Symbiosis vs. crowding-out: the interaction of formal and informal credit markets in developing countries

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Abstract

It is a common observation in many developing countries that enterprises are active borrowers in both formal and informal credit markets. We propose a model in which the formal sector’s superior ability in deposit mobilization is traded off against the informational advantage that lenders in the informal sector enjoy. The formal sector can screen borrowers by providing only partial financing for projects, thereby forcing borrowers to resort to the informal sector for the remainder of the loan. We use the model to predict how the market structure responds to changes in the environment, and we consider the policy implications of various forms of government intervention. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Despite the long-standing recognition that informal lenders exist on a widespread basis in most low income countries, there is still much debate over their role in the development process.¹ This paper shows how the informal sector can help to

¹ For a discussion of this debate, see Ghate et al. (1992), Besley (1995), and the studies collected in Adams and Fitchett (1992) and Von Pischke et al. (1983).
solve certain problems faced by formal sector lenders. It is a common observation in many developing country financial markets that enterprises are active borrowers in both the formal and informal sectors. \(^2\) We suggest an explanation for this activity in terms of the informational differences between those sectors. We consider an environment in which informal lenders have better information about borrowers than formal lenders, but the opportunity cost of funds is lower for the formal sector. In such an environment, the formal sector can screen borrowers by providing only partial financing for projects, thereby forcing borrowers to resort to the informal sector for the remainder of the loan. Thus, the formal sector’s superior ability in deposit mobilization due to economies of scale and scope, and the security of deposit insurance is balanced against the informational advantage that the informal sector enjoys. We characterize this trade-off precisely.

This paper also contributes to the literature on price-discrimination by a monopolist with imperfect information (Maskin and Riley, 1984). The screening problem of the formal sector (which, for reasons discussed later, we model as a monopolist bank) can be likened to that which an imperfectly informed price-discriminating monopolist faces, in deciding the menu of price-quantity combinations to offer. The ability of our monopolist bank to successfully ‘price-discriminate’, in the sense of sorting borrowers by type, can actually be welfare-enhancing. The intuition is straightforward: by imposing a co-financing requirement, the bank may be able to screen out bad borrowers whose presence might otherwise have led to a breakdown of the loan market.

Two sorts of explanations have previously been offered for the simultaneous activity of borrowers in both the formal and the informal sector. The first is to treat it as a disequilibrium phenomenon, and to suggest that it occurs as a consequence of exogenously imposed controls on the formal sector (Bell et al., 1997; Bell, 1990; Kochar, 1997). While this approach may be appropriate in certain contexts, we show how this simultaneous activity might occur as an equilibrium phenomenon, even in markets where these distortions are missing. The second approach is to argue that for the reasons suggested in the literature on credit rationing, banks ration borrowers, and the informal sector serves those borrowers who are rationed out by banks. Credit rationing models, however, do not consider the existence of the informal sector. \(^3\) If banks are aware that rejected borrowers will seek recourse to informal lenders, then why do they not incorporate this knowledge in their lending decision? This paper attempts to address this issue by explicitly incorporating the existence of the informal credit sector in the bank’s decision rule.

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\(^2\) See, for example, the surveys by Timberg and Aiyar (1984) and Das-Gupta et al. (1989) for India, Ghate et al. (1992) for several southeast Asian countries, the account by Cole and Park (1983) of the Korean experience, and the study of Japan, Korea and Taiwan by Patrick and Park (1994).

\(^3\) See, for example, Jaffee and Russell (1976), Stiglitz and Weiss (1981, 1986), Bester (1985), and Milde and Riley (1988).
A recent literature has analyzed vertically segmented credit markets, where middlemen borrow in the formal sector and onlend in the informal sector. In contrast, we are interested here in analyzing markets in which a given borrower is funded by both sectors, as occurs in the small and medium scale enterprises sector of many developing countries (described in Section 2). Thus, in our model, the formal and informal sectors are in direct competition. We derive conditions under which the formal sector dominates and, alternatively, conditions under which a firm obtains partial financing by both sectors.

The paper is organized as follows. Section 2 briefly describes the institutional structure of the market that we model. Section 3 lays out the model, and derives our basic result: that there exist situations in which there will be firms partially financed by both sectors. This basic framework is then used to characterize the conditions under which the formal sector will use partial financing. Section 4 applies the model to the analysis of the effects of government regulation on informal sector activity and on interest rate ceilings. Section 5 provides conclusions. All proofs are relegated to Appendix A.

2. The structure of informal credit markets

The specific context that we shall model in detail is motivated by the market for credit for small and medium scale enterprises in India. However, the ideas apply more generally: a number of studies have commented on the remarkable persistence of informal institutions, even in the urban ‘modern’ manufacturing and services sector. Thus, for example, Cole and Park (1983), in their study of the financial development of Korea, note the dependence of even quite large enterprises on the informal financial sector to meet needs of working capital and short-term loans. Biggs (1991) reports that much of the financing for small and medium enterprises in Taiwan is raised in the informal sector. We choose to focus
on the Indian context—in part, this is motivated by the larger body of evidence available in studies of urban informal credit markets in India. The formal sector consists of (usually government-owned) commercial banks, and specialized lending agencies catering to the industrial sector. Urban informal credit is provided by a variety of institutions. Prominent among these institutions are specialized ‘indigenous bankers’, who usually (though not always) accept deposits, use their own funds, or borrow from friends or relatives, to finance their lending (for a description, see Timberg and Aiyar, 1984). Some idea of the relative importance of the two sectors can be gained from the available empirical evidence, which is summarized below.

In a survey of small industry, traders and transport operators done in 1977–1978, reported in RBI (1979, 1981), and summarized in Table 1, the ratio of formal to informal credit used by small scale industry, for instance, was about 65%. The importance of the two sectors, as reflected in this ratio, varied across markets, ranging from 29% in retail trade to about 188% in the case of transport operators. The data reported is at the aggregate rather than the firm level, but it is indicative of the fact that both sectors are active lenders in this market. Das-Gupta et al. (1989) conducted several surveys of specific industries. For example, they found

