$ also prefers private to public provision.

A similar argument establishes that if a household with $y^{\prime}$ prefers public provision, then so do all households with $y<y^{\prime}$.

[^2]

Fig. 2

Let $E^{*}(t)$ be educational expenditure per household for those attending public school when all households make utility-maximizing choices. We will call this the Government Budget Constraint (GBC). We can now develop its key properties.

For ( $E, t$ ) such that all households choose public education, $E^{*}=t Y$ from (1). We consider ( $E, t$ ) such that some households strictly prefer private schooling. Let $\hat{y}$ be the income of the household indifferent between public and private schooling. Then $\hat{y}(E, t)$ satisfies

$$
\begin{equation*}
U(E, \hat{y}(1-t))=v(\hat{y}(1-t)) . \tag{6}
\end{equation*}
$$

Corollary 1 implies:

$$
\begin{equation*}
N(E, t)=F(\hat{y}(E, t)) \tag{7}
\end{equation*}
$$

Using (1), $E^{*}(t)$ is then defined implicitly in

$$
\begin{equation*}
E^{*}=\frac{t Y}{N\left(E^{*}, t\right)} \tag{8}
\end{equation*}
$$

Now, from (6) and (7) and using the envelope and implicit function theorems, we can find the tax elasticity of the number of public school users:

$$
\epsilon_{N, t} \equiv \frac{\partial N}{\partial t} \frac{t}{N}=\frac{t \hat{y} f(\hat{y})}{(1-t) N}
$$

Likewise, we can find the expenditure elasticity of the number of public school users:

$$
\epsilon_{N \cdot E} \equiv \frac{\partial N}{\partial E} \frac{E}{N}=\frac{\tilde{U}_{1} E f(\hat{y})}{(1-t) N\left(U_{2}^{*}-\tilde{U}_{2}\right)},
$$

where a tilde and an asterisk denote the consumption bundles for $y=\hat{y}$ associated with public and private schooling, respectively. Both elasticities are positive, $\epsilon_{N, i}$, obviously, and $\epsilon_{N, E}$ because DMU implies $U_{2}^{*}-\tilde{U}>0$. Both elasticities are of unrestricted magnitude since $f(\hat{y})$ is unrestricted in magnitude. Finally, we can find the elasticity of the GBC from (8):

$$
\begin{equation*}
\frac{\mathrm{d} E^{*}}{\mathrm{~d} t} \frac{t}{E^{*}}=\frac{1-\epsilon_{N, t}}{1+\epsilon_{N, E}} . \tag{9}
\end{equation*}
$$

Lemma 2. $E^{*}(t)$ is continuous for all $t \in(0,1)$ and differentiable over this range except at the point where the highest income household is indifferent between the public and private alternatives.

Proof. If all households prefer public to private provision at point ( $\left.E^{*}(t), t\right)$, then the claims follow trivially from (1). For ( $E^{*}(t), t$, having some households choose the private alternative, $E^{*}(t)$ is defined by (6)-(8) with the derivative described in (9). The positivity of the denominator on the right-hand side of (9) and the implicit function theorem imply that $E^{*}(t)$ is continuous and differentiable over this range. For $t$ such that $\hat{y}=\bar{y}$, continuity follows from $F(\bar{y})=1$. It is straightforward to check that $E^{*}(t)$ is not, however, differentiable at this point.

The one non-differentiability in $E^{*}(t)$ for $t>0$ will be inconsequential to our analysis. If the minimum income in the population is positive, then $E^{*}(t)$ is also discontinuous at $t=0$, jumping from zero to a positive value. Likewise, this is not of consequence, and our figures below h: e $E^{*}(t)$ go through the origin when $y=0$.

Perhaps surprisingly, $E^{*}(t)$ need not be everywhere increasing, by (9). One effect of a higher tax rate is to increase aggregate public expenditure, attracting more students into the public sector so long as per student expenditures rises. This effect is captured by the denominator in (9) and cannot cause $\mathrm{d} E / \mathrm{d} t<0$. However, the relative preference for public over private education at any $E$ rises with the tax rate because the marginal utility of disposable income rises. The numerator of (9) captures this effect, which can be sufficiently strong in some ranges of $t$ for some income distributions to cause $\mathrm{d} E / \mathrm{d} t \leqslant 0$. We must then contend with a poorly behaved GBC, as illustrated, for example, in Fig. 5 below.

The restriction on preferences that permits our development of the
properties of voting equilibrium is now described. Let the slope of an indifference curve of $U(E, y(1-t)$ ) in the ( $E, t$ ) plane be denoted by $M(E, y, t)$. Hence,

$$
\begin{equation*}
M(E, y, t)=\frac{U_{1}(E, y(1-t))}{y U_{2}(E, y(1-t))} . \tag{10}
\end{equation*}
$$

It will be assumed for all $y$ that the slope of the $U(E, y(1-t))$ function in the ( $E, t$ ) plane is monotone in $y .{ }^{4}$ In particular, we assume that one of the following alternatives holds:

Assumption $A 3$ (SDI). $\partial M(E, y, t) i \partial y \leqslant 0$ for all $y$.
Assumption $A 4$ (SRI). $\partial M(E, y, t) / \partial y \geqslant 0$ for all $y$.
For ease of reference, we adopt the mnemonics SDI (slope deciining in income) and SRI (slope rising in income) to refer to these assumptions. Income changes affect the marginal willingness to bear a proportional tax rise for increased public education (i.e. $M(\cdot)$ ) through an income and substitution effect. Assumption A1 ensures a positive income effect. Under proportional taxation, countering this is the effective price increase of public education associated with rising income. Following Kenny`s (1978) analysis, SRI results if the income elasticity of the implied demand for public education exceeds the (absolute value of the) price elasticity of the same, and SDI results if the reverse holds. ${ }^{5}$ Most of the empirical analysis of the demand for education finds income elasticities that exceed (tax) price elasticities (e.g. see Denzau and Grier, 1984, and Fischer, 1988), but estimation is controversial owing to the possibilities of Tiebout biases and household production of education (see Rubinfeld and Shapiro, 1989). It is prudent to examine both possibilities theoretically.