<table>
<thead>
<tr>
<th>Nature of activity</th>
<th>Percentage of units</th>
<th>Ratio of formal to informal credit (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small scale industry</td>
<td>100.0</td>
<td>65.28</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>85.59</td>
<td>75.56</td>
</tr>
<tr>
<td>Job work</td>
<td>10.24</td>
<td>64.61</td>
</tr>
<tr>
<td>Other</td>
<td>4.16</td>
<td>99.49</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>100.00</td>
<td>52.12</td>
</tr>
<tr>
<td>Food and beverages</td>
<td>29.40</td>
<td>44.94</td>
</tr>
<tr>
<td>Textile trade</td>
<td>23.76</td>
<td>36.83</td>
</tr>
<tr>
<td>Other</td>
<td>46.84</td>
<td>71.03</td>
</tr>
<tr>
<td>Retail trade</td>
<td>100.00</td>
<td>28.91</td>
</tr>
<tr>
<td>Food and beverages</td>
<td>42.83</td>
<td>23.31</td>
</tr>
<tr>
<td>Textiles and ready-made garments</td>
<td>12.78</td>
<td>28.84</td>
</tr>
<tr>
<td>Other</td>
<td>44.39</td>
<td>30.67</td>
</tr>
<tr>
<td>Transport operators*</td>
<td>92.84</td>
<td>188.57</td>
</tr>
<tr>
<td>Motor passenger transport</td>
<td>27.82</td>
<td>148.75</td>
</tr>
<tr>
<td>Motor freight transport</td>
<td>24.98</td>
<td>210.68</td>
</tr>
<tr>
<td>Animal/animal vehicle/hand</td>
<td>40.04</td>
<td>205.67</td>
</tr>
</tbody>
</table>

*Omitted group of transport operators: water transport.
Source: Das-Gupta et al. (1989) (p. 121).
that of 19 garment exporters around Delhi who were being financed by banks, 13 were also borrowing from the informal sector. Similarly, of 17 powerloom units and master weavers in the Surat area, all were active in the informal sector, while 11 had access to bank finance. (Also, they report that interest costs were lower, on average, for firms that were active in both sectors.) Finally, they report that a survey of 35 road construction firms showed that 27% of their sample received formal credit, 24% received informal credit, and an additional 21% borrowed from both sectors. Timberg and Aiyar (1984) estimate that informal finance accounts for 10–30% of the capital requirements of small-scale producers. While the small-scale sector has been reasonably well-served by bank credit directed towards it, \(^7\) nevertheless, most units ‘find it necessary to go into the informal market, at least during their busy season’ (Timberg and Aiyar, 1984). They also report that interest rates in the informal sector are about 2–4% higher than those in the formal sector, and in fact, in some markets, the informal sector rate is quoted as a premium over the formal sector rate. They also cite the results of a survey by the Banking Commission of 1981 which reported that indigenous bankers provided one-twelfth to one-half of all credit to different categories of industrial units.

As each of these studies points out, and the survey by Srivastava (1992) of urban commercial lenders in Delhi confirms, the problem of asymmetric information about borrower quality is endemic in these markets. The respondents in these surveys stress the importance of informal lenders’ personal knowledge about their clients, and the importance of clients’ general reputability. As these and earlier studies (e.g., Rosen, 1962) argue, the lack of information about borrowers, and the differences in the degree of information possessed by lenders, are clearly important determinants of lenders’ behavior. In the formal model below, we focus on these informational differences in order to draw out the implications for the likely structure of credit arrangements in these markets.

The evidence presented above supports the considerable anecdotal evidence that the market for small enterprise credit is one characterized by the active participation of both the formal and informal credit sectors. Furthermore, individual firms are often active in both markets.

3. The model

We consider a three-sided model, consisting of borrowers (entrepreneurs), the informal credit sector (moneylenders), and the formal credit sector (the bank). Since the distinction between ‘informal’ and ‘formal’ is a notoriously hard one to draw, we skirt the issue by focusing on (possibly government-owned) commercial

\(^7\) See Little et al. (1987) for a description of credit programs directed at the small-scale enterprise sector.
banks at one extreme, and moneylenders or financiers at the other. We also assume that laws pertaining to incorporation, bankruptcy, and seizure of collateral are generally enforced, and that the transactions costs of loan processing are not significant, relative to the size of the loan. The non-enforceability of contracts is frequently cited as an explanation for missing or incomplete markets—by abstracting from these difficulties, we hope to focus more closely on the asymmetries in information and size in these markets.

We abstract from all considerations of risk-aversion by assuming that all individuals and firms are risk-neutral. In practice, the loan agreement between the borrower and the informal lender may have an element of insurance, in that the moneylender may allow state-contingent repayment, and not bankrupt the borrower when the project outcome is low. This is especially likely when social ‘connectivity’ between the moneylender and the borrower is high; loans from ‘friends and relatives’ are an example. These agreements are akin to equity participation in the project by the ‘insurer’, so by ignoring considerations of risk-trading, we can focus on debt rather than equity contracts.

Borrowers are heterogeneous, and this heterogeneity can be modeled in various ways. For simplicity, we let borrowers be of two types $i = a, b$, indexed by the projects that they are endowed with. Each borrower is endowed with a project which requires a fixed investment of $K$. Projects are denoted by $(X_i, p_i)$, where $X_i$ is the return to a successful project, and $p_i$ is the probability of success. Failed projects earn 0. We also assume that project $a$ is safer, in that it has a higher probability of success, $p_a > p_b$, and it has a higher expected return, $p_a X_a > p_b X_b$. We assume that a proportion $\gamma$ of the borrowers have the ‘good’ project, $a$. All borrowers are assumed to have no capital, and can offer no collateral. They finance their projects by borrowing from the bank, and/or the informal credit market. For analytical simplicity, the bank is modeled as a monopoly. This assumption does not alter the basic insights to be garnered from this model, and allows us to set up the problem as a simple profit-maximization exercise. In practice, this is not an unreasonable assumption. Government intervention in credit markets has often taken the form of setting up specialized institutions catering to a specific sector, such as Small Industries Promotion Corporations, or by designating a Lead Bank, as in the schemes in India and the Philippines, through which

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8 Adams (1992) and other articles in the same volume give some indication of the wide variety of institutions that comprise the informal financial sector.

9 Udry (1990) provides evidence for the use of credit as insurance in a rural Nigerian context.

10 An interesting area for future research would be to consider the implications of inequality in the initial distribution of wealth, and the evolution of that distribution as borrowers are able to self-finance a portion of their investment (Banerjee and Newman, 1993).

11 It also allows us to ignore problems of potential non-existence of equilibria in screening models (Rothschild and Stiglitz, 1976; Wilson, 1977; Riley, 1979).

12 See Webster (1991) for a list of several countries with specialized institutions catering to the small and medium enterprises sector.
credit is channelled. The bank’s cost of funds, or the rate at which it raises deposits, is \( c \). However, the bank is unable to identify the ‘type’ of the borrower to whom it lends funds, other than by offering self-selecting contracts, \( (q_i,r_i) \) and \( (q_x,r_x) \), where \( r_i \) is the gross rate of interest charged on a loan of size \( K - q_i \).