The following lemma establishes that if the utility function $U(\cdot)$ satisfies the single-crossing-condition SDI, then the indifference curves of the utility function defined in Eq. (4) also satisfy the same single-crossing condition.

Lemma 3. If SDI holds, then any indifference curve of the utility function $V\left(E, y^{\prime}(1-t)\right)$ crosses any indifference curve of $V\left(E, y^{\prime \prime}(1-t)\right)$ at most

[^3]once." If a crossing occurs, and $y^{\prime \prime}>y^{\prime}$, then the indifference curve of $y^{\prime \prime}$ crosses the indifference curve of $y^{\prime}$ from above.

Proof. To establish the first claim, let us suppose the contrary. We consider households with incomes $y^{\prime}$ and $y^{\prime \prime}$. There are two cases that must be ruled out. One of these is illustrated by the solid curves in Fig. 3. Here an indifference curve $V^{\prime \prime}$ of household $y^{\prime \prime}$ crosses an indifference curve $V^{\prime}$ of household $y^{\prime}$ twice, once along the flat part and once along the upwardsloping part. The latter crossing and SDI imply $y^{\prime \prime}>y^{\prime}$. To establish the contradiction, we draw the indifference curve of $y^{\prime}$ that has its flat part coinciding with the flat part of $V^{\prime \prime}$. This is illustrated by the dashed curve $\hat{V}^{\prime}$ in Fig. 3. From Eq. (5), the corner of $\hat{V}^{\prime}$ (point $A$ ) is downward and to the right of the corner of $V^{\prime}$ (point $B$ ), and hence to the right of the corner of $V^{\prime \prime}$ (point $C$ ). Since $y^{\prime \prime}>y^{\prime}$, this contradicts Lemma 1 .


Fig. 3

[^4]The other case to be ruled out is that in which the flat parts of the indifference curves of $y^{\prime}$ and $y^{\prime \prime}$ are distinct or overlap, while the upwardsloping parts cross many times. This is trivially a violation of SDI.

Lemma 1 implies that the upward-sloping part of an indifference curve of $y^{\prime \prime}>y^{\prime}$ cannot intersect the flat part of an indifference curve of $y^{\prime}$. It follows from SDI that any crossing must be one in which an indifference curve of $y^{\prime \prime}$ crosses an indifference curve of $y^{\prime}$ from above.

Definition. An allocation is a majority-voting equilibrium if it is on the GBC and it garners $50 \%$ or more of the vote in a binary comparison against any alternative on the GBC.

Proposition 1. When SDI holds, a maiority-voting equilibrium exists, and the median-income voter is decisive. ${ }^{7}$

Proof. The proof is based on Roberts (1977). Here we adapt the geometric proof developed in Epple and Romer (1991). Let ( $\tilde{E}, \tilde{t})$ be the point on the GBC most preferred by the voter with median income, and assume for now that $(\bar{E}, \tilde{t}) \gg(0,0)$ and is unique. We draw the median-income voter's indifference curve through this point (see the curve labeled $\tilde{V}$ in Fig. 4). There can be no points on the GBC below $\tilde{V}$ since any such point would be


Fig. 4

[^5]preferred to ( $\tilde{E}, \tilde{t}$ ) by the median-income voter. Next, we consider any points on the GBC that lie above $\tilde{V}$ and the horizontal line through $\tilde{t}$ (points in region $A$ ). Lemma 3 implies that all voters with income $y \geqslant \tilde{y}$ prefer ( $\tilde{E}, \tilde{t}$ ) to any point in this region (e.g. the voter with dashed indifference curve $V^{\prime}$ shown in Fig. 4). Since $\tilde{\tilde{E}}$ is the median, then at least half the electorate prefers $(\tilde{E}, \tilde{i})$. Hence, $(\tilde{E}, \tilde{t})$ defeats all points in region $A$. An analogous argument establishes that $(\tilde{E}, \tilde{i})$ defeats all points above $\tilde{V}$ and below the line through $\tilde{t}$ because all voters with $y \leqslant \tilde{y} \operatorname{prefer}(\tilde{E}, \tilde{t})$ to any point in this region.

If $\tilde{E}=\tilde{\boldsymbol{t}}=0$, a briefer version of the same argument establishes it as an equilibrium. Here the horizontal line through $(\tilde{E}, \tilde{t})$ is the abscissa, and no region below it exists to consider.

We have shown that the median-income household's most preferred choice is always an equilibrium which establishes existence. Since the arguments apply to any point in that household's most preferred set. multiple equilibria are theoretically possible. However, a point not a member of this set cannot be an equilibrium. If the median voter preferred a point on the GBC with both lower $E$ and $t$ than the candidate point, then, by Lemma 3, so too would all households with higher incomes. By the continuity of $V(\cdot)$ in $y$, a positive measure of households with incomes below but close to the median would also have such a preference. Hence, the candidate point would not garner a majority. An analogous argument rules out points where the median-income voter has a preference for a point on the GBC with both higher $E$ and $t$. If there exists a point on the GBC with (weakly) higher $E$ and (weakly) lower $t$ than a candidate point, then there is unanimity of preferences for the former. Only a most preferred point of the median-income household will be an equilibrium.
Remarks. (1) Note that this result follows from the properties of the induced utility function defined in Eq. (4) and does not rely on any properties of the GBC. Intuitively, the result occurs for the following reason. If no private alternative existed and all households consumed the publicly provided good, the most-preferred level of provision would be inversely related to income under SDI. The equilibrium would be the choice of the median-income household. When a private alternative is available, it is consumed by the highest-income segment of the population. While this reduces the level of the publicly provided good that this high-income segment prefers relative to the case of no private alternative, it does not change the inverse relationship between income and the mosi-preferred level of public provision. Hence, the median-income household remains decisive.
(2) If the median-income voter's highest attainable indifference curve is achieved at more than one point on the GBC, then all such points are equilibria. Since such multiple equilibria are knife-edge cases, a unique equilibrium is the generic outcome.


Fig. 5
(3) An interesting implication of Proposition 1 is the following. Suppose that the median-income voter has a slight preference for private schooling over public schooling. Fig. 5 illustrates such a case where $V$ and $V^{\prime}$ are indifference curves of the median-income household. Then the tax rate will be set equal to zero and there will be no public provision. Under these conditions, a small perturbation may lead to large changes in the level of public provision. For example, suppose that courts mandate that there be a minimum level of public provision. Then the level of public provision actually provided may be significantly greater than that mandated by the court. If the mandated minimum level exceeds $E_{\mathrm{m}}$ in Fig. 5, then the equilibrium has public provision of $\tilde{E}$. ${ }^{8}$

Next, we turn to a consideration of the voting equilibrium when the monotonicity condition SRI holds. For ( $E, t$ ) values where two households prefer the public alternative and their indifference curves cross, the higher income household's indifference curve crosses the lower income household's indifference curve from below. Taking account of the private alternative and using Lemma 1 to draw indifference curve mappings, it is simple to confirm that single crossing will fail to hold, as we illustrate shortly. Under weak as monptions, the median voter's preference will no longer determine public expenditure in a voting equilibrium. An equilibrium may also fail to exist. If it exists, a lower tax-expenditure choice than the median-income voter's

[^6]preference will prevail. A middle income group will prefer tax-expenditure increases, but an equal-sized coalition of rich and poor households will prefer the opposite.

Making two realistic assumptions simplifies the presentation and avoids uninteresting cases.

Assumption A5. The median-income household's most preferred choice (or choices) on the GBC has (have) $E>0$.

The appendix shows that a sufficient condition for Assumption 5 is that the median income is below the mean income in the population.

Assumption A6. The highesi-income individual strictly prefers a private alternative if (any of) the median-income household's most preferred choice(s) on the GBC prevails.

Assumption A6 will preclude an equilibrium with no private consumption of education. (The effect of relaxing this assumption is discussed later in footnote 9.) We have:

Proposition 2. The median-income voter's most preferred choice(s) cannot be a majority voting equilibrium.

Proof. In Fig. 6, $V^{\mathrm{m}}$ is an indifference curve of the median-income voter and $\bar{V}$ an indifference curve of the highest-income $(\bar{y})$ voter. Hence, point $\frac{A}{V}$ represents the median-income voter's most-preferred alternative and $\bar{V}$


E
Fig. 6
conforms to Assumption A6. (That $\bar{V}$ recrosses the GBC is not relevant to the argument but will be used to make another point below. We ignore points $B$ and $C$ for now.) A marginally lower tax-expenditure combination is preferred by a majority to point $\boldsymbol{A}$. All households with incomes below the median income have such a preference by Corollary 1 and SRI. Likewise the highest income household has such a preference as do some households with incomes near $\bar{y}$, the latter by continuity of $V(\cdot)$ in $y$. The same argument applies to each of the median-income household's most preferred choices if a multiplicity of such points exists.

The fact that point $A$ in Fig. 6 is not a majority voting equilibrium relates to the failure of single crossing. Using SRI, a preference for lower taxes by those households with incomes near, but below, the median income is countered by a preference for higher taxes by those with incomes near, but above, the median income. However, let us consider households with incomes sufficiently high that they make a private choice at the medianincome household's most preferred choice. Indifference curves of such high-income households cross $V^{m}$ at point $A$ from above. They, too, prefer tax decreases, breaking $A$ as an equilibrium. Similar logic precludes any point above $\boldsymbol{A}$ on the GBC from being an equilibrium:

Corollary 2. A majority voting equilibrium, if it exists, entails less public expenditure than the (minimum of) the median-income household's most preferred choice(s).

Proof. We refer to Fig. 6 where point $A$ is the median-income household's most preferred choice. (hir a muitiplicity exists, let point $\boldsymbol{A}$ be the medianincome household's minimum-expenditure most preferred choice.) Households with incomes below the median income stricily prefer point $\boldsymbol{A}$ to any point on the GBC above $A$, by Corollary 1 and SRI. By the continuity of $V(\cdot)$ in $y$, point $A$ is also preferred by a positive measure of incomes in the vicinity of $\bar{y}$ to all points above $A$ on the GBC, except points on or in the vicinity of the arc $B C$. This establishes that points above $A$ on the GBC other than those on or in the vicinity of the arc $B C$ are defeated by point $A$. Point $A$ is also preferred by a positive measure of households with incomes above and in the vicinity of the median income to all points on or in the vicinity of the arc $B C$, again by continuity of $V(\cdot)$ in $y$. Matched against points on or in the vicinity of the arc SC, point $A$ would again garner a strict majority. If the GBC wiggles in such a way that other arcs preferred by $\bar{y}$, like $B C$, are present, then points on or in the vicinity of these arcs can be rejected as equilibria analogously. Combining these results with Proposition 2 implies that any majority-voting equilibrium must have less public expenditure than at point $A$.

Proposition 3 contains the paper's most novel result, namely necessary conditions for an interior majority voting equilibrium (i.e. one with $E>0$ ) under SRI. The candidate point is majority preferred to local deviations on the GBC under these conditions.

Proposition 3. If $(\tilde{E}, \tilde{i})$ is an interior najority voting equilibrium under SRI, then:
"(i)" there exists a household with income $y_{t}$ that weakly prefers public consumption at point ( $\tilde{E}, \tilde{t}$ ) to public consumption at all other points on the GBC;
" (ii)" there exists a household with income $y_{\mathrm{h}}$ that is indifferent between public and private consumption at point ( $\hat{E}_{\dot{i}}^{\tilde{)}}$ );
"(iii)" $y_{t}<y_{\mathrm{h}}$; and
" (iv)" $\rho \equiv \int_{y_{t}}^{y_{\mathrm{n}}} f(y) \mathrm{d} y=0.5$.
Proof. The proof coasists of two parts. First, we show that at points on the GBC where householas $y_{i}$ and $y_{\mathrm{h}}$ satisfying (i)-(iii) exist, $\rho$ must equal 0.5 for the point to be an equilibrium. The secoad part shows that all candidate points for an equilibrium must have such househoids.