If borrowers are able to raise \( q_i \) in the informal sector (in bankers’ parlance, ‘meet the margin requirement’), then the project goes through. Otherwise, the bank does not disburse \( K - q_i \). Implicit in this formulation is the assumption that the bank can observe the size of the borrower’s project. This analytical simplification allows us to concentrate on informational differences having to do with the type of the borrower, rather than the characteristics of the project.\(^{13}\) It is also an accurate representation of the markets in question: as we argued above, the bank’s informational disadvantage arises not so much from its lack of knowledge about project characteristics (such as size) per se, but rather about the ability of a particular entrepreneur to implement a proposed project. And it is precisely this knowledge about borrowers that is the main source of the informal lenders’ competitive advantage vis-a-vis the bank. Further, given that the bank can observe each borrower’s project size, we can simply model all projects as being ‘lumpy’, with a required investment of \( K \).\(^{14}\)

Finally, the bank cannot write contracts that specify repayment in the good state as contingent on the realized outcome. Otherwise, the bank could trivially screen borrowers by specifying repayment as a function of \( X_r \). In other words, the bank is restricted to offering debt contracts. Repayment, except in default states, is fixed in advance. The informal credit market is perfectly informed as to the types of all borrowers—for simplicity, we assume that it is a perfectly competitive market.\(^{15}\)

The cost of funds to informal lenders is denoted by \( m \), where \( m \geq c \), i.e., the informal lender faces a higher cost of funds than the bank does. This assumption captures the notion that the formal sector can raise funds more cheaply, due to the scale economies associated with its size, and because of the security it offers its depositors. Finally, we assume that the ‘good’ project is socially worthwhile, i.e., that \( p_x X_a \geq Kc \).

\(^{13}\) For an interesting example of how loan size might be used as a screening instrument in an environment where borrowers differ in their willingness to accept larger loans, see Milde and Riley (1988).

\(^{14}\) This simplifies the model in two ways, without significant loss of generality. First, since the size of each borrower’s project contains no information about the type of the borrower, we can assume that all borrowers have the same sized project. Second, since the bank can verify the borrower’s investment in the project, it can specify, as a condition for lending, the project size to be undertaken. Thus, rather than specifying a schedule of returns corresponding to every potential project size, we can simply require a ‘lumpy’ investment of \( K \) for all projects.

\(^{15}\) The precise structure of the market is unimportant for what follows, other than in determining the division of the surplus between the borrower and his moneylender. At the other extreme from the one we have chosen here, if each borrower faces only one moneylender, then all the residual gains from the project are appropriated by the informal lender.
3.1. The bank’s maximization problem

The bank’s problem is to choose contracts \((q_a, r_a)\) and \((q_b, r_b)\) in order to maximize expected profits. Or, using \(R_i\) to denote the amount of repayment in the event that the project is successful (i.e., \(R_i = (K - q_i)r_i\)), the bank’s problem can be written as:

\[
\max_{q_a, r_a, q_b, r_b} \pi = \gamma(p_a R_a - (K - q_a)c) + (1 - \gamma)(p_b R_b - (K - q_b)c)
\]

subject to the appropriate non-negativity and technological constraints,

\[q_a, q_b \geq 0, \quad 0 \leq R_a \leq X_a, \quad 0 \leq R_b \leq X_b,\]

and the voluntary participation and incentive compatibility constraints of the borrowers. The voluntary participation constraints require that each borrower must receive at least as much expected utility (or payoff) from taking his contract, as he would from non-participation. If the borrower chooses not to participate, then either he is able to fund the project entirely via the informal sector, or the project is not implemented.

\[VP:\]

\[a: p_a(X_a - R_a) - mq_a \geq \max\{0, p_a X_a - mK\}\]

\[b: p_b(X_b - R_b) - mq_b \geq \max\{0, p_b X_b - mK\}\]

The incentive compatibility constraints require that each borrower prefer his ‘own’ contract to that offered to the other borrower.

\[IC:\]

\[a: p_a(X_a - R_a) - mq_a \geq p_a(X_a - R_b) - mq_b\]

\[b: p_b(X_b - R_b) - mq_b \geq p_b(X_b - R_a) - mq_a\]

A feasible contract is defined by a set \(((q_a, R_a), (q_b, R_b))\), which satisfies the four constraints above. A pooling solution is defined by: \((q_a, R_a) = (q_b, R_b)\), i.e., all borrowers are offered the same contract.

We use \(u_i\) to denote the utility, and \(S_i\) to denote the expression for the reservation utility of type \(i\), \(\max\{0, p_i X_i - mK\}\), for \(i = a, b\). Fig. 1 shows the indifference curves corresponding to the reservation utility of each type, for the case when the bad project is not socially worthwhile, \(p_b X_b \leq Kc\), while the good project is good enough to be financed even by the informal sector, \(p_a X_a \geq Km\).\(^{16}\)

In that case, type \(a\)’s reservation utility is greater than 0, and is represented by the indifference curve \(u_a = S_a\), which lies to the southwest of \(u_b = 0\). Moves to the southwest, toward a lower repayment \(R\) and lower co-financing requirement \(q\),

\(^{16}\)While the construction of the figures, and the accompanying discussion, are for the case \(X_b > X_a\), i.e., the good projects second-order stochastically dominate the bad projects, the analysis is essentially unchanged for the case \(X_b \leq X_a\).
are utility-increasing for both types of borrowers. In contrast, moves to the northeast increase profits for the bank. 17

The solution procedure for the bank’s maximization problem follows a fairly standard path (for an example, see Besley and Coate, 1992). Start by observing that borrowers’ indifference curves (defined over the \((q,R)\) space of contracts) satisfy the single-crossing property. This can be easily seen in Fig. 1—since borrowers care only about expected income, each borrower type has straight line indifference curves, though the slope of the bad type’s indifference curve is higher. Intuitively, good borrowers are less willing to accept a higher repayment obligation in return for a lower co-financing requirement, since they do not find the co-financing requirement as onerous as the bad borrowers do. In other words, bad borrowers, who find it hard to obtain financing from the informed informal sector, are more willing to pay a higher interest rate to get more financing from banks.

To simplify the bank’s maximization problem, we make three observations, all of which are characteristic of screening models in which agents’ indifference curves satisfy the single-crossing property. For a formal derivation, please see Appendix A.

**Remark 1.** In any solution, \(q_b = 0\).

In Fig. 1, this means that \(b\)’s contract must lie on the vertical axis. If the bank is going to finance the bad types at all, it should do so in such a manner as to minimize the need for outside finance, since the bank can provide that financing more cheaply itself, and the bad types are willing to pay a higher \(R\) to compensate. As is standard in screening models, the screening is to prevent the bad types from mimicking the good. Hence, since the good types don’t want to mimic the bad, there is no screening advantage to be had from imposing a co-financing requirement for the bad types.