Fig. 7 illustrates a potential equilibrium, where $V_{i}$ and $V_{\mathrm{h}}$ are the indifference curves of households with incomes $y_{t}$ and $y_{\mathrm{h}}$, respectively. Households with incomes $y \in\left(y_{t}, y_{\mathrm{h}}\right)$ strictly prefer the public alternative at ( $\tilde{E}, \tilde{\boldsymbol{t}}$ ) using Lemma 1 and Corollary 1 . SRI implies that, relative to ( $\tilde{E}, \tilde{t})$, these households prefer marginal expenditure-tax increases along the GBC. If $\rho>0.5$, then a marginal expenditure-tax increase defeats $(\tilde{E}, \tilde{t})$.


Fig. 7

Relative to ( $\tilde{E}, \tilde{t})$, households with incomes below $y_{t}$ prefer marginal tax-expenditure decreases along the GBC, by Corollary 1 and SRI. Households with incomes above $y_{\mathrm{h}}$ have the same preference, by Lemma 1. If $\rho<0.5$, then a marginal tax-expenditure decrease from ( $E, \tilde{t}$ ) defeats it. Hence, given the existence of households satisfying (i)-(iii) with incomes $y_{t}$ and $y_{h}$ at a point on the GBC, $\rho=0.5$ is necessary for it to be an equilibrium.

We now consider the consequences for an equilibrium of the potential non-existence of such households. Corollary 2 ruled out as a candidate equilibrium those points abeve and including the (minimum) preferred choice of the median voter, i.e. points above $A$ in Fig. $6 .{ }^{9}$ We restrict our attention to points below $A$ on the GBC. At all such points a household of type $y_{h}$ exists that satisfies (ii). The alternative leads to a contradiction. Assumption A6 and Expression (5) imply that the highest-income household chooses the private alternative at all points below $A$ on the GBC. Lemma 1 implies that the corners of the indifference curves shift to the left as income declines. Then, if no $y_{\mathrm{h}}$ type exists at such points, then all households must likewise choose the private alternative. By (1), $E=x$, which contradicts Lemma 2. A type $y_{\mathrm{b}}$ will then exist at all candidate points. It also becomes clear momentarily that, for an equilibrium, the right-hand slope of $V_{h}$ at ( $\tilde{E}, \tilde{t}$ ) must exceed that of the GBC (as Fig. 7 illustrates).

Let us now consider the existence of $y_{t}$ types. By Corollary 1, all and only households with incomes less than $y_{\mathrm{h}}$ strictly prefer the public alternative at the point in question, and they make up the set of candidate households. Hence, (iii) must be satisfied. There are two possibilities for the nonexistence of a type $y_{i}$, as defined by (i). One has no household with a tangency at the point in question, with two subcases. The first subcase is illustrated in Fig. 8, where the lowest income ( $y$ ) household's indifference curve ( $(V)$ is fatter than the GBC at the point in question. The right-side slope of the $y_{\mathrm{h}}$ type's indifference curve must then also be flatter than the GBC, otherwise some $y \in\left(y, y_{\mathrm{h}}\right)$ with a tangency exists, by , SRI and the continuity of $V(\cdot)$ in $y$. Such a point cannot be an equilibrium because i unanimous preference for marginally reduced taxes is implied. Note also that a slightly modified version of the latter argument can be used to reject any points on any non-increasing ranges of a GBC (i.e. points where $\mathrm{d} E^{*} / \mathrm{d} t \leqslant 0$ ).

The second subcase of no tangency, illustrated in Fig. 9, presumes the lowest income household's indifference curve $\left(V^{\prime}\right)$ is steeper than the GBC at the point in question. The properties of the indifference mappings imply

[^7]

Fig. 8
that all $y \in\left(y, y_{\mathrm{h}}\right)$ prefer a marginally higher tax, and all $y>y_{\mathrm{h}}$ prefer a marginally lower tax. Point $B$ cannot be an equilibrium if $\because s$ is other than the median income. If $y_{\mathrm{h}}$ equals the median income, then points like $C$ will defeat $B$. All households with incomes below $y_{\mathrm{h}}$ prefer $C$ to $B$, as does a positive measure of households with incomes greater than, and in the vicinity of, $y_{h}$.

The remaining possibility of the non-existence of a $y_{t}$ type, illustrated in Fig. 10, presumes a tangency at the candidate point ( $B$ ), but fails to satisfy the requirement that the point is a most preferred point of public consumption of the $y_{t}$ household. The arguments we make apply whether or not the GBC is concave at $B$; Fig. 10 illustrates the 'more difficult' case. If $\rho \neq 0.5$,


Fig. 9


Fig. 10
then $B$ is defeated by a local deviation on the GBC by the first argument of the proof. If $\rho=0.5$, then the $y_{t}$ household's most preferred public alternative can be above point $B$ (e.g. point $C$ ) or below point $B$ (e.g. point $D)$. In the former case, $C$ defeats $B$, since all households with incomes $y \in\left[y_{t}, y_{\mathrm{h}}\right)$ prefer $C$ to $B$, as do a positive measure of households with incomes above and in the vicinity of $y_{\mathrm{h}}$. This is confirmed by drawing the indifference curves of such types through $B$, using their required properties. In the latter case, $D$ defeats $B$, since all households with $y \leqslant y_{t}$ and $y>y_{h}$ prefer $D$ to $B$, as do a positive measure of households with incomes below and in the vicinity of $y_{h}$ -