**Remark 2.** In any solution, \(b\)’s incentive compatibility constraint binds.

In a pooling solution, this is trivially true. In a separating solution, if \(b\)’s IC constraint were slack, then the bank could increase profits by offering a new contract, targeted at the good borrowers, with higher repayment and lower

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17 Mathematically, an indifference curve for type \(i\) is given by: \(p_i(X_i - R_i) - mq_i = \pi\), so that its slope is: \(-m/p_i\). Note that the bank’s iso-profit line from lending to type \(i\), given by \(p_iR_i - c(K - q_i) = \pi\), has a slope of \(-c/p_i\). The rate at which the bank trades off co-financing vs. higher repayment differs from the rate at which borrowers view that trade-off. The intuition is straightforward: assuming the bank can identify borrowers via self-selection, its opportunity cost of lending to the borrower, \(c/p_i\), is lower than the opportunity cost to the borrower of raising that funding from the informal sector, \(m/p_i\).
co-financing requirements. Unless the bad borrowers’ IC constraint is binding, they too will take up the new contract, since their willingness to pay a higher repayment for a lower co-financing requirement is greater than that of good borrowers. But so long as bad borrowers are content with their ‘old’ contracts (i.e., their IC constraint remains slack), the bank can increase its profits from the good borrowers. Graphically, Fig. 1 illustrates that in a separating solution, \((q^*, R^*)\) lies...
on a line with slope $-(m/p_b)$ (the cost to the informal sector of lending to bad types), through the point $(q_{gb},R_b)$ on the vertical axis. If $(q_{ga}, R_a)$ lay outside the $u_a = 0$ line, then a move to the northwest along $a$’s indifference curve would (till the $u_a = 0$ line is reached) increase bank profits, since $a$’s indifference curves are steeper than the bank’s iso-profit lines from the $a$ types.

**Remark 3.** In any solution, $a$’s participation constraint is binding.

If the good types were enjoying some surplus, the bank could always increase its profit by taking away some of that surplus in such a way as to leave the bad types unaffected. In Fig. 1, this means that the solution contract for type $a$ must lie on the line $u_a = S_a$. If the good types’ contract lies somewhere to the southwest of that line, then a move toward the northwest along the bad type’s indifference curve (so as to keep his utility unchanged) increases profit for the bank, since its iso-profit lines from the good types are less steep than the good type’s reservation indifference curve $u_a = S_a$. Thus, the bank should always force the good types down to their reservation utility.

Using these three conditions, and relegating the intervening steps to Appendix A, we can rewrite the bank’s problem in a reduced form as:

$$\max_{q_a} \pi = \gamma\{p_aX_a - mq_a - S_a - (K - q_a)c\}$$

$$+ (1 - \gamma)\{(p_aX_a - mq_a - S_a)(p_b/p_a) + mq_a - Kc\},$$

subject to

$$0 \leq q_a \leq \frac{p_a p_b (X_b - X_a) + p_b S_a - p_a S_b}{(p_a - p_b)m}$$

The maximand is a linear function of $q_a$, subject to the constraint that $q_a$ lie in a certain interval. Thus, the maximum value must lie at one of the extremes of the permissible interval. This leads immediately to our first proposition.

**Proposition 1.** (1.1) The profit-maximizing set of contracts is either: (i) **Pooling** Both borrowers are offered the same contract—full financing by the bank (i.e., $q_i = 0$, $i = a,b$) and a repayment, if the project is successful, that pushes the good type to his reservation utility (i.e., $R_i = X_a - (S_a/p_a)$, $i = a,b$). (ii) **Separating** The contract for ‘$b$’ offers full financing by the bank, and, in the event the project is successful, a repayment sufficient to drive him down to his reservation utility, i.e., $q_b = 0$, $R_b = X_b - (S_b/p_b)$. The contract for ‘$a$’ offers partial finance and a repayment less than the full return from a successful project (i.e., $q_a > 0$, $R_a < X_a$). Both types are pushed to their reservation utilities, $S_a$, $S_b$. (1.2) It is possible that both pooling and separating contracts yield negative profits, even when there is a set of potentially profitable projects. In that case, the bank makes no loans in this market, and neither does the informal sector.
Proposition 1 implies that the bank need consider only two elements of the set of solution contracts. In one case (Proposition 1.1(i)), the bank pools all borrowers and offers both types of borrowers the same contract. From Remark 3, we know that in any solution, the participation constraint of the good type must be binding, hence the repayment amount must be sufficient to drive type $a$ down to his reservation utility level, $S_a$. From Remark 1, we know that if the bad types are financed at all, they must be fully financed. Hence, in the pooling solution, the bank fully finances every project brought before it, and chooses a level of repayment calculated to push the good type down to his reservation utility. This is the point $(0, R_s)$ in Fig. 1, and represents the ‘crowding out’ of the informal sector by the formal sector.

The more interesting case is the one in which the bank uses the loan size (i.e., the co-financing requirement) as an instrument in screening bad borrowers (Proposition 1.1(ii)). At the $(q_a, R_s)$ contract in Fig. 1, banks only offer partial financing for projects, and bad borrowers are unable to finance $q_a$ from the informal sector. In this (‘symbiotic’) separating solution, the formal and informal sectors co-finance good borrowers.

How should we interpret Proposition (1.2)? If the returns to the good projects are sufficiently low, then there may be no financing at all, even though the projects are socially worthwhile (i.e., $p_X G K c$). Fig. 2 illustrates a case where the bank cannot make a profit from either pooling or separating borrowers. A necessary condition for this is that the project returns be so low that the informal sector would be unwilling to fully finance even the good project, i.e., $p_X G K c mK$. Hence, Fig. 2 is drawn to the specification that the reservation utility of both types is 0, i.e., $S_a = S_b = 0$. The bank’s break-even line on loans to the good types is given by the line $u_a = 0$, and type $a$’s participation constraint is given by the line $u_a = 0$. The

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18 As is characteristic of screening models, the bad type emerges from the pooling solution with some surplus. The pooling solution gives type $b$ an expected utility of $p_b(X_b - X_a + (S_a/p_b))$. It is easily checked that this exceeds $S_b$.
19 If $p_X G K c$ (as is assumed in Fig. 1), then bad borrowers are ‘screened out’—the bank offers them a contract slightly worse than $(q_a, R_s)$, so that bad borrowers are better off not participating. If $p_X G K c$, then the bank ‘screens in’ bad borrowers, by offering them a full-financing contract that gives them slightly more expected payoff than their reservation utility from non-participation, $S_b$. Thus, bad borrowers are either fully financed or not financed at all.
20 Recall that our formal sector is modeled as a monopolist bank. If borrowers’ indifference curves satisfy the single-crossing property, as they do here, then a pooling equilibrium with competitive banks will not exist (Rothschild and Stiglitz, 1976). This strengthens the case for our result that there will be situations in which borrowers are co-financed. In order to have a Nash equilibrium with competitive banks, there must be symbiosis; there cannot be crowding out of the informal sector. I am grateful to an anonymous referee for pointing this out.
21 It is easily checked that $p_X G K c mK$ implies that profits in the separating solution are non-negative. For a proof, see Appendix A.
key feature of Fig. 2 is that the separating contract \((q_R, R)\), which is at the intersection of \(a\)'s participation constraint and \(b\)'s participation constraint, lies below the bank’s break-even line. In other words, the bank makes a loss, even on the good types, even when it successfully separates them from the bad types.

As we shall demonstrate later, if the bank can make positive profits by pooling borrowers, it can always make positive profits by separating them too. In other
words, a sufficient condition for maximum profit to be negative is that the profit-maximizing separating solution yield negative profits. Thus, in the situation depicted in Fig. 2, the bank makes no loans at all in this market. Furthermore, no projects are financed by the informal sector either. As mentioned above, a necessary condition for maximum profits to be zero is that \( p_a X_a \leq mK \), and a necessary condition for the informal sector to finance projects independently is that the expected return to the project, \( p_a X_a \), exceed the costs, \( mK \). Thus, if the formal sector is unwilling to finance any part of the project, then the informal sector will not finance the whole project. In this situation, there is no lending in this market.

3.2. Crowding out vs. co-financing

For the bank, there is a trade-off in the choice of projects to finance. The pooling option has the virtue that it maximizes the surplus from the ‘good’ projects, and since the bank can extract all the surplus on good projects (by Remark 3 above), it maximizes its payoffs from the good types. However, the pooling solution involves fully financing bad projects too, and the losses from those projects may be sufficiently great as to make the separating solution more profitable. In the separating case, the ability to screen out bad borrowers comes at a cost—good projects are only partially financed, and the surplus from them is correspondingly lower, since the share \( q_a \) financed by the informal sector comes at a higher cost of funding, \( m \). The following proposition gives a condition for the resolution of this trade-off.

**Proposition 2.** The profit-maximizing separating solution will yield higher profits than the pooling solution when the proportion of bad borrowers is high, the riskiness of the bad contracts is large relative to that of the good contracts, and the cost difference between the two sectors is small, i.e., when \((1 - \gamma)\) and \((p_a - p_b)\) are high, and \((m - c)\) is low.

The intuition for this is straightforward—the informal sector provides a screening function, since it has better information about borrowers. This information can be extracted by forcing the informal sector to risk enough of its own capital \((mq_i)\) to deter it from financing bad projects. This information extraction is costly in terms of surplus foregone by using the relatively expensive sector to cover part of the investment in good projects. The higher the relative inefficiency of the informal sector in funds procurement (the larger is \(m - c\)), the greater the loss associated with co-financing—hence, the greater the attractiveness of the pooling solution. Analogously, the value of the informal sector’s superior information is greater when the difference in the success probabilities of the two types \((p_a - p_b)\) is high. Thus, using the informal sector to separate borrower types is more attractive when the cost disadvantage is not too great, and when its informational advantage is greater (i.e., \( p_a \gg p_b \)).
When $p_Y X_b > K c$, i.e., even the bad borrowers’ projects are worthwhile, then the bank fully finances all bad borrowers. The trade-off between pooling and separating is a little different now, since all borrowers yield positive expected profits. Thus, separating is a less attractive option in this case than it is when bad projects are not socially worthwhile. The separating solution has one other interesting feature. Since banks fully finance all bad borrowers, and both sectors partially finance good borrowers, the bank’s portfolio has a higher component of risky projects than that of the informal sector. This appears to be consistent with the available evidence (see Timberg and Aiyar, 1984) that informal lenders generally have lower default rates than the formal sector.

3.3. Comparative statics

The profit-maximizing solutions that we have derived so far have depended on the parameters of the problem—the respective probabilities of success, the relative efficiency of the sectors in intermediating funds, and the proportion of each of the two types in the borrower population. We now use these insights to try to predict the structure of credit markets as we vary these parameters. Fig. 3 illustrates, for the case $p_Y X_b < m K$, how the profit-maximizing solution changes as the proportion of good borrowers, $\gamma$, and the bank’s cost of funds, $c$, are varied. The lines $\pi^p = 0$ and $\pi^s = 0$ denote $(c, \gamma)$ combinations for which the profit-maximizing pooling and separating solutions, respectively, are zero. Points to the left of each line yield positive profits. Note that the parameter space in which the bank can break even by using the pooling solution is a subset of the corresponding parameter space for the separating solution. In other words, if the bank can
profitably lend by pooling borrowers, it can always lend profitably by separating
them using the co-financing requirement. The efficiency-enhancing ‘symbiosis’
between the two sectors can be seen most easily in the space between the lines
\( \pi^p = 0 \) and \( \pi^s = 0 \)—co-financing allows the bank to profitably (partially) finance
borrowers when it would have been unable to make non-negative profits as the
lone financier in the pooling region. It is worth noting that this is a feature
common to screening models: in the observed asymmetric information separating
solutions, we see institutions using instruments that would be wasteful in a perfect
information world—work requirements in income maintenance programs in the
case of Besley and Coate (1992), incomplete coverage (e.g., by using ‘deductibles’)in insurance markets (Rothschild and Stiglitz, 1976), and costly informal sector
credit in our model.

When both solutions offer positive profits, the bank’s choice will depend on
which one offers higher profits. The line \( \pi^p = \pi^s \) maps the \((c, \gamma)\) combinations
for which profits in the profit-maximizing pooling and separating solutions are
equal, and divides Fig. 3 into regions according to which solution is the profit-
maximizing one. In the pooling region, banks crowd out informal lenders, while in
the separating region, both types of lenders co-exist and lend jointly. However, if
\( c \) is high enough, then no market exists, even though good projects may be
socially worthwhile. 22 Thus, banks are more likely to opt for partial financing
when the proportion of bad borrowers is high (so that there are benefits from the
informal lenders’ screening), and when the cost of funds of the informal sector is
relatively low (so that the ‘loss’ from the informally financed portion is low). 23

4. Applications and extensions

4.1. The effect of interest rate ceilings

Almost all governments have used interest rate ceilings at one time or another,
either as a check on usury, typically in agrarian settings, or as a means of directing
credit to particular sectors, or as a means of mitigating moral hazard by lenders in
a setting with government provided deposit insurance. There is much debate on the
extent to which interest rates in informal markets fall within the purview of
government regulation. 24 Much of the empirical literature on the efficacy of
monetary policy in the presence of active informal credit markets is premised on

22 Recall the discussion of Proposition 1.2.
23 We have implicitly assumed that the transaction costs associated with loan applications to banks
(e.g., of loan processing) are small relative to the size of the project. If these costs are high enough,
then in a simple extension of the model, the bank may be unable to make a profit net of transaction
costs. Further, if \( m \) is not much greater than \( c \), then the informal sector may still be able to lend
profitably to good borrowers. Thus, in this case, we would observe the crowding out of the formal
sector by the informal sector.
24 For a discussion, see Ghate et al. (1992).
the difficulty of implementing interest rate regulations in informal markets. We follow this literature in assuming that the interest rate ceiling $\rho \geq c$ is applicable only to loans made by the formal sector.