Remarks. (1) This result has much intuitive appeal for services such as education. When a private alternative is available, high-income households prefer low public school expenditure because they do not use public schools. Low-income households prefer low public school expenditures because they are less willing than higher-income households to substitute public school expenditures for other goods. Middle-income households, by contrast, use public schools and prefer that they be of relatively high quality. Hence, a coalition of middle-income households prefers higher public school expenditure at the margin, while a coalition of high- and low-income households prefers a reduction. In equilibrium, these two coalitions are equal in size and balance each other in voting. ${ }^{19}$

[^8](2) The proposition provides a necessary condition for an interior equilibrium. The condition ensures that local deviations will not defeat the candidate point on the GEC. If preferences obeyed single crossing, then satisfaction of the local condition for a voting equilibrium at the pivotal voter's most preferred choice would imply that the point defeats all alternatives. The presence of double crossings eliminates the guarantee of no majority-preferred alternatives. Let us consider, for example, a 'large' tax increase from the candidate point. Some higher income households that choose a private alternative at the candidate point and would vote against marginal tax increases, would favor a tax increase that causes them to switch to the public alternative. Under single crossing, no alternative points like the latter could exist. These 'switchers' are, however, countered by some middle-income households who would prefer a marginally better public alternative to the candidate point and would vote for marginal tax increases, but would vote against a large tax increase. Equilibrium requires that the size of the latter group exceeds the former, and analogously for all 'large' tax deviations. Whether a candidate point passes the global test will then depend on the specifics of preferences and the distribution of income. It may be surprising that, in our computational analysis reported below, we found the point that satisfies the necessary condition to be an equilibrium in all cases.
(3) Epple and Romano (1996) and Gouveia (1996) have independently analyzed the alternative dual provision environment where a proportional income tax finances the provision of a good consumed by all households, but who can frictionlessly supplement consumption with private market purchases. These papers show that a voting equilibrium exists generally, and, assuming SRI and a median income below the mean, an equilibrium is characterized by the ends-against-the-middle property. The middle-income group that favors a tax increase consists of the half of the population that has an income below but closest to the mean. In contrast, the present paper shows that when the public and private alternatives cannot be jointly consumed, the middle-income group that favors a tax increase has no fixed upper bound; rather, it depends on preferences, technology, and income distribution.
(4) A corner equilibrium at the origin (i.e. $t=E=0$ ) or the non-existence of an equilibrium are, of course, other possibilities. We can probably contrive cases with a multiplicity of equilibria. Since the alternative equilibria would need to tie, empirical relevance is unlikely.

We have assumed that households can purchase as many units of private school services as they desire at a constant unit price, $p$. In reality, of course, there are a finite number of private schools, each offering a given level of educational services per pupil. We might then consider voting over public school inputs when there are a discrete number of private
alternatives. We have shown (Epple and Romano, 1994) that all the results above extend easily to such a case.

## 3. Computational model

We develop a computational model with two objectives. One is to investigate, for a range of parameter values, whether the point satisfying the necessary condition in Proposition 3 is a majority-voting equilibrium. The other is to explore implications of the model for a policy that provides vouchers for private education.

The computational model requires a specification for the income distribution and the utility function. We assume that household incomes are $\log$-normally distributed, $\ln y \sim \mathbf{N}\left(\mu, \sigma^{2}\right)$. Given our one-jurisdictional model and the likelihood of Tiebout sorting over multiple jurisdictions in the real world, our presumed distribution is admittedly a crude approximation. In 1989, mean and median U.S. household incomes were $\$ 36,250$ and $\$ 28,906$, respectively. Measuring income in thousands, these imply $\mu=3.36$ and $\sigma=0.68{ }^{11}$

We assume that preferences are given by the following CES function:

$$
U(x, b)=\left[\beta x^{-\rho}+(1-\beta) b^{-\rho}\right]^{(-1 / \rho)} .
$$

When $\rho<0$, this function satisfies assumption SDI. When $\rho>0$, assumption SRI is satisfied. ${ }^{12}$

We calibrate the utility function as follows. Expenditure per student in U.S. public schools in 1988 was $\$ 4,222$ and there were 0.5 students per household (i.e. expenditure per household was $\$ 2,111$ ). In our calibration, we require that the parameters be such that the necessary conditions for voting equilibrium are satisfied at a public expenditure of $\$ 4,222$ per student. An additional condition is obtained by fixing the value of the price elasticity of demand for education (evaluated at the point satisfying the necessary conditions for equilibrium). Values for the two utility function parameters, $\rho$ and $\beta$, are determined by these two conditions.

Results are reported in Table 1 for four different price elasticities. We chose a broad range of price elasticities in order to illustrate how outcomes

[^9]Table 1

| $\boldsymbol{7}_{\text {b }}$ | $\rho$ | $\beta$ | $E$ | $t$ | \% Public | $y_{1}$ | $y_{6}$ | $\frac{\varepsilon}{E} \frac{\mathrm{~d} E}{\mathrm{~d} t}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Voucher $=0$ |  |  |  |  |  |  |  |  |
| -0.5 | 1.2 | 0.006 | \$2.110 | 0.043 | 74.9 | \$18.190 | \$45.519 | 0.59 |
| -0.67 | 0.54 | 0.02 | 2.110 | 0.051 | 88.0 | 23.497 | 62.241 | 0.74 |
| -1.25 | -0.21 | 0.10 | 2.110 | 0.057 | 97.8 | 28.854 | NA | 0.93 |
| -1.5 | -0.35 | 0.134 | 2.110 | 0.057 | 98.9 | 28.854 | NA | 0.95 |
| Voucher $=\mathbf{\$ 1 . 0 0 0}$ |  |  |  |  |  |  |  |  |
| -0.5 | 1. 2 | 0.006 | \$2,098 | 0.045 | 71.4 | \$16.815 | \$42.366 | 0.68 |
| -0.67 | 0.54 | 0.02 | 2.131 | 0.053 | 85.9 | 22,607 | 59.982 | 0.79 |
| -1.25 | -6. 21 | 0.10 | 2.100 | 0.056 | 96.8 | 28.854 | NA | 0.91 |
| -1.5 | -0.35 | 0.134 | 2.090 | 0.057 | 98.2 | 28.854 | NA | 0.95 |
| Voucher $=\$ 2.000$ |  |  |  |  |  |  |  |  |
| -0.5 | 1.2 | 0.006 | \$2.056 | 0.047 | 65.7 | \$14.555 | \$37.995 | 0.82 |
| -0.67 | 0.54 | 0.02 | 2,166 | 0.054 | 83.2 | 21.473 | 55.498 | 0.83 |
| -1.25 | -0.21 | 0.10 | 2.111 | 0.056 | 95.1 | 28.854 | NA | 0.91 |
| -1.5 | -0.35 | 0.134 | 2,080 | 0.056 | 96.9 | 28,854 | NA | 0.94 |