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25 See, for example, Acharya and Madhur (1983) and Sundaram and Pandit (1985).
This constraint can be mapped on our \((q, R)\) diagram as the equation \(R_i \leq (K - q_i)\rho\). In Fig. 4, this is a straight line labeled the IRC (for interest rate ceiling) constraint. One implication is immediately obvious. If \(\rho \leq c/p_s\), then the bank’s break-even line for lending to good borrowers falls everywhere above the interest-rate-ceiling (IRC) constraint, and the bank does not lend at all. Thus, a very restrictive ceiling may drive formal lenders out of the market altogether. Also, if \(p_i X_g \geq mK\), good projects will still be funded (wholly) by the informal sector. If \(\rho \geq c/p_s\), i.e., the interest rate ceiling is high enough for the bank to profitably finance at least the good borrowers, then a new separating solution can be established in Fig. 4 at \((q, R_i)\), the intersection of the IRC constraint and type \(b^*\)’s participation constraint. 26

In the interest-rate-constrained solution, the bank compensates for its inability to screen using the interest rate instrument by using the loan size instrument more intensively. The bank’s profit is less, measured geometrically as the distance \(AC\) in Fig. 4, than it was without the interest rate ceiling. Part of this loss, \(BC\) in Fig. 4, is picked up by type \(a\) borrowers, who are able to extract some surplus from the project. However, the remainder, distance \(AB\), is the deadweight loss resulting from the co-financing of the project from the more costly informal sector. This analysis suggests that government programs to provide cheap credit to certain sectors may in fact lead to a transfer in surplus, though it may come at some cost. The intuition for this is clear. Since the interest rate is an instrument for surplus extraction as well as for screening, any restrictions placed on it will affect not only the distribution of claims to the surplus generated by investment, but also the efficiency with which those investments are financed.

4.2. Government regulation of informal sector activity

For the reasons discussed in Section 4.1, governments have found it difficult to regulate the terms of the contracts that the informal sector offers. However, governments are able to impose some restrictions on activities, by requiring registration or licensing, incorporation, etc. In the Indian context, for example, several classes of informal institutions are regulated to some extent. 27 In short,

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26 When \(\rho \geq c/p_s\), the bank’s iso-profit lines from the good types are flatter than the IRC constraint. Therefore, a move northwest up the IRC constraint means higher profits for the bank. However, in order to maintain the separation of the two classes of borrowers, points on the IRC constraint that lie to the left of \(b^*\)’s participation constraint cannot be offered. Thus, the intersection of the two constraints is the profit-maximizing separating contract.

27 Ghate et al. (1992) and Nayar (1992) mention that ‘finance companies’ fall within the purview of the Companies Act, which requires registration and regulates deposit-taking; chit funds (rotating savings and credit associations) are required to be licensed and their by-laws registered under the Chit Funds Act; and shroffs’ (indigenous bankers) use of indigenous financial instruments (such as the hundi, a type of demand draft) is restricted by the Income Tax law that requires all payments over a certain amount to be made by crossed check or bank draft.
the government can ‘tax’ the informal sector by raising the costs of regulatory avoidance. This is effectively the same as raising $m$ in our model, and we outline its effect on the co-financing solution, in which banks separate borrowers by type. For simplicity, consider the case $S_a = 0$, i.e., where good projects would not be financed by the informal sector alone.

A rise in $m$ reduces the co-financing requirement $q_u$ (see the expression for $q_u$ in the proof of Proposition 1.1(ii) in Appendix A), so that the bank can raise the repayment $R$, while keeping the good borrowers at their reservation utility. Since the bank trades off repayment against co-financing at a lower rate than borrowers (i.e., its iso-profit curves are flatter than the indifference curves of borrowers), its profits increase. The good types are no better or worse off, but there is a social loss from the imposition of the higher costs on the informal sector. On balance, however, the gains to the bank outweigh the costs imposed on the informal sector. Formally, the bank’s gain is $d\pi = (m_0 - c)q_u^0 - (m_1 - c)q_u^1$, where the superscripts 0 and 1 denote the pre- and post-intervention situations. The loss to society from higher intermediation costs in the informal sector is $(m_1 - m_0)q_u$. A comparison of the two indicates that social efficiency of investment actually increases as the government drives up the informal sector’s costs.

The intuition for this is as follows: informal sector financing involves some social welfare loss relative to the information-unconstrained first best situation. An intervention that raises $m$ allows the formal sector to accomplish its screening more effectively, by requiring less co-financing from the more costly informal sector. Thus, by facilitating the efficient selection of good projects, government regulation of informal financial markets may actually improve welfare. Obviously, this conclusion will not hold for markets in which the formal sector is not active—there, a rise in $m$ merely acts as a deadweight loss, without any concomitant improvement in the ability of lenders to choose better projects. This emphasizes the importance of a point made previously—any consideration of government regulation of credit markets must pay the closest attention to the precise structure of the market, and the role that various types of lenders play in that market.

5. Conclusion

This paper has attempted to analyze the interaction of the formal and informal credit sectors, motivated by the observed behavior of firms, banks and other lenders in various countries. It is not uncommon, in developing country credit markets, to observe borrowers who are active in both sectors. In such settings, it is reasonable to suppose that each sector takes account of the other’s actions in planning its own. We propose a simple model that incorporates this, and use it to derive implications about the structure of credit markets as a function of the underlying environment. The findings are broadly consistent with the stylized facts reported in the literature.
At a time when the liberalization of financial markets is being actively pursued or considered in several countries, this research emphasizes the importance of understanding the roles that the formal and informal sectors play, in conjunction with or in exclusion of each other. As we point out, the effects of government regulation on social welfare depend crucially on whether and how this regulation enhances the efficiency of project selection. An important area for further research is to identify, for specific markets, the precise nature of the interaction between the two sectors.

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Appendix A

We begin by proving the three observations in Section 3.1.

**Remark 1.** $q_b = 0$ in any solution.

Proof: Suppose not. Then, let $\{(q_0^b, R_0^b), q_a, R_a\}$ be a solution. Now, choose $q_b^1 < q_b^0$ and $R_b^1 > R_b^0$ such that $u_b(q_b^1, R_b^1) = u_b(q_b^0, R_b^0)$. This implies that

$$p_b X_b - p_b R_b^0 - mq_b^0 = p_b X_b - p_b R_b^1 - mq_b^1,$$

$$\Rightarrow p_b(R_b^1 - R_b^0) = m(q_b^0 - q_b^1) \quad (A.1)$$

Check the constraints, in order to verify that $(q_b^1, R_b^1)$ is a solution. $b$’s participation constraint is still satisfied, since $u_b(q_b^1, R_b^1) = u_b(q_b^0, R_b^0) \geq 0$. $a$’s participation constraint is unaffected by changes in $b$’s contract. $b$’s incentive compatibility is maintained, since his utility remains unchanged, by definition of $(q_b^1, R_b^1)$. If we can show that $u_a(q_a^0, R_a^0) \geq u_a(q_a^1, R_a^1)$, then that will establish that $a$’s IC constraint holds for $(q_b^1, R_b^1)$ too.

$$u_a(q_a^0, R_a^0) \geq u_a(q_a^1, R_a^1) \Leftrightarrow p_a X_a - p_a R_a^0 - mq_a^0 \geq p_a X_a - p_a R_a^1 - mq_a^1 \Leftrightarrow p_a(R_a^1 - R_a^0) \geq m(q_a^0 - q_a^1)$$
Substituting for the right hand side from Eq. (A.1), this becomes

$$p_a(R_b^1 - R_b^0) \geq p_a(R_b^1 - R_b^0).$$

which is true since $p_a \geq p_b$, and $R_b^1 > R_b^0$. Hence, $a$’s IC constraint is satisfied.

It remains to show that replacing $(q_b^0, R_b^0)$ by $(q_b^1, R_b^1)$ yields a higher profit to the bank. The bank’s profit is

$$\pi((q_a, R_a), (q_b, R_b)) = \gamma(p_a R_a - (K - q_a)c)$$

$$+ (1 - \gamma)(p_b R_b - (K - q_b)c)$$

So,

$$\pi((q_a, R_a), (q_b, R_b)) - \pi((q_a, R_a), (q_b^0, R_b^0))$$

$$= (1 - \gamma)(p_b R_b^1 + cq_b^1 - p_b R_b^0 - cq_b^0)$$

$$= (1 - \gamma)(p_b(R_b^1 - R_b^0) - c(q_b^0 - q_b^1))$$

$$= (1 - \gamma)(m - c)(q_b^0 - q_b^1)$$

using Eq. (A.1), $> 0$, since $m > c$, and $q_b^0 > q_b^1$.

Hence, for any proposed solution in which $q_b$ is not equal to zero, the bank can always find a pair of contracts with $q_b = 0$ that yields higher profits and that satisfies the incentive compatibility and participation constraints of the borrowers.

\[\diamond\]

**Remark 2.** $b$’s IC constraint must be binding in any solution.

Proof: Suppose not. Suppose that, at a solution $((q_a, R_a),(0, R_b))$, $b$’s IC constraint is slack, i.e., that $u_b(0, R_b) > u_b(q_a, R_b)$. Since $a$ and $b$’s indifference curves fulfill the single-crossing property, there must exist a contract $(q^*, R^*)$, where $q^* < q_a$ and $R^* > R_a$, such that

(i) $u_b(q_a, R_a) = u_b(q^*, R^*)$

(ii) $u_b(0, R_b) = u_b(q^*, R^*)$

It is easily seen that this new contract, by its definition, satisfies both sets of IC and VP constraints. It remains to check that replacing $(q_a, R_a)$ by $(q^*, R^*)$ yields higher profits to the bank.

$$\pi((q^*, R^*),(0, R_b)) - \pi((q_a, R_a),(0, R_b))$$

$$= p_a(R^* - cK + cq^* - p_a R_a + cK - cq_a$$

$$= p_a(R^* - R_a) - c(q_a - q^*)$$

$$= m(q_a - q^*) - c(q_a - q^*) \quad(\text{since } (i) \Rightarrow p_a(R^* - R_a) = m(q_a - q^*))$$

$$= (m - c)(q_a - q^*)$$

$$> 0, \quad \text{since } m > c \text{ and } q_a > q^*. \diamond$$
Remark 3. a’s VP constraint is binding with equality in any solution.

Proof: Suppose not. Suppose \( ((q_a, R_a), (0, R_b)) \) is a solution contract, and \( u_a(q_a, R_a) > S_a \). Again, using the single-crossing property, there exists a point \((q^*, R^*)\), where \( q^* < q_a \) and \( R^* > R_a \), such that

\[
\begin{align*}
\text{(i) } u_a(q^*, R^*) &= S_a \\
\text{(ii) } u_b(q^*, R^*) &= u_b(q_a, R_a) = u_b(0, R_b), \text{ from Remark 2}
\end{align*}
\]

It is easily seen that this new contract, by its definition, satisfies b’s IC and VP constraints. Property (i) guarantees that a’s VP constraint is satisfied. Thus, it remains to be checked that a’s IC is still satisfied, and that the bank earns higher profits from this new contract.

a’s IC constraint requires that:

\[
u_a(q^*, R^*) \geq u_a(0, R_b)
\]

\[
\iff p_a X_a - p_a R^* - mq^* \geq p_a X_a - p_a R_b
\]

\[
\iff p_a(R_b - R^*) > mq^*
\]

(A.3)

Note that property (ii) implies that \( p_b(R_b - R^*) = mq^* \), so that Eq. (A.3) becomes:

\[
p_a(R_b - R^*) > p_b(R_b - R^*)
\]

which is true, since \( p_a > p_b \) and \( R_b > R^* \).

Finally, we check that the bank makes higher profits by replacing \((q_a, R_a)\) with \((q^*, R^*)\).

\[
d\pi = \pi((q^*, R^*), (q_a, R_b)) - \pi((q_a, R_a), (q_a, R_b))
\]

\[
= p_a R^* - Kc + c q^* - p_a R_a + Kc - c q_a
\]

\[
= p_a(R^* - R_a) - c(q_a - q^*)
\]

(A.4)

Property (i), \( u_a(q^*, R^*) < u_a(q_a, R_a) \), implies that \( p_a(R^* - R_a) > m(q_a - q^*) \).

Substituting this in Eq. (A.4) we get:

\[
d\pi > m(q_a - q^*) - c(q_a - q^*) \Rightarrow d\pi > (m - c)(q_a - q^*),
\]

since \( m > c \) and \( q_a > q^* \).  

A.1. Simplifying the bank’s maximization problem

Next, simplify the bank’s maximization problem in Section 3.1. Using the three observations above, we can rewrite the constraints as:

VP:

\[
\begin{align*}
a: p_a(X_a - R_a) - mq_a &= S_a \quad \text{(VP.a)} \\
b: p_b(X_b - R_b) &\geq S_b \quad \text{(VP.b)}
\end{align*}
\]
IC:

\[ a: p_a(X_a - R_a) - mq_a \geq p_s(X_a - R_s) - mq_b \]  \hspace{1cm} \text{(IC.a)}

\[ b: p_b(X_b - R_b) = p_b(X_b - R_a) - mq_a \]  \hspace{1cm} \text{(IC.b)}

Eq. (IC.b) can be rewritten as: \( mq_a = p_b(R_b - R_a) \).