change as the elasticity is changed. Price elasticities less than one in absolute value correspond to assumption SRI, while those greater than one in absolute value correspond to SDI (see footnote 12). Hence, the first two rows of results correspond to SRI and the second two rows correspond to SDI.

When SDI holds, Proposition 1 establishes that a voting equilibrium exists. When SRI holds, Proposition 3 provides the necessary conditions. ${ }^{13}$ We checked whether the allocation in Proposition 3 was an equilibrium for each case in which the parameters correspond to SRI. We did this by computing the proportion of voters favoring ( $\tilde{E}, \tilde{t}$ ) against a dense grid of alternatives along the GBC. For an allocation to be an equilibrium, it must garner at least half the votes against every aiternative. Fig. 11, plotted for

[^10]

Fig. 11. Percentage of voters that favor an equilibrium outcome against alternative expenditure levels.
parameters in the highlighted row of the case with a voucher equal to zero in Table 1, is typical of the results we obtained. In every case reperted in the table, the allocation satisfying Proposition 3 is an equilibrium. This is one key finding of our computational analysis. While we believe that there may be circumstances under which an allocation satisfying Proposition 3 fails to be an equilibrium, the results in Table 1 suggest that failure of existence may not be a significant problem for empirically interesting cases.

We now turn to a discussion of the substantive results of our computations. Comparing the first four rows of the computational results, we see that the main effect of changing the price elasticity of demand is in public school enrollment. The greater the price elasticity (in absolute value), the larger is public school enrollment. This is intuitively plausible. The higher the price elasticity, the greater the sensitivity that households have to the price differential required for private schooling compared with free public education.

In Table 1, we have highlighted the results corresponding to a price elasticity of $\mathbf{- 0 . 6 7}$. We do this for two reasons. First, the equilibrium public school attendance with this elasticity is $88 \%$, and this is the observed U.S. percentage. Second, this value is within the range found in econometric studies. ${ }^{1+}$

A much-debated policy issue with regard to education is the effect of introducing vouchers. How will a subsidy for private education affect public

[^11]school attendance and public school expenditures? We consider a voucher of exogenously specified magnitude, $s$, funded from the same tax base as expenditure on public schools and available to all households that choose private schooling. We investigate how the equilibrium public expenditure per capita is affected by this voucher. One motivation for this approach is to suppose that a voucher is mandated by a higher level government. When voting over local expenditures, voters take this federal policy with respect to vouchers as given. In equilibrium, the tax rate must be high enough to fund local expenditures on public schoois and the cost of the vouchers. The utility of households that choose private school is now $v(y(1-t)+s)$ and the government budget constraint satisfies $t Y=N E+(1-N) s$. The model is the same otherwise, and the equilibrium results carry over. ${ }^{15}$

The middle panel of Table 1 reports results with a voucher of $\$ 1,000$ per student, and the bottom panel a voucher of $\$ 2,000$ per student. The most striking feature of these computations is that expenditure per student in public schooling is remarkably insensitive to the introduction of vouchers. We might have thought that a voucher as large as $\$ 2,000$ would draw students from public schooling into puate schooling, and that this in turn would lead to a substantial reduction an the amount voted for public education. The first part of this intuition is correct, but the second part proves to be incorrect. For example, for our benchmark price elasticity of -0.67 , per student expenditure on public schooling rises slightly from $\$ 4,222$ to $\$ 4,332$ as the voucher is increased from 0 to $\$ 2,000$. This increase occurs despite the fall in public school attendance and the associated fall in income of the voter whose indifference curve is tangent to the GBC at the equilibrium allocation.

Increasing the voucher affects a household's indifference mapping only by shifting out the $\hat{E}(\cdot)$ locus. While the voucher increases the likelihood of private school choice, preferences over ( $E, t$ ) are unchanged as long as public schooling remains the household's optimal choice. Changes in the voucher then impact the equilibrium only through the effect on the GBC and on the changing identity of $y_{l}$ in the case of SRI, and only through the former effect in the case of SDI. Raising the voucher from 0 to $\$ 2,000$ in the benchmark case of Table 1 does lower the income of the pivotal voter $y_{l}$, implying a decreased preference for public expenditure. Offsetting this, however, is an increased tax elasticity of the GBC. This elasticity rises because the voucher attracts some students to private schooling, reducing the incremental revenue required for a given per student increase in public schooling expenditure. Loosely, a voucher tends to flatten the GBC in ( $E, t$ ) space. This is illustrated in the last column of Table 1. In our benchmark

[^12]case, the elasticity of per student public schooling expenditure with respect to the tax increases from 0.74 to 0.83 as the voucher increases from 0 to \$2,000.

While expenditure per public school student is not very sensitive to the voucher regardless of the price elasticity, the same is not true of public school enrollment. When the price elasticity is low in absolute value ( -0.5 ), enrollment drops from $75 \%$ to $66 \%$ when the voucher is increased from 0 to $\$ 2,000$. However, if the price elasticity is as high as -1.5 , the effect of vouchers on enrollment is quite small. Enrollment drops from $\mathbf{9 9 \%}$ to $\mathbf{9 7 \%}$ when the voucher is increased form 0 te $\$ 2,000$.