Use Eqs. (IC.b) and (VP.a) to solve for \( R_a, R_b \) in terms of \( q_a \).

\[ R_a = \frac{p_a X_a - mq_a - S_a}{p_a} \]  \hspace{1cm} \text{(R.a)}

\[ R_b = \frac{p_a X_a - mq_a - S_a}{p_a} + \frac{mq_a}{p_b} \]  \hspace{1cm} \text{(R.b)}

Substitute the above expressions for \( R_a \) and \( R_b \) in the inequality constraints, Eqs. (IC.a) and (VP.b), to get the reduced set of constraints on the bank’s maximization problem.

\[ q_a \geq 0 \]

\[ q_a \leq \left( \frac{p_a p_b X_b - X_a) + p_b S_a - p_a S_b}{m(p_a - p_b)} \right) \]

Use Eqs. (R.a) and (R.b) to substitute for \( R_a, R_b \) in the expression for the bank’s profit function, and write it as a function of one variable, \( q_a \).

A.2. Proof of Proposition 1

Since \( \pi \) is a linear function of \( q_a \), its maximum occurs at one of the extreme values of the permissible interval.

(i) At \( q_a = 0 \), \( R_a = R_b = X_a - (S_a/p_a) \) (from Eqs. (R.a) and (R.b) above). All borrowers are offered the same contract \( X_a - (S_a/p_a) \) and \( \pi(X_a - (S_a/p_a)) = p_a X_a - p_b(X_a - (S_a/p_a)) = S_a \).

(ii) At \( q_a = (p_a p_b X_b - X_a) + p_b S_a - p_a S_b)/(m(p_a - p_b)) \), \( R_a = (p_a X_a - p_b X_b - S_a + S_b)/(p_a - p_b) \). This is the separating contract. \( R_a < R_b, q_a > q_s(= 0) \). \( q_a \leq K \), and is equal if and only if \( p_b X_b \geq mK \).

(iii) If the maximum attainable profit in the pooling and separating cases is negative, then the market breaks down, and the bank does not lend.

If \( p_a X_a \geq mK \), then the bank’s profit in the separating solution, \( \pi^s \), is positive (from Section 3.1).

Proof: It suffices to show that at the separating solution, the bank is making non-negative profits from the good types, since in the separating solution, the bank can always screen out the bad borrowers if it is incurring losses on them. We want to show that profits from good borrowers are positive, i.e., that

\[ p_a R_a - c K + cq_a \geq 0 \]

\[ \iff p_a X_a - S_a - mq_a + cq_a \geq c K \]  \hspace{1cm} \text{(using (VP.a))}

\[ \iff mK - c K \geq (m - c) q_a \]  \hspace{1cm} \text{(since \( S_a = p_a X_a - mK \geq 0 \))}

\[ \iff K \geq q_a \], which is true.
A.3. Proof of Proposition 2

Proposition 2: There are two cases to be considered: (i) \( p_b X_b \leq K_c \); (ii) \( p_b X_b > K_c \).

Formally, a necessary and sufficient condition for the separating solution to yield a higher profit than the pooling solution, when \( p_b X_b \leq K_c \), is:

\[
(1 - \gamma) m(p_a - p_b) \geq \gamma (m - c) \frac{p_a p_b X_b + p_b (S_a - p_a X_a)}{p_a K_c + p_b (S_a - p_a X_a)}
\]

When \( p_b X_b > K_c \), the necessary and sufficient condition that determines whether separating is more profitable than pooling is:

\[
(1 - \gamma) m(p_a - p_b) \geq \gamma (m - c) p_a c.
\]

Proof: Use \( \pi^s \) and \( \pi^p \) to denote the maximum profits under the separating and pooling solutions, respectively. Consider each case in turn.

(i) \( p_b X_b \leq K_c \). So \( S_b = 0 \). In this case, the bank screens out the bad borrowers in the separating solution—thus, we need to consider only the profits it makes from good borrowers in evaluating \( \pi^s \).

\[
\pi^p = \gamma(p_a X_a - S_a) + (1 - \gamma) p_b (X_a - (S_a / p_a)) - Kc
\]

\[
\pi^s = \gamma(p_a R_a - K_c + q_a c) = \gamma(p_a X_a - S_a - mq_a + q_a c - Kc)
\]

So, \( \pi^s \geq \pi^p \) iff \((1 - \gamma)(p_b(X_a - (S_a / p_a)) - Kc) \leq \gamma q_a (c - m)\).

Substitute for \( q_a \), the separating co-financing requirement for the good borrowers, and multiply both sides by \( p_b (p_a - p_b) m \) to get:

\[
(1 - \gamma)(p_a - p_b) m[p_b(p_a X_a - S_a) - p_b K_c]
\]

\[
\leq \gamma p_a (c - m)(p_a p_b (X_b - X_a) + p_b S_a)
\]

Rearranging, this yields the necessary and sufficient condition for \( \pi^s \geq \pi^p \), when \( p_b X_b \leq K_c \):

\[
(1 - \gamma) m(p_a - p_b) \geq \gamma (m - c) \frac{p_a p_b X_b + p_b (S_a - p_a X_a)}{p_a K_c + p_b (S_a - p_a X_a)}
\]

(ii) \( p_b X_b > K_c \). In this case, the bank screens in bad borrowers in the separating solution, i.e., it gives them full financing and charges them a repayment, \( R_b = X_b - (S_b / p_b) \).

\[
\pi^p = \gamma(p_a X_a - S_a) + (1 - \gamma) p_b (X_a - (S_a / p_a)) - Kc
\]

and

\[
\pi^s = \gamma(p_a R_a - K_c + q_a c) + (1 - \gamma)(p_b R_b - K_c)
\]

\[
= \gamma(p_a R_a + q_a c) + (1 - \gamma) p_b R_b - Kc
\]
So, \( \pi^s \geq \pi^p \) iff
\[
\gamma(p_a R_a + q_a c) + (1 - \gamma)(p_a X_b - S_b) \geq \gamma(p_a X_a - S_a) + (1 - \gamma)p_b(X_a - (S_a/p_a))
\]
\[
\Leftrightarrow \gamma(p_a R_a + q_a c) - \gamma(p_a X_a - S_a) \geq (1 - \gamma)\left[p_b(X_a - (S_a/p_a)) - (p_b X_b - S_b)\right]
\]
\[
\Leftrightarrow \gamma c q_a (c - m) \geq (1 - \gamma)(p_b p_a X_a - p_a S_a - p_a p_b X_b + p_b S_b) \quad \text{(using (R.a))}
\]
Substituting for \( q_a \), and multiplying both sides by \((p_a - p_b)m\), this reduces to
\[
(1 - \gamma)m(p_a - p_b) \geq \gamma(m - c) p_a.
\]

References