## 4. Conclusion

Our goal in this paper is to understand the determination of public choice and patronage for a service when a private alternative is present. Education is an important example. We find it plausible that, for educaiion, highincome households are more willing to substitute public education for other goods than low-income households (i.e. SRI applies). We show for this case that the pivotal voter has a below-median income (Proposition 2) and that an equilibrium is characterized by a balancing of a middle-income coalition preferring higher public expenditure against a coalition of high- and lowincome households preferring lower expenditure (Proposition 3). To the best of our knowledge, no previous majority-voting model has this ends-against-the-middle property as the generic feature of an equilibrium.

We also characterize the voting equilibrium (Proposition 1) when lowincome households are more willing than high-income households to increase taxes to pay for the publicly provided good. This is a plausible characterization of preferences for services such as public bus transportation when private transportation is an alternative. In this case we show that the median-income voter is decisive.

We develop a computational model to investigate a voucher policy. Our results suggest that public school expenditure per student is relatively insensitive to the introduction of vouchers, but that public school attendance is relatively sensitive to their introduction.

The model we study in this paper is relatively spartan. It is a useful structure to illuminate the issues involved in characterizing a voting equilibrium. At the same time, however, the model abstracts from potentially important features of competition between public and private providers (e.g. peer-group effects in education). We believe, however, that it will prove feasible to generalize the structure to incorporate such features. We see this as an avenue for research on educational competition and education policy.

## Acknowledgements

This research was supported by the National Science Foundation. We thank Larry Kenny and two anonymous referees for helpful comments. The first author thanks the Keliogg Graduate School of Management at Northwestern University where parts of this research were conducted.

## Appendix

Here we show that a median income $\left(y_{m}\right)$ below the mean income ( $E[y]$ ) is sufficient for Assumption 5. Recall that we have normalized the number of consumers in the population to 1 , implying $E[y]=Y$, the aggregate income. We find the hypothetical government production frontier, $\overline{G B C}$, where everyone always consumes the public alternative. Along it, $E=t Y$, as depicted in Fig. 12. We show that the median-income household's indifference curve through the origin ( $V_{m}$ ) cuts above the GBC. Then, since the actual GBC is nowhere above GBC (and is the same only where no households consume the private alternative or $t=0$ ), Assumption 5 is implied.

The median-income household's utility at the origin is given by $U\left(y_{m}-\right.$ $\left.x^{*}\left(y_{\mathrm{m}}\right), x^{*}\left(y_{\mathrm{m}}\right)\right)$. Let $t_{\mathrm{m}}$ denote the tax along the GBC that yields per household public expenditure equal to $x^{*}\left(y_{\mathrm{m}}\right)$. It satisfies $t_{\mathrm{m}} Y=t_{\mathrm{m}} E[y]=$ $x^{*}\left(y_{m}\right)$. Then, $V_{m}$ cuts above GBC, if $U\left(y_{m}\left(1-t_{m}\right), x^{*}\left(y_{m}\right)\right)>U\left(y_{m}-\right.$ $\left.x^{*}\left(y_{m}\right), x^{*}\left(y_{\mathrm{m}}\right)\right)$, or if $t_{\mathrm{m}} y_{\mathrm{m}}<x^{*}\left(y_{\mathrm{m}}\right)$. The latter is implied by $y_{\mathrm{m}}<$ $E[y]$.


Fig. 12

## References

Atkinson, A.B. and J.E. Stiglitz, 1980, Lectures on public economics (McGraw-Hill, New York).
Barzel, Y., 1973. Private schools and public schools and public school finance. Journal of Political Economy 81, 174-186.
Caillaud, B., R. Guesnerie, P. Rey and J. Tirole, 1988. Government intervention in production and incentives theory: A review of recent contributions. The Rand Journal of Economics 19. 1-26.
Cooper, R., 1984, On allocative distortions in problems of self-selection. The Rand Journal of Economics 15, 568-577.
Denzau. A. aind K. Grier, 1984. Determinants of local school spending: Some consistent estimates, Public Choice 44. 375-383.
Ellickson, B., 1971, Jurisdictional fragmentation and residential choice, American Economic Review Papers and Proceedings 61, 334-339.
Epple. D. and R.E. Romano, 1994, Ends against the middle: Determining public service provision when there are private alternatives, Working paper, University of Florida.
Epple, D. and R.E. Romano, 1996, Public provision of private goods, Journal of Political Economy 104, 57-84.
Epple. D., R. Filimon and T. Romer, 1984, Equilibrium among local jurisdictions: Toward an integrated treatment of voting and residential choice. Journal of Public Economics 24, 281-308.
Epple, D. and T. Romer, 1991, Mobility and redistribution. Journal of Political Economy 99. 828-858.
Fernandez, R. and R. Rogerson, 1996, Income distribution, communities and the quality of public education, Quarterly Journal of Economics 111, 135-164.
Fischer, R., 1988, State and local public finance (Scott-Foresman, Glenview, IL).
Gans, J.S. and M. Smart, 1996, Majority voting with single-crossing preferences. Journal of Public Economics 59, 219-237.
Glomm, G. and B. Ravikumar, 1996, Opting out of publicly provided services: A majority voting result, Social Choice and Weifare, in press.
Goodspeed. T.J., 1986, Local income taxation: Equilibrium. social optimality, and welfare loss in a general equilibrium metropolitan model. Ph.D. dissertation, University of Maryland, MD.

Goodspeed, T.J., 1989. A re-examination of the use of ability to pay taxes by local governments, Journal of Public Economics 38, 319-342.
Gouveia. M., 1996. Majority rule and the public provision of a private good. Public Choice. in press.
Ireland, N.J.. 1990. The mix of social and private provision of goods and services. Journal of Public Economics 43. 201-219.
Kenny, L., 1978, The collective allocation of commodities in a democratic society: A generalization, Public Choice 33, 117-120.
Matthews. S. and J. Moore, 1987, Monopoly provision of quality and warranties: An exploration in the theory of multidimensional screening. Econometrica 55. 441-468.
Roberts, K.W.S., 1977, oting over income tax schedules, Journal of Public Economics 8. 329-340.
Rubinfeld, D. and P. Shapiro, 1989. Micro estimation of the demand for schooling: Evidence from Michigan and Massachusetts, Regional Science and Urban Economics 19. 381-398.
Stigler, G.J.. 1970, Director's law of public income redistribution. Journal of Law and Economics 13, 1-10.

Stiglitz, J.E., 1974, The demand for education in public and private school systems, Journal of Public Economics 3. 349-385.
Westhoff, F.. 1977. Existence of equilibria in economies with a local public good. Journal of Economic Theory 14. 84-112.


[^0]:    * Corresponding author. Tel.: (904) 392-4812; fax: (904) 392-7860; e-mail: romanor@dale.cba.ufl.edu.

[^1]:    'Such conditions are used extensively in screening models (excellent references include Cooper. 1984; Mathews and Moore. 1987; and Caillaud et . 1988) and in the analysis of multi-community equilibrium (Ellickson, 1971: Westhoff, 1977; Epple et al.. 1984; Epple and Romer. 1991; Goodspeed. 1989: Fernandez and Rogerson. 1996). Their power in voting models was first established by Roberts (1977).
    ${ }^{2}$ Fernandez and Rogerson (1596) analyze public education and assume that the direction of a single crossing is opposite that of Glomm and Ravikumar (1996). The empirical evidence and related theory is discussed below in Section 2.

[^2]:    ${ }^{3}$ This concavity is proved in Westhoff (1977, p. 87). Our model is in the spirit of Westhoff's. but differs in presuming that public educational services are congested and private alternatives are available.

[^3]:    ${ }^{\text {* }}$ The exploitation of 'single crossing ${ }^{\text {r }}$ restrictions on preferences to analyze voting problems is on the increase. Gans and Smart (1996) provide a very general analysis and interesting applications, including a synthesis of earlier research.

    An adaptation of Kenny's analysis to our model is in Epple and Romano (1994). Alternatively, see Goodspeed (1986).

[^4]:    ${ }^{6}$ The pcesibility that indifference curves coincide over ranges (e.g. along the horizontal segments) or are tangent may create confusion about what we mean by a crossing'. Any 'touching' of indifference curves that violates either SRI or SDI must entail two crossings. Hence, if indifference curve $V$ meets indifference curve $V^{\prime}$, say from below. coincides with it over a range, and then $V$ rises above (falls back belc $w$ ) $V^{\prime}$, then this counts as one (two) crossing(s). A tangency of indifference curves would ccerrespond to two crossings. The simple hueristic to determine the number of crossings in any unclear case is this: shift marginally one indifference curve in a way to create the maximum number of crossings.

[^5]:    ${ }^{7}$ This generalizes the result of Glomm and Ravikumar (1996), who prove a similar result by invoking restriciions on the functional form of preferences and the distribution of income.

[^6]:    ${ }^{8}$ Alternatively, suppose a policy change dictates some minimum level of expenditure financed outside of the jurisdiction (see, for example. 'Can Big Money Fix Urban School Systems? A Test Is Underway?', Wall Street Journal, 7 January 1992. p. 1). This results in a rightward shift in the GBC and, similarly, can lead to large changes in public expenslitures.

[^7]:    "An equilibrium with the median-income household pivotal may result if the highest-income household weakly prefers public consumption at point $A$ (i.e. if Assumption 6 is dropped). Such an equilibrium satisfies a corner version of the necessary conditions in Proposition 3.

[^8]:    ${ }^{10}$ The ends-against-the-middle property of an equilibrium is reminiscent of Director's $I$ aw of redistribution. Public redistribution is from the rich and the poor to the middle class according to the Law (see Stigler, 1970). Our model provides some theoretical support for this empirical phenomenon.

[^9]:    ${ }^{11}$ If $\ln (x) \sim \mathrm{N}\left(\mu, \sigma^{2}\right)$, the mean of $x$ is $E(x)=\exp \left(\mu+\left(\sigma^{*} / 2\right)\right)$, and the median of $x$ is $\mathrm{e}^{\mu}$. Given the mean and median of $x$, these can be solved for $\mu$ and $\sigma^{2}$.
    ${ }^{12}$ This function is homothetic, implying an income elasticity of demand equal to one. The use of such a homothetic function greatly simplifies the computations. While this income elasticity is consistent with macro estimates of the demand for education, it is considerably larger than results obtained from micro studies (Rubinfeld and Shapiro, 1989). Consistent with the discussion in Section 2, $\rho>(<) 0$ implies a price inelastic (elastic) demand.

[^10]:    ${ }^{13}$ All computations were done using the algorithm for solving non-linear simultaneous equations in Gauss. These equations simultaneously specify the GBC, the slope of the GBC, the slope of voter $y_{t}$ 's indifference curve, equality of the latter two slopes, the indifference of voter $y_{\mathrm{h}}$ over public and private consumption, and one-half the population between $y_{\text {; }}$, and $y_{\mathrm{n}}$ When a point satisfying the necessary conditions is found, a second program is then used to calculate the vote favoring that point against each of a dense grid of points along the budget constraint. More details are available from the authors upon request.

[^11]:    ${ }^{14}$ For example. Rubinfeld and Shapiro (1989) report price elasticity estimates ranging from -0.43 to -0.719 using various specifications applied to data from Massachusetts and Michigan. Our benchmark price elasticity of -0.67 lies near the upper end of this range.

[^12]:    ${ }^{15}$ The GBC has a $t$-axis intercept, $s / Y$, which increases the chance of a conner equilibrium with $E=0$, but this never occurs in our simulation results.

